Analogous Quantities of Electrodynamic, Electrostatic and Magnetic Circuits ‡

ELECTRODYNAMIC				ELECTROSTATIC				MAGNETIC			
Quantity	*Name of Unit*	*Symbol*	*Formula*	*Quantity*	*Name of Unit*	*Symbol*	*Formula*	*Quantity*	*Name of Unit*	*Symbol*	*Formula*
Electric Current	*Ampere*	I		**Dielectric Flux**	*Coulomb*	Q	Q – I × T	**Magnetic Flux**	*Line of Force or Maxwell*	ϕ	ϕ = E × T
Current Density	*Ampere per Sq Inch*	U	U = I A	**Dielectric Flux Density**	*Coulomb per Square inch*	D	D = Q – A	**Magnetic Flux Density**	*Lines per Sq inch*	B	B = ϕ – A
Resistor				**Elastor**				**Reluctor**			
Resistance	*Ohm*	R		**Elastance**	*Daraf*	S	S = E – Q	**Reluctance**	*Rel*	$\mathcal{R}$	$\mathcal{R}$ = M – ϕ = 1 – $\mathcal{P}$
Resistivity	*Ohm per Inch Cube*	ρ	ρ U L	**Elastivity**	*Daraf per Inch Cube*	σ	σ G – D	**Reluctivity**	*Rel per Inch Cube*	ν	ν – H – B = 1 – μ
Electromotive Force (*or Voltage*)	*Volt*	E	E = I × R	**Electromotive Force** (*or Voltage*)	*Volt*	E	E – C – Q	**Magnetomotive Force**	*Ampere Turn*	M	M = n × I
Voltage Gradient (*or Electric Intensity*)	*Volts per Foot* (*or Volts per Inch*)	G	G = E –	**Voltage Gradient** (*Electric Intensity or Dielectric Stress*)	*Volts per Foot* (*or Volts per Inch*)	G	G = E	**M M F Gradient** (*or Magnetic Intensity*)	*Ampere Turns per Inch*	H	H = M – l
Conductor				**Permittor** (*or Condenser*)				**Permeator**			
Conductance	*Mho*	g	g = 1 – R	**Permittance** (*Capacity or Capacitance*)	*Farad*	C	C – Q – E = 1 – S	**Permeance**	*Perm*	$\mathcal{P}$	$\mathcal{P}$ ϕ – M
Conductivity	*Mho per Inch Cube*	γ	γ = U – G = 1 – ρ	**Permittivity** (*or Dielectric Constant*)	*Farad per Centimeter Cube*	K	K = D – G = 1 – σ	**Permeability**	*Perm per Inch Cube*	μ	μ = B – H
Quantity of Electricity	*Coulomb* (*Ampere Second*)	Q	Q = I × T								

‡ *Based on Tables in Karapetoff's* "THE ELECTRIC CIRCUIT" *and his* "THE MAGNETIC CIRCUIT" *McGraw-Hill Book Company*
ρ (Pronounced Rho) – Resistivity. γ (Pronounced Gamma) Conductivity. σ (Pronounced Sigma) – Elastivity. ϕ (Pronounced Phi) Magnetic Flux.
ν (Pronounced Nu) = Reluctivity. μ Pronounced Mu – Permeability

PRACTICAL
ELECTRICITY

BOOKS BY

TERRELL CROFT

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PRACTICAL ELECTRICITY

BY

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PREFACE

Seven years in the making, is the record for *Practical Electricity*. Some of the material was written during the years from 1910 to 1913 for the *American Electrician's Handbook*, but it developed that the text thus prepared was not suitable for a handbook. Hence, it was stored away temporarily and has been incorporated in the present volume. Since June, 1916, a very considerable portion of the author's time and effort has been devoted to this work.

Practical Electricity was written for certain definite purposes. Primarily, its object is to present the fundamental facts and theories relating to electricity and its present-day applications in a straight-forward, easily-understood way for study by any man, of little mathematical training, who desires to acquire a working knowledge of the subject. If a man understands arithmetic he should be able to get the meat out of this book. Secondarily, the book was designed for university graduates who desire a medium whereby they can, with minimum effort, review, refresh and reconstruct in line with modern theory and practice their concepts of electric and magnetic phenomena. While it has not been found necessary to use the higher mathematics, the statements and explanations are technically accurate.

There is theory, "practical theory," in the book because one cannot retain and cannot effectively apply his practical information unless he understands the vital principles which must, whether he knows it or not, underlie such information. In explaining the theories, familiar analogies have been employed wherever possible. Much expense and exertion have been expended in preparing the illustrations. In so far as feasible, the pictures have been so made as to tell their own stories. Numerical examples have been used frequently, because it is only by the solution of concrete problems that one can obtain a real appreciation of any physical subject.

Now as to the method of treatment: Since, as is explained in the opening chapter, *electricity is the stuff of which everything around us—matter—is made*, considerable space has been devoted to explaining, in a semi-popular way, the relation between matter and electricity. Experience has shown that if the student obtains a proper conception of this relation, the ideas associated with the practical applications of electricity can be grasped much more easily.

Also the modern electron theory has been examined and has been used frequently as a vehicle whereby explanations of things, which ordinarily appear to be difficult of comprehension by the reader, are readily understood. After this question of what electricity is has been covered, each of the succeeding fifty-two sections treats of some important sub-division of the electric or magnetic phenomena. Thus, in the opening sections, the basic ideas, concepts and units are developed. Then magnetism, electro-magnetism, the magnetic circuits and their uses are discussed. Wherever desirable, the electron theory has been utilized in explaining these things. Following, are sections treating of the development of the electromotive forces by the different methods and after this material comes that having to do with the different forms of electromagnetic induction and inductance. Now the reader is in position to understand direct-current generators and motors, which are, therefore, given attention. Finally come the sections relating to alternating currents and alternating-current equipment, both single-phase and polyphase.

In arranging the sections the endeavor has been to have their order such that the development of the different ideas is consecutive and logical.

It is the intention of the author and the publisher to make this work a permanent thing. That is, we propose to revise it as frequently as conditions permit or demand and to enlarge it as becomes necessary. The author hopes that he may have the coöperation of the readers in working out these revisions and enlargements. It will be the greatest assistance if the readers can advise the author of (1) *typographical errors*, (2) *things which they do not understand that are now in the book for which the explanations should be made simpler*, (3) *things which should be added to the book, and* (4) *things which are now in the book and which should be eliminated.* All suggestions which are thus received

will be carefully preserved and will, if possible, be incorporated when the future editions are issued. Where the volume is used as a text, the teacher can, if he so desires, afford most effective and valuable coöperation.

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September, 1917.

ACKNOWLEDGMENTS

If a complete list of acknowledgments were given, it would include practically every book now in print, in English, on electrical theory and practice. *Practical Electricity* is based on electrical information which has been obtained during the author's lifetime in every conceivable way. Thus, if the list were complete, the author would have to mention the name of nearly every person whom he has ever heard talk on electrical subjects or to whom he has talked. Ideas, which have been worked out and enlarged upon in the book, have come from all sorts of sources. Many of these suggestions came from the professors and instructors in the author's student days. Others who have given thoughts for development comprise linemen, wiremen, dynamo tenders, practising electrical engineers, lawyers, students in schools where the author has lectured, and ordinary every-day people whom the author has met in connection with his engineering practice. A number of specific acknowledgments have been made in footnote form throughout the book.

Considerable of the material originally appeared as articles or discussions by the author in the following periodicals: *The Jovian, Electrical Review and Western Electrician, The National Electrical Contractor, Power, Everyday Engineering, Practical Engineer, The Power Plant* and *Electrical Age.*

L. W. Helmreich, who is head of the electrical department of the David Ranken, Jr., School of Mechanical Trades in Saint Louis, read the galley and final proofs. He located a number of errors and made valuable suggestions for improvements—additions or revisions—in the text, which were, in so far as possible, followed.

Among the concerns which coöperated in supplying text data and material for illustrations are the *Allis-Chalmers Manufacturing Company, The General Electric Company, The Westinghouse Electric & Manufacturing Company, The Western Electric Company* and *the Wagner Electric & Manufacturing Company.*

CONTENTS

PRACTICAL ELECTRICITY

SECTION 1

MATTER AND THE ELECTRON THEORY

1. Electricity Is the Stuff of Which Everything Tangible Is Made, at least this is the doctrine which is now generally accepted. That is, everything that we see and feel around us is made up of minute particles or corpuscles of negative electricity which have been named "electrons;" and of similarly-minute nuclei of positive electricity. These are described in detail further on, Art. 4. All experimental evidence is in favor of this "*electron theory.*" No valid arguments have been raised against it. The scientists who have given the situation the most searching study now regard this principle as the correct one. *Hence, to obtain an understanding of the conditions affecting, and the probable reasons for and explanations of, electrical phenomena it will be necessary to get right down to the bottom of things and learn something concerning the structure of "matter"*—which is the technical name for everything that is tangible.

2. "Matter" is anything—except the "æther," defined below (Art. 35) which occupies space. Substances are different kinds of matter. Anything which has weight and dimensions is matter. In general, the existence of matter may be detected by the senses—by seeing, feeling, touching and smelling. Some kinds of matter are invisible—for example pure air, illuminating gas and oxygen—but since they occupy space and have weight they are matter.

EXAMPLES of matter are shown in Fig. 1. Other familiar examples: Air, water, butter, a book, the human body, a building, a waste basket, a copper wire, a newspaper, a locomotive, a fish, a suit of clothes, are all made up of "matter," that is, they are matter.

3. The Construction of Matter, that is, how it is made up or built, is, in a general way, now fairly well understood. It is known that matter is not a continuous, homogeneous structure. On the contrary it is composed of myriads of discrete—distinct

or separate—material particles with non-material spaces between them. In fact it is now generally accepted that matter is made up of very small particles called molecules (Art. 22). There are as many different kinds of molecules in the universe as there are

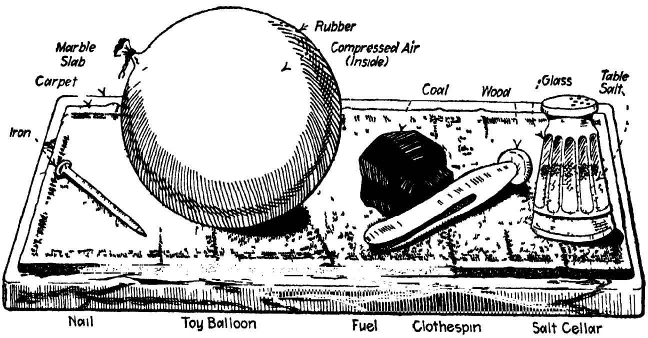


FIG. 1.—Some familiar examples of matter.

different kinds of substances—an almost limitless number. These molecules are made up of smaller particles called atoms (Art. 12). Only about 80 different kinds of atoms have been discovered but experimental evidence* tends to indicate that there are just 92 different kinds of atoms—no more, no less. Atoms are made up of the very-much-smaller electrons (which are, probably, particles of negative electricity) and of charges of positive electricity. There is, it is believed, only one kind of electron—all electrons are just alike. The distinction between these three sorts of matter constituents and the properties of each will be discussed later.

FIG. 2.—Imaginary picture of molecules of water enlarged many thousand times. Each molecule comprises an atom of hydrogen and two atoms of oxygen.

EXAMPLE.—A drop of water comprises an almost inconceivably great number of molecules of water (Fig 2). The attractive force which is called cohesion binds these molecules together. Every molecule of water comprises 2 atoms of hydrogen gas and 1 atom of oxygen gas—which are held together by some electrical attractive action to form the molecule. Then each of the atoms is composed of electrons and positive electricity.

* Robert S Millikan, RADIATION AND ATOMIC STRUCTURE, *Science*, Apr. 6, 1917, p 322.

4. An Electron is a minute, but very active, particle or corpuscle of negative electricity—so diminutive that it is very difficult to appreciate how infinitesimally small one really is. It is, possibly, the smallest object known to science. Formerly it was thought that the atom was the smallest indivisible grain of matter, but now the electron holds this distinction. However, it is hardly accurate to state that an electron is composed of matter, because, as suggested above and as will be shown, matter is, probably, made up of electrons.

4A. The Distinguishing Property of Electrons is that They Tend to Repel One Another with Relatively Enormous Forces and *will* thus repel each other unless restrained by some counter force. Electrons always separate as far as possible away from one another unless there is some restraining force preventing such separation. The two important things to remember about

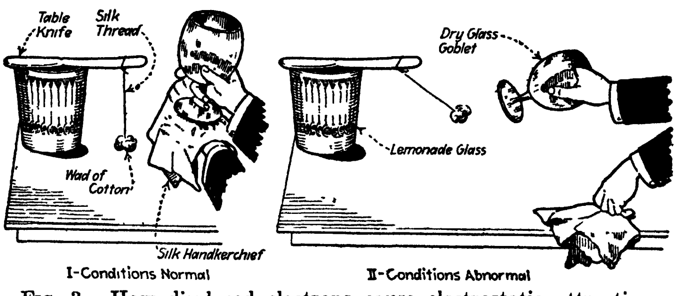


FIG. 3.—How displaced electrons cause electrostatic attraction.

electrons are, then: (1) *They are almost inconceivably small.* (2) *They tend to exert powerful repelling forces on one another.*

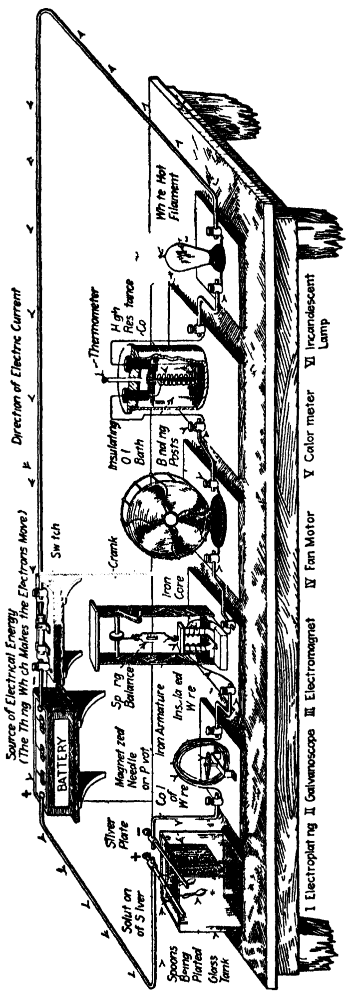
NOTE.—Practically all electrical phenomena can be satisfactorily explained on the basis of the electron theory. It can be shown that it is probable that an excess of electrons at rest on the goblet of Fig. 3,*II*, causes the goblet, after being rubbed with the silk handkerchief "to attract" the wad of cotton. Try this experiment yourself. Furthermore it is believed that a flow or continuous movement of electrons within a conductor constitutes an electric current (Art. 36C) whereby all of the remarkable phenomena depicted in Fig. 4 may be produced.

NOTE.—The electrons "locked up" in normal atoms cannot exert these great repelling forces because, in a normal atom, the repelling effect due to the negative electricity of the electrons is exactly neutralized by the positive electricity of the nucleus of the atom.

5. All Electrons Are Alike, so it is believed; that is, all are of the same size, have the same mass (weight) and embody the

same quantity of electricity. They have been derived from many different kinds of matter (from many different substances), but in every case their properties have been found to be identical.

6. The Methods of Deriving or Isolating Electrons cannot be described in detail here. Normally, in matter of most kinds, some of the electrons are held, bound in the atoms which they compose, by powerful restraining forces due to the attractive effect of the "positive" nucleus of the atom, which will be considered later. Also, there are, so it is believed, electrons in some atoms, which can be separated from their atoms with comparative ease. However, there are methods of neutralizing or overcoming the restraining forces. This having been done, the electrons will shoot away from their atoms like "bullets out of a machine gun," except that the electrons travel at far greater velocities. Such streams—torrents—of electrons are called *cathode rays*. The behavior and properties of electrons may be determined by experimentally studying these streams.



I Electroplating II Galvanoscope III Electromagnet IV Fan Motor V Calorimeter VI Incandescent Lamp

FIG 4 —Some of the more important effects produced by moving electrons (or an electric current)

EXAMPLES —Electrons are thus projected from many objects—particularly from some of the metals—when ultra-violet light rays are permitted to impinge on the object or if it is heated to incandescence. If a glass tube, from which the air has been almost exhausted, be arranged as shown in Fig 5, and a high voltage from an induction coil or other source be impressed across *DE*, with the polarities as shown, a stream of electrons (cathode rays) will be projected from the cathode *C* as indicated in the diagram. Electrons are shot off spontaneously by certain kinds of matter, which are termed radioactive substances.

7. The Size or "Diameter" of an Electron is such a small quantity that it is difficult to designate it in a way which will be understood. The diameter of an electron is about (2×10^{-13})* a five million, millionth of a centimeter $\left(\frac{1}{5{,}000{,}000{,}000{,}000}\text{ cm.}\right)$ or about a thirteen million, millionth of an inch $\left(\frac{1}{12{,}700{,}000{,}000{,}000}\right.$ in.). An electron is about 100,000 times smaller in volume than the average atom.

EXAMPLE.—As outlined below, a molecule is such a small thing that one can not be detected even with the most sensitive microscope. However, if a drop of water were magnified to 100,000 times the size of the earth its molecules would be about 100 times the size of the earth, its atoms would be about the size of the earth and its electrons would be about the size of baseballs.

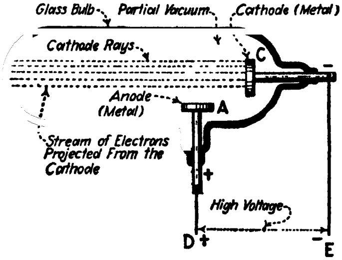


FIG. 5.—Cathode rays in a partial vacuum.

8. The Mass or Weight of an Electron is also so minute that it is difficult to comprehend. The mass of an electron is, so experimental evidence indicates, about $[0.61 \times 10^{-27}$ grams* $= 0.61 \times (0.035 \times 10^{-27})$ oz.$]$ = a forty-six, billion, billion, billionth of an ounce =

$$\left(\frac{1}{46{,}100{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000}\text{ oz.}\right).$$

An electron weighs only about one seventeen hundredth $\left(\frac{1}{1{,}700}\right)$ as much as does an atom of hydrogen.

9. The Forces of Repulsion Which Electrons Tend to Exert Between One Another Are Relatively Enormous.—It has been estimated† that any pair of electrons placed at a distance of approximately 3/8 in. (1 cm.) from one another in a vacuum repel each other with a force of 1.16×10^{-19} dynes. This is approximately equivalent to a million, million, million millionth of a pound. Though this sounds like a small force it is simply prodigious—considering the exceedingly minute size of the electron —as is indicated in the following example.

EXAMPLE.‡—Assume that it would be possible to collect about 7/100 oz.

* Fournier, THE ELECTRON THEORY, p. 23.
† Fournier, THE ELECTRON THEORY, p. 29.
‡ Fournier, THE ELECTRON THEORY, p. 24.

(2 grams) of pure electrons and to form them into two equal spheres, each weighing 1 gram, Fig. 6. Then, if these two spheres of electrons were held about ⅜ in. (1 cm.) apart, as diagrammed in the picture, they would repel each other with a force of three hundred and twenty million, million, million, million tons, that is with a force of 320,000,000,000,000,000,000,000,000 tons (31.4×10^{34} dynes).

10. The Quantity of Electricity of an Electron has been quite accurately determined. The coulomb (Art. 122) is the commonly used unit of quantity or amount of electricity, just as the gallon is the commonly used unit of quantity or amount of liquids. One electron of electricity* is equal to $\left(\frac{22}{10^{20}}\right) = \frac{1}{455,000,000,000,000,000}$ of a coulomb. That is, 1 coulomb of electricity contains approximately five million, million, million electrons.

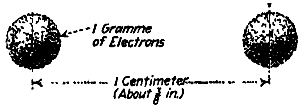


FIG. 6.—To illustrate the repulsive force between electrons.

11. The Amount of Electricity—That Is the Number of Electrons—in the Universe Is Constant and Unvarying.—Electricity, then, can be neither created nor destroyed. Electrons can, as is shown later (Art. 102), be set in motion and caused to move from one location to another, thus producing what are known as electric phenomena. But electricity—electrons—can be neither made nor eradicated.

NOTE.—It is therefore evident that electricity—electrons—can not be "secured," "produced" or "generated," in spite of the fact that the term "generation of electricity" is frequently used. When the statement is made that "electricity is generated by a battery or dynamo" what is really meant is that the battery or dynamo forces some of this electricity—electrons—which is already in existence, to move. A battery or dynamo does not generate electricity in the wires connected to it any more than a pump, which is impelling a stream of water in a pipe, generates the water.

12. An Atom is the smallest particle into which matter can be divided by chemical separation—it is the chemist's unit of matter. Until about the beginning of the present century the atom was the most minute bit of matter known to science. It was then considered the ultimate indivisible unit of matter But now it is, as previously intimated, reasonably well established that atoms are made up of the very much smaller electrons.

13. How Atoms Are Built up from Electrons is—if the generally accepted conception is correct—illustrated in an approximate

* Fleming, THE ELECTRONIC THEORY OF ELECTRICITY.

diagrammatic way in Figs. 7 and 8. The positive nucleus is not shown in these pictures. It should be understood that these illustrations are merely qualitative diagrams because, as detailed below, an electron is many thousand times smaller than the atom of which it forms a part. It would be impossible

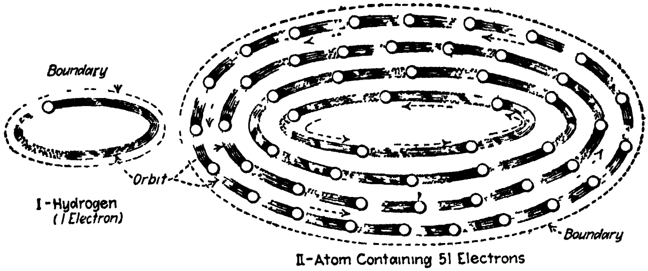


FIG. 7.—Illustrating diagrammatically the probable construction of an atom.

to draw to scale an atom with its constituent electrons. It is believed that an atom is similar in a way to an "ultra-minute" solar system. An atom, very likely, comprises from one to a number of electrons which are interlocked and revolving at in-

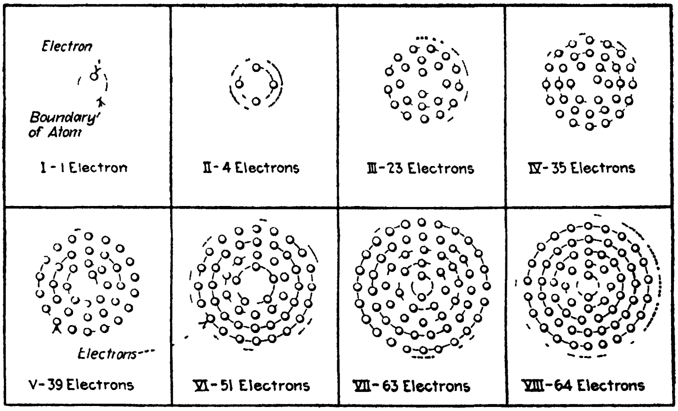


FIG. 8.—Diagram indicating some groupings (which may be possible) of electrons into atoms.

conceivably great speeds in regular, circular orbits around the positive nucleus—in somewhat the same manner as that in which the eight satellites of the planet Saturn (Fig. 9) rotate around in their orbits. Thus it is apparent that energy (Art. 169) is associated with and locked up in every atom.

NOTE.*—An atom consists of a nucleus charged with positive electricity around which revolve in fixt orbits negative electrons, as planets about a central sun. There are exactly ninety-two chemical elements, and the essential difference between them is in the electrical charge of the nucleus (which differs always by the same amount from one element to the next in the series) and in the number of electrons which revolve about the nucleus. Hydrogen, the lightest element, has only one electrical element in its nucleus, and uranium, the heaviest, has ninety-two. The orbits draw nearer to the nucleus, as it is heavier and more powerful, but they always remain at the same distances for the same substance. When an atom radiates light or heat an electron jumps from one orbit to the next, so that radiation is a series of little explosions and not a continuous process. Of this surprisingly complicated atom, Professor Millikan says that it is really much simpler than physicists have for years been expecting. They have long known that

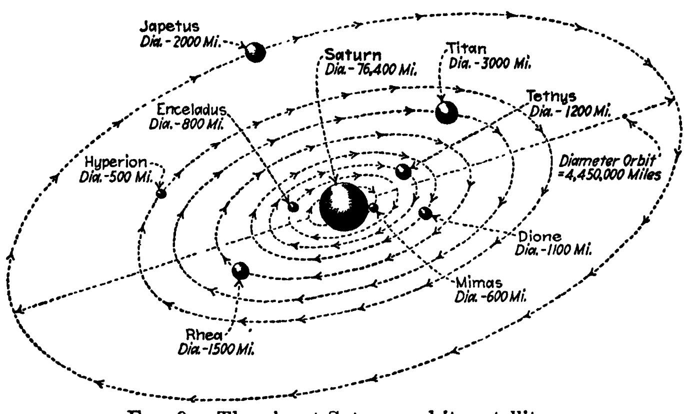


FIG. 9.—The planet Saturn and its satellites.

atoms were not simple, but they had no means of ascertaining in what way and to what extent their structure was complex.

NOTE.—It is essential that the reader understand that electrons are not packed solidly in their atoms. An atom is, in a general way, as much larger than an electron as the auditorium of a good-sized church or theatre is larger than the head of a pin. Olive Lodge† says: "Electrons occupy the otherwise empty region of space which we call the atom in the same sense that a few scattered but armed soldiers can occupy a territory—occupying it by forceful activity, not by bodily bulk."

14. The Reason Why Its Electrons Tend to Stay Within an Atom may be explained thus: Under normal conditions, the electrons are maintained within their atoms—in their orbits—by the attractive force of the positive nucleus. Thus it is assumed that in every atom there must be a positively charged "nucleus."

* Robert S. Millikan, RADIATION AND ATOMIC STRUCTURE, *Science*, Apr. 6, 1917.
† MODERN VIEWS OF MATTER, p. 11.

The positive charge of the nucleus just equals, hence neutralizes the total of the negative charges of the electrons composing the normal atom. Therefore every atom, which has its normal—usual—complement of electrons, exhibits no indication of electrification. The positive and negative electricities in it neutralize one another.

It is believed that the "positive" nucleus can not be removed from an atom. So far as is known no positively charged nucleus has ever been isolated. Electrons have been separated—isolated—from atoms hundreds of times and by many different methods. The nucleus may be thought of as a charge of positive electricity concentrated at a point, at the center of the atom, around which the electrons—restrained in their orbits by it—revolve.

By the application of suitable electric or other forces it is possible to detach electrons from atoms. As above suggested an atom with its normal complement of electrons does not exhibit any evidence of external electrification because any such tendency is "counterbalanced" by a neutralizing agency, often referred to as a "positive nucleus," the exact nature of which is not understood.

15. Electrons Form Themselves into Different Kinds of Atoms.—About eighty (80) different sorts of "elements" or atoms have thus far been discovered but it is believed that there are ninety two (92). It is believed that the different kinds of atoms vary only in the number and in the arrangement of the electrons which comprise them—there is only one kind of electron—and in the charge on the positive nucleus. Thus the difference between an atom of carbon and one of copper is due principally to a difference in the number and in the arrangement of the constituent electrons. Each of the 92 different elements has, probably, atoms of different sizes and weights and each of the 92 has its own, pronounced, and distinguishing properties or characteristics. In Figs. 7 and 8 are diagrammed, enormously enlarged, but not to scale, arrangements that may represent the constitutions of some of the different sorts of atoms.

EXAMPLES of some of the different elements are iron, carbon, copper, hydrogen, chlorine, zinc, tungsten and oxygen. A lump of pure iron is built up wholly of iron atoms; a volume of hydrogen is built up wholly of hydrogen atoms, etc.

16. The Number of Electrons in an Atom Varies with the Kind of Atom, as does also the value of the positive charge of its

nucleus, that is, the heavier atoms or elements comprise many more electrons than do the lighter ones. In fact it is probable that the number of electrons in an element is proportional to the number representing the atomic weight of the element. It follows that some kinds of atoms or elements are heavier than others. But in every ounce (or gram) of every kind of matter there is about the same number of electrons, there being likely (6×10^{23} per gram)* about seventeen million, million, million, million, that is, 16,800,000,000,000,000,000,000,000 electrons in every ounce of every kind of matter.

EXAMPLE.†—Thus, the atomic weight of hydrogen, the lightest known element, is about 1, and it is probable that there is but 1 rotating electron in an atom of hydrogen. Vanadium, which is about 51 times heavier than hydrogen has an atomic weight of about 51 and likely contains 23 electrons. Zinc with an atomic weight of 65.37 probably contains 30 electrons.

NOTE.‡—"The number of negative electrons in an atom is equal to about half the atomic weight of the atom. The atoms are built up one from the other by the successive addition of one and the same electron to the nucleus."

17. The Size of an Atom can be best appreciated by the consideration of analogies. While it is true that an atom is a very, very small fragment of matter it is super-gigantic as compared with the size of its constituent electrons. Lodge states** that: "If an electron is represented by a sphere an inch in diameter, the diameter of an atom on the same scale is a mile and a half. The spaces between the electrons are enormous as compared with their size—as great relatively as are the spaces between the planets in the solar system. An average atom is about a thousand millionth of an inch in diameter, that is it has a diameter of about $\frac{1}{1,000,000,000}$ in."†† Compare this with the diameter of an electron as given above and note that the atom is about a thousand times larger in diameter than an electron. The 92 different atoms of the 92 different elements are probably all of somewhat but of not widely different diameters, but little is now known as to their relative sizes.

18. Atoms in Matter Are Always on the Move—they are oscillating and quivering constantly. And they cannot—even in a good vacuum—travel far without bumping into neighboring

* Crehore, PHIL. MAG.
† Comstock and Troland, THE NATURE OF MATTER AND ELECTRICITY
‡ Robert S. Millikan, RADIATION AND ATOMIC STRUCTURE, *Science*, Apr. 6, 1917.
** Lodge, MODERN VIEWS OF MATTER, pp. 7 and 9.
†† Lodge, MODERN VIEWS OF ELECTRICITY, p. 429.

atoms. It has been estimated* that in the ordinary air every atom collides with another about six thousand million times per second (6,000,000,000 times per sec.). An atom can not move through even the minutest distance without colliding with another. It should be understood that the distances between atoms may be equal to many times the diameter of an atom yet such a distance is, strictly speaking, exceedingly small.

19. It Has Not Been Definitely Established That Atoms Are Spherical in Shape, although they are probably approximately spherical. This form is usually assumed to facilitate explanation and computation.

20. The Phenomena Which We Call Light Is, There Is Reason to Believe, Produced by Electrons as They Move in Their Atoms.—This doctrine is accepted as the correct one by those who are in the best position to judge. As the electrons shift from one orbit to the next at prodigious speeds they produce vibrations or waves in the æther (see following paragraph for *æther*). These æther waves constitute what is known as light. If the æther waves are of low frequency they are then known as radiant heat. The different colors are produced by æther waves of different frequencies, that is by waves which have different rates of vibration. This situation is explained at some length in the author's PRACTICAL ELECTRIC ILLUMINATION.

21. Atoms of Matter May Combine to Form Molecules.—A molecule may, as indicated in the following example, comprise 1, 2 or more atoms of the same kind or it may comprise 2 or more atoms of different kinds. In any event, it is believed that when atoms combine or arrange themselves into an aggregation or group of atoms, which we call a molecule, the unbalanced electric forces of the constituent atoms of the group are so neutralized by the electromagnetic interaction of the member atoms that the group or molecule becomes, usually, a relatively stable arrangement. It should be understood that the member atoms of the molecule do not probably, normally, touch each other. They are separated but are held in their positions in the group by the forces just referred to.

EXAMPLE.—Thus two atoms of hydrogen (H) will combine to form a molecule (Fig. 10) of hydrogen, H_2. Two atoms of oxygen (O) may combine to form a molecule of oxygen, O_2. Other examples are given immediately following the succeeding paragraph.

* Lodge, MODERN VIEWS OF ELECTRICITY, p. 429.

22. A Molecule is the smallest portion of any substance which can not be subdivided further without its properties being destroyed. It is the smallest complete and normal unit of any substance. A molecule is an aggregation of atoms which are bound together into one group by some kind of an electric attraction. The number of atoms in a molecule varies with the substance. In a molecule of common salt there are 2 atoms; in a molecule of alum there are about 100 atoms and in a molecule of albumin (the white of an egg) there are about 1,000 atoms. As previously suggested different kinds and combinations of atoms can be arranged in an endless variety of ways to form different substances —different kinds of matter. There are in the universe as many different kinds of molecules as there are different kinds of substances. All are made up from about 92 different kinds of atoms or "elements"—and from one kind of electron. Although it is usually assumed, in making computations, that molecules are spherical it is known that such is not the case.*

EXAMPLE.—An atom of the gas chlorine and an atom of the semi-plastic metal sodium may unite and produce a molecule of common salt. An atom of oxygen, a gas, and 2 atoms of hydrogen, a gas, unite and the result is a molecule of water (Fig. 2). The diagrams of Fig. 2 should not be taken too literally because it is not probable that molecules are actually built up as there shown. Howevei, this picture will enable one to form a general idea of the situation.

23. The Holding Power of an Atom Is Called Its Valency.—That is, valence is that property of any element by virtue of which one of its atoms can hold in combination, to form a molecule, a certain definite number of other atoms. Atoms of different kinds have the power of holding in combination different numbers of atoms. An element, an atom of which can never hold more than 1 other atom in combination, is called a *univalent* element and its atoms may be called unit atoms. An element, an atom of which can hold in combination 2 unit atoms is a *bivalent* element. One that can hold 3 is *trivalent*. One that can hold 4 is *quadrivalent*—and so on. While most elements have valencies of one of the four classes just recited, there are certain other of the elements which can hold as many as 7 unit atoms in combination.

EXAMPLES.—Hydrogen (H) is univalent as is also chlorine (Cl), thus an atom of hydrogen will combine with an atom of chlorine and form a molecule

* ENC. BRIT., Vol. XVIII, p. 656.

of hydrochloric acid, HCl. Oxygen (O) is bivalent, hence 2 atoms of hydrogen combine with 1 of oxygen to form 1 molecule of water, H_2O (Fig. 2). Sodium (Na) is univalent, hence an atom of sodium and an atom of chlorine (Cl) will combine to form a molecule of common salt, NaCl. Silver is univalent, zinc is bivalent and copper is bivalent, gold is trivalent.

24. A Single Molecule May Be Made up of Many Different Kinds of Atoms.—Remember that there are only 92 different kinds of atoms or elements. The chemical properties of substances are determined by the kind, number and arrangement of the atoms which compose the molecules of the substance.

EXAMPLE.—A molecule of water consists of 2 atoms of hydrogen and 1 atom of oxygen.

25. The Size of a Molecule* can best be appreciated by the consideration of a specific case. If a grain of common table salt (sodium chloride, NaCl), be cut with a very sharp knife into the smallest fragments that one can see, every one of these fragments will still be salt. If, by using a delicate cutting instrument and a microscope, one of these fragments be further divided, it might be cut into minute particles about a hundred thousandth inch $\left(\frac{1}{100,000}\text{ in.}\right)$ in diameter. This is about the smallest particle that would be visible through the microscope—and it would still be salt. Now if one of these minute particles of salt was again cut up into a hundred equal spheres, then each sphere would be a molecule of salt or at least about the size of one. That is a molecule of salt is about a ten millionth of an inch $\left(\frac{1}{10,000,000}\text{ in.}\right)$ in diameter. Then if this salt molecule were further divided, the resulting portions would no longer be salt but would be an atom of the metal sodium or an atom of the gas chlorine which are the chemical constituents of salt. Obviously neither sodium or chlorine have the properties of salt. Consider this: If with some powerful instrument a drop of water could be magnified to the size of the earth, then the molecules of water would appear to be about the size of a baseball.

EXAMPLES INDICATING THE MINUTENESS OF MOLECULES.—Odors, scents and smells are due to some sort of action by molecules on the nerves in our nostrils. It is evident that these molecules must be very small since they can not be seen either with the naked eye or with a microscope. The distance between molecules of water* is between the two thousand millionth

* Fleming, ELECTRONIC THEORY OF ELECTRICITY, p. 5.

and the ten thousand millionth of an inch (between $\frac{1}{2,000,000,000}$ in. and $\frac{1}{10,000,000,000}$ in.). The average diameters in inches of the molecules of some of the gases are:*

Hydrogen 2.03×10^{-8} cm. $= 0.798 \times 10^{-8}$ in. $= \frac{0.798}{100,000,000} = \frac{1}{125,000,000}$.

Carbon monoxide 2.85×10^{-8} cm. $= 1.14 \times 10^{-8}$ in. $= \frac{1.14}{100,000,000} = \frac{1}{87,720,000}$.

Nitrogen 2.92×10^{-8} cm. $= 1.15 \times 10^{-8}$ in. $= \frac{1.15}{100,000,000} = \frac{1}{86,900,000}$.

Air 2.83×10^{-8} cm. $= 1.12 \times 10^{-8}$ in. $= \frac{1.12}{100,000,000} = \frac{1}{89,000,000}$.

Oxygen 2.70×10^{-8} cm. $= 1.06 \times 10^{-8}$ in. $= \frac{1.06}{100,000,000} = \frac{1}{94,300,000}$.

Carbon dioxide 3.33×10^{-8} cm. $= 1.31 \times 10^{-8}$ in. $= \frac{1.31}{100,000,000} = \frac{1}{76,300,000}$.

The number of molecules in a cubic inch of air† at ordinary temperatures and pressures is between sixteen million billion and sixteen thousand million billion (that is between 16×10^{18} and 16×10^{21} or 16,000,000,000,000,000,000 and 16,000,000,000,000,000,000,000). The number of molecules in a cubic inch of hydrogen is at freezing temperature and ordinary-atmospheric-pressure about three hundred and twenty million million million (that is 32×10^{19} or 320,000,000,000,000,000,000). To appreciate the meaning of this enormous number it should be realized that if all the molecules in a cubic inch of air were placed side by side in a row, small as each one is, the row would extend nearly twice the distance from the earth to the sun.

With a very delicate measuring instrument, a variation in length of one-millionth $\left(\frac{1}{1,000,000} \text{ in.}\right)$ of an inch in a metal bar may be detected. In this short length about 100 molecules could be placed in a row close together. With a good microscope about the smallest object that can be seen is one having a diameter of a hundred thousandth of an inch. In a small box this size, sixteen million (16,000,000) molecules can be packed close together. The smallest weight (or strictly speaking, mass) which can be weighed on a very good chemical balance is one-hundredth of a milligram. A million million (1,000,000,000,000) molecules of hydrogen would therefore be just detectable on such a balance.

26. The Molecular Structure of Matter is such that no two molecules are in permanent contact with one another. An inconceivably small space separates each molecule from its neighbors. Every molecule is, at ordinary temperatures, quivering and oscillating to and fro—many million times a second—in its small, restricted space between its fellows. It bounds and rebounds back and forth between them. If a body is warmed, its mole-

* ENC. BRIT., Vol. XVIII, p. 656.
† Fleming, ELECTRONIC THEORY OF ELECTRICITY, p. 6.

cules vibrate more actively. They pound harder on the adjacent molecules and thereby push them away. This makes the body become larger as it is warmed. Thus the expansion of things as they are heated, may be explained.

FIG. 10.

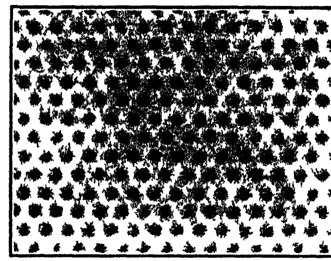


FIG 11.

FIG. 10—Imaginary, greatly-enlarged view of molecules of a gas, such as oxygen, wherein two atoms compose a molecule

FIG. 11—The atoms of a solid in which the arrangement is regular (Probably in some solid materials, the atoms are arranged higgledy-piggledy as in Fig 12 *I* while in other substances the arrangement is, it is likely, regular, as shown above)

27. There Are Three States of Matter: (1) *Solid,* (2) *liquid,* (3) *gaseous.* In solids,* the motion of each molecule is like that of a man in a dense crowd (Figs. 11 and 12,*I*) where it is almost or quite impossible for him to leave the space he occupies between

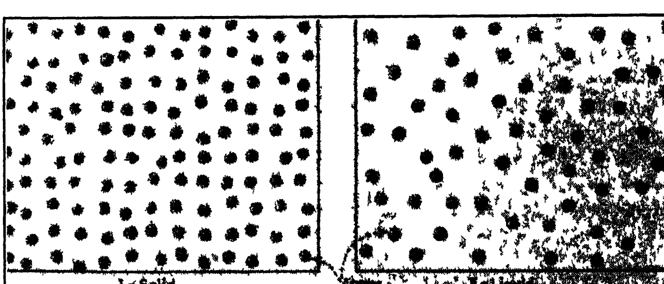


FIG 12—Imaginary microscopic views of portions of a solid and a liquid a "few hundred millionths of an inch wide"

his neighbors; yet he may turn around and have some motion from side to side. In solids the molecules make only very small excursions from their average positions. In liquids (Fig. 12,*II*) the motion of the molecules is like that of men moving on a

* Gage, ELEMENTS OF PHYSICS, p 19.

crowded thoroughfare. In gases (Fig. 10) the molecules are thought to be in motion like gnats in the air.

In solids the attractive force between molecules is very great; it requires considerable pressure to alter the shape of solids. In liquids the attraction between molecules is relatively small—the cohesive force is almost lost; hence liquids will assume the shape of any vessel into which they are poured. In a gas there is practically no attraction between molecules—the cohesive force is almost entirely absent; they are bounding and traveling around endeavoring to knock one another apart. Hence force is required to keep the molecules of a gas from separating entirely and wandering around every which way. It is probable that

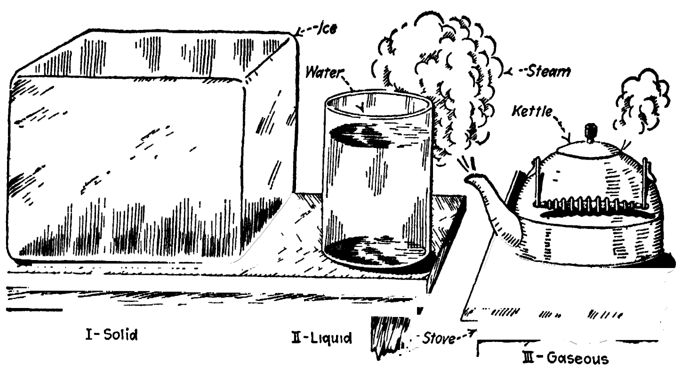


FIG. 13—Three states of matter and using water as an example.

the state (solid, liquid or gaseous) of any kind of matter is determined by its temperature and the pressure to which it is subjected.

EXAMPLE.—Water molecules (Fig. 13) may exist in: (*a*) *A solid state*, ice, Fig. 14; (*b*) *a liquid state*, water, Fig. 2; (*c*) *a gaseous state*, steam, Fig. 15. Water can be changed from any one of these three states to the other by heating or cooling it as the case may be. Zinc (Fig. 16) we ordinarily see in its solid state; by heating it is changed into its liquid state and heating it still further vaporizes it. All liquids (except possibly alcohol) have been solidified—frozen—by applying low temperatures and high pressures. Every liquid has been changed into a gas—volatilized. Every gas has been both liquefied and solidified. Sandstone, diamonds and quartz can be liquefied—melted—and vaporized by the intense heat of the electric arc.

It follows that any solid element is merely the "frozen" or congealed state of that kind of element. Any liquid is the melted state of that kind of element. Any gas is the vaporized state of

that kind of element. Note then that in the change of matter from one state to another, the molecules themselves are not changed, it is merely the spacings between and the spaces occupied by the molecules in their vibrations and wanderings that

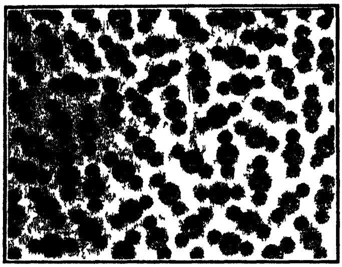


FIG 14 —Ice molecules, an imaginary view greatly enlarged Each molecule is shown as being composed of an atom of hydrogen and 2 ohms of oxygen

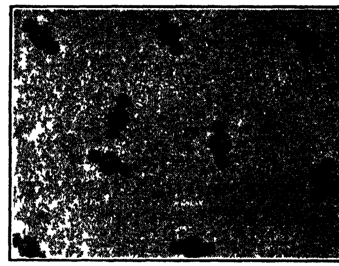


FIG. 15 —Greatly enlarged inaginary view of molecules of steam (water vapor).

have been changed The molecules remain intact as do the atoms composing the molecules and the electrons composing the atoms.

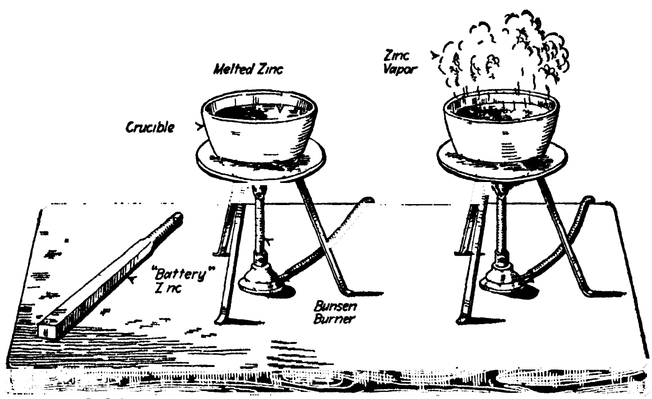


FIG. 16 —Zinc in the solid, liquid and gaseous states.

28. Equal Volumes of All Gaseous Substances Contain the Same Number of Molecules under the same conditions of temperature and pressure. This is AVOGADRO'S law. It constitutes one of the most important physical concepts by means of which many physical and chemical phenomena can be explained.

29. When a Substance Expands, the Distances between Its Molecules Increases; when it contracts, the distances between its molecules decrease—the molecules themselves do not change in size. This might be inferred from the statement in a preceding paragraph. Fig. 17 illustrates this truth in a diagrammatic way. The molecules of a compressed gas are relatively close together, as in *I*. If the gas is permitted to expand, its molecules will be further apart as in *II*. There is precisely the same number of "molecules" in diagram *I* as in diagram *II*. Similarly, the molecules of a substance are somewhat closer together when the substance is cold than they are when it is heated.

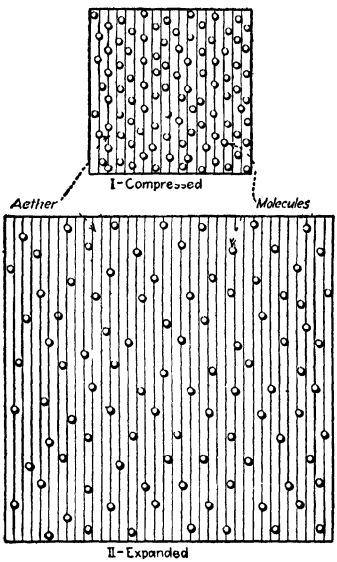


FIG 17.—How the distance between molecules of a gas increases if the gas is permitted to expand.

30. The Difference between an Atom and a Molecule, it being first understood that a molecule is made up of atoms, may be explained thus: It has been stated that the molecule is the physicist's unit of matter or the structural unit of physics while the atom is the chemist's unit. The meaning of this is that during any ordinary physical change such as evaporation, heating, cooling, expansion, contraction and the like the constitution of the molecule is not affected. In fact these phenomena are due to changes in the positions or relations of the molecules. On the other hand, when chemical phenomena occur, the molecule is broken up into its constituent atoms and new molecules are formed. No division of an atom occurs during any ordinary physical or chemical change. But atoms can, as elsewhere suggested, be partially broken up into electrons by the application of certain methods.

NOTE.—In an "element" all the atoms are alike. In a "compound" all the molecules are alike. In a "mixture" there are different kinds of molecules.

EXAMPLE.*—If two forms of matter, namely, two kinds of sand, black and white, are mixed together, each keeping its individual properties unchanged,

* Rowland & Ames, PHYSICS.

so that it is possible to separate them again, the change is called a physical one. Whereas, if, as the result of bringing two things together, namely, a piece of coal and the oxygen of the air (this occurs when coal burns), the properties of each are lost and an entirely new substance appears, it is called a chemical change.

31. A Chemical Change, then, is really nothing more than a transfer or change in grouping of atoms. Old groupings (molecules) are disrupted and new groupings (molecules) are formed. In chemical changes neither the atoms themselves nor the electrons composing them are altered in any way. For example, the aggregation of electrons whirling around a positive nucleus, which we call an atom of copper, remains, during a chemical change always an atom of copper. However, certain atoms do sometimes lose electrons as described in the following paragraph.

32. The Atoms of "Radio-active" Substances Are Continually Shooting off Electrons.—The reason for this is, so it is believed, that the arrangement of the electrons in these radio-active atoms is an unstable one. Hence—since the general properties of a substance are determined by the number and grouping of the electrons which compose its atoms—the properties of these substances are gradually changing—though very slowly—as the number of electrons composing them decreases. Thus, due to electron emanations, one kind of radio-active element may change spontaneously into another kind of element.

33. Furthermore, Atoms of Any Substance May Lose an Electron—or possibly a few electrons—by the electrons being brushed or rubbed off by main force or friction. Or atoms may be caused to give up electrons by virtue of some properly directed electric agency. However, in practically all such cases, the total number of electrons rubbed off or pushed off by such forces—frictional or electric—is an exceedingly small proportion of the total number of electrons in the substance, probably only one electron in a million millions of them. It is such a small proportion that no change in the appearance, constitution or weight, of the substance as a whole, is detectable even by the application of the most delicate and refined methods and measurements. But because of the enormous repulsive forces of free electrons (Art. 9), a relatively few electrons in excess or in deficit on a substance can account for many of the wonderful electrical phenomena which are constantly being observed and which are difficult of explanation on any other basis.

34. How Matter Is, Possibly, Built up from Electrons, Atoms and Molecules is suggested in Fig. 18. The spheres represent the centers of atoms. The lines connecting the atoms are merely imaginary lines to indicate that the atoms are disposed, in the substances shown, in regular geometric arrangements. There are no such lines connecting the atoms of actual matter. By making certain measurements on crystals with X-ray beams gifted scientists (the Braggs and Mosley) have been able to determine that, in crystals of the substances indicated, the atoms are actually located as diagrammed in the illustration. Crystalline substances have doubtless a relatively simple or regular structure. Probably the structures of a compound substance like wood or cloth would be much more complicated than those illus-

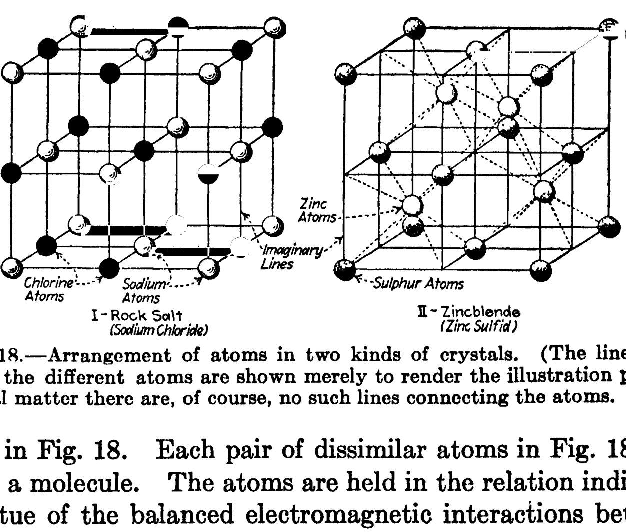


FIG. 18.—Arrangement of atoms in two kinds of crystals. (The lines connecting the different atoms are shown merely to render the illustration plainer; in actual matter there are, of course, no such lines connecting the atoms.

trated in Fig. 18. Each pair of dissimilar atoms in Fig. 18 constitute a molecule. The atoms are held in the relation indicated by virtue of the balanced electromagnetic interactions between them.

35. The Æther (sometimes spelled "ether," not to be confused with the anesthetic, an entirely different thing) is the invisible stuff which is assumed to fill the voids in space between the electrons and positive nuclei which compose matter. It may be that it also fills the space occupied by the electrons themselves and hence is absolutely continuous without voids or gaps! All space—the entire universe—is, it is assumed, permeated by this medium "æther." It has been called "*the all-pervading æther.*" It exists in those portions of space which are apparently empty. It is probably at rest—stationary. Very little is definitely known

about æther except that it—or something equivalent—exists. How it is known that it exists is outlined in the example following.

36. Æther Is Not Matter because matter is made up of electrons and nuclei atoms and molecules. It is known that the æther offers no resistance to things passing through it—in this sense it is frictionless—because the earth on which we live is whirling through it at an enormous speed without the people on the earth being aware of its (the æther's) existence. On the other hand, in certain respects æther appears to be similar to a semi-rigid, jelly-like substance. The æther is, so it is believed, the vehicle whereby light is transmitted and heat is radiated and it is also an important factor in certain electrical and magnetic

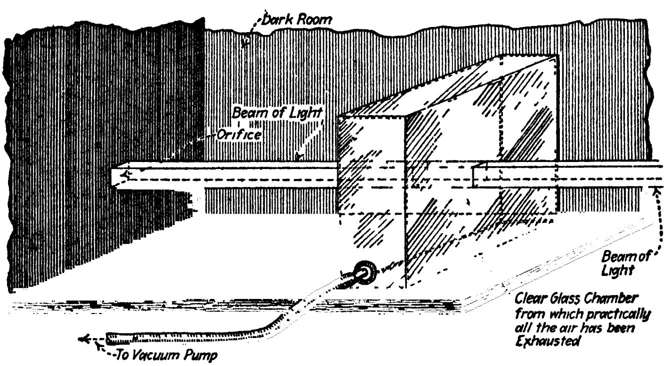


FIG. 19.—Light beam—æther waves—passing through a partial vacuum in the glass chamber.

phenomena. Æther is susceptible to the effects of certain stresses and strains and in this particular is similar to ordinary matter.

NOTE.—"Strain" and "Stress" are two words which have certain definite meanings in technical parlance: A *strain* is an alteration in form, size or volume due to the application of a stress. A *stress* is the force, pressure or other agency which produces the strain.

EXAMPLE.—If the air is, in so far as possible, exhausted from a glass vessel, Fig. 19, a beam of light will then pass through the vessel as well as or even better than it would with air in the vessel. There must be something besides the few remaining molecules of air in this apparently empty vessel to transmit the light waves. There is; that something may be called æther. Heat can be radiated through an almost perfect vacuum as in Fig. 20. The æther, which is made to vibrate, is, it is assumed, the medium whereby the heat waves are radiated through the practically "empty" space. The sun warms an object—in a hot house for example—on which its rays fall on a cold day without warming the intervening air. This is because the sun's heat is transmitted by radiation through the æther between it and the earth.

When light falls on the vanes of a radiometer (Fig. 21), which operate in a partial vacuum, they rotate by virtue of energy which is transmitted to them through the æther. If a thermometer in a bottle (Fig. 22) from which the air has been exhausted is brought near a cake of ice its mercury will fall. This indicates that energy has been transferred from the mercury to the atmos-

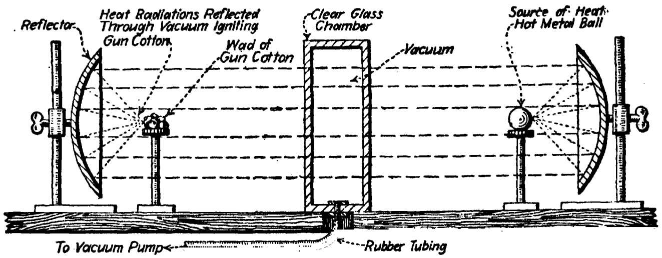


FIG. 20.—Heat being radiated through a partial vacuum by æther waves.

phere outside of the bottle through the medium of the æther. Our eyes are sensitive to the æther vibrations which are called light as is explained in detail in the author's PRACTICAL ELECTRIC ILLUMINATION.

36A. Ions and Ionization.—Every normal atom comprises a certain number of electrons in combination with sufficient posi-

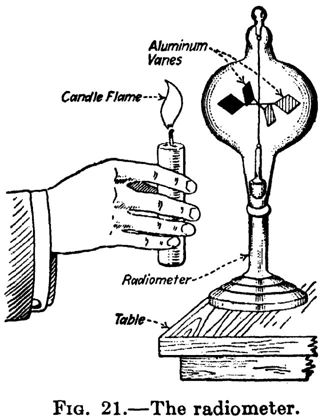


FIG. 21.—The radiometer.

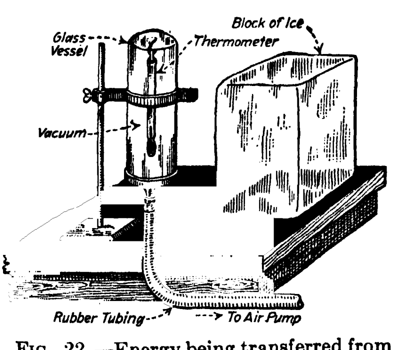


FIG. 22.—Energy being transferred from mercury in thermometer bulb to the outside atmosphere.

tive electricity to just neutralize the negative effect of the electrons. Normally, atoms exhibit no unusual electrical properties because the positive electricity in them neutralizes the negative. But if an atom has an electron too many or an electron too few, then it does exhibit unusual electrical properties, which can

be detected by the electrostatic attractive and repulsive effects (Fig. 22A) thereby produced, and the atom is then said to be ionized. Thus* *when an electron is taken from or added to a previously-neutral atom or molecule, the charged particle which is thus formed is called an ion.* The process is that of ionization. In other words, an ion is what is left after an electron has been knocked from a neutral atom or molecule. Or an ion is what then exists, after an electron is added to a previously-neutral

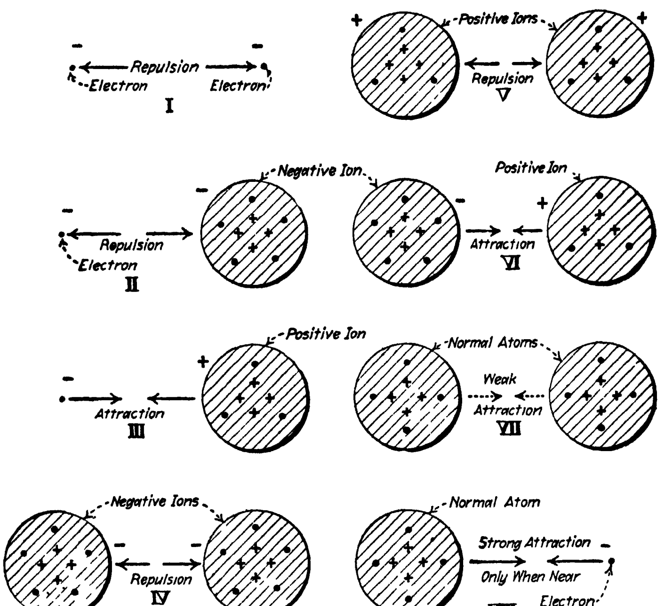


FIG. 22A.– Illustrating the differences, and the attractive and repulsive forces, between atoms and positive and negative ions. (It should be understood that the above are merely diagrams and are not intended to show the actual construction of atoms. This illustration adapted from Comstock and Troland.)

atom or molecule. If an electron is knocked from a normal atom, the atom then becomes a positive ion (sometimes called a kation). If an electron is added to a normal atom, that atom then becomes a negative ion (sometimes called an anion).

36B. Positive and Negative Electrification.—When there is an excess of electrons associated with a thing it then acquires certain remarkable properties and is said to be negatively charged or electrified. When a thing has associated with it less than the

* THE NATURE OF MATTER AND ELECTRICITY, Comstock and Troland, *D. Van Nostrand Co.*, New York City.

normal number of electrons, it has certain other properties and is said to be positively charged or electrified.

EXAMPLE.—If a glass rod (see Fig. 3) is rubbed with a piece of silk, the rod becomes "electrified" and will attract light-weight objects such as pith balls or bits of paper. The explanation is that rubbing the rod with the cloth knocked some of the constituent electrons from the rod. There is, then, a deficit (less than the normal number) of electrons associated with the rod, and the pith balls are attracted to it because any object having a deficit of electrons will attract an object which has its normal complement of electrons. Electric attractions and repulsions are always such that they tend to restore an electrical balance, see Fig. 22A.

Thus, if a thing has an excess of electrons, it will repel any other thing which has an excess of electrons. Or if one thing has an excess of electrons and another a deficit, they will attract one another—tend to restore the electrical balance. If one thing has an excess of electrons and another thing its normal number, they will attract one another. And, if a thing has a deficit of electrons and another its normal number, they will attract one another.

36C. Electron currents always Flow from the Negative Pole to the Positive, in spite of the fact that it is usually assumed that *electric currents* flow from positive to negative. This unfortunate state of affairs is due to the fact that the pioneer electrical experimenters before the nature of electricity, as we understand it now, was appreciated, arbitrarily named an electrification involving a deficit of electrons a positive electrification. However, this apparent difficulty works no real harm inasmuch as in practical work we will continue to assume that what we call an *electric current* flows from positive to negative, whereas we actually know that the current which does flow (the electron current) moves from what we call negative, to positive.

37. To Summarize the Present-day Ideas as to the Construction of Matter.—Matter is made up of molecules, which are in turn composed of atoms. Atoms are groups of electrons in stable orbital motion around a common center of mass. Electrons are particles or corpuscles of electricity.

SECTION 2

MAGNETISM

38. Magnetism.—We do not know precisely what magnetism is. But we do know how to produce and control it. Magnetism and electricity are not the same things by any means. They are closely related as a study of the following pages will prove.

39. The Nature of Magnetism (see Art. 68) is not thoroughly understood. It may be a stream or current of something through the æther, somewhat similar to a displacement current (Art. 105) of electricity in a conductor—though it is not likely that this is the real explanation. On the other hand, it is altogether probable that magnetism is a phenomenon which occurs whenever electrons (Art. 4) are set into rotation. In any case, it is known that the phenomenon of magnetism is always accompanied by a certain kind of stress in the æther—or by what is equivalent to such a stress.

39A. The Electron Theory of Magnetism is explained in a general way in Art. 218A, in the Electromagnetism Section of this book. To understand this theory, one must first understand electromagnetism.

40. A Magnet is a body having that remarkable property of *polarity* and of *attraction* and *repulsion* found in nature in the lodestone. Every magnet, (except the ring magnet (Art. 62), has at least two opposite (positive and negative) poles (Art. 46).

41. Classes of Magnets.—Magnets may be divided into three classes: (1) *Natural magnets* (Art. 43) of lodestone which are pieces of magnetic oxide of iron; (2) *permanent magnets* (Art. 80) which are bars of hardened steel which have been permanently magnetized; and (3) *electromagnets* (Art. 218) which are soft iron bars, wound round with a coil of insulated wire. When electricity flows through the coil the bar is magnetized and when the flow ceases the bar loses its magnetism.

42. The Importance of a Thorough Understanding of Magnetism can scarcely be overestimated. The principles involved are applied in nearly all electrical apparatus—from the simplest electric bell to the largest electric generator.

43. The Lodestone. Natural Magnets.—An iron ore, now called *magnetite*, was discovered centuries ago near Magnesia, in Asia Minor. It was found that some of it had the wonderful property of attracting iron (Fig. 23,*I*). It was also found that if a piece was suspended by a thread (Fig. 23,*II*), a certain one of its ends always pointed *north* and the other *south*. Thus

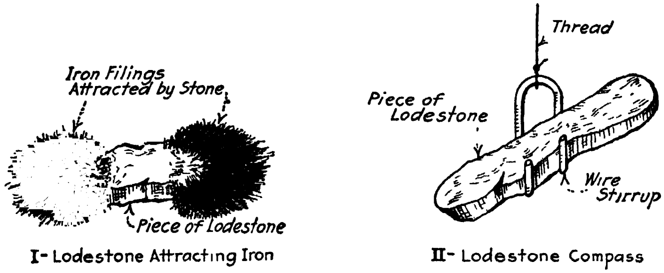


FIG. 23.—Lodestone magnets.

arranged it constituted a compass (Art. 50). Ships were then navigated by using suspended pieces of lodestone, which means "leading-stone." The stones were called, from the name of the town, magnets and their property was termed magnetism. Magnetite is also found, among other places, in the State of Arkansas, in Sweden and in Spain.

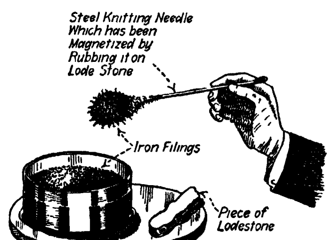


FIG. 24.—Piece of steel darning needle (which has been magnetized) attracting iron filings.

44. Artificial Magnets.—If a darning needle or other piece of steel (Fig. 24,*I*) be stroked with a piece of lodestone and then dipped into iron filings, the filings will cling to the ends of the needle in tufts. Obviously the needle is now also a magnet. A second needle stroked with the first will become a magnet too. Thus artificial magnets can be prepared, but it can be done most effectively by electromagnetism (Art. 204).

45. Magnetic and Non-magnetic Substances.—A magnetic substance is one that is forcibly attracted by a magnet—or capable of being temporarily magnetized. A *magnetizable substance* is one that will retain magnetism and there are only a few: steel, lodestone, nickel and impure iron. Very few substances are

pronouncedly magnetic and most of these are not magnetizable. Iron and steel are decidedly magnetic. *Pure,* soft iron is not magnetizable; a piece of it will be attracted by a magnet equally well at any point. It has no poles (Art. 46). Steel is very magnetizable—hard steel more so than soft—and ordinary commercial iron, which is always impure, and nickel are so to a certain extent. Iron and steel are the only substances used in practice where magnetic properties are necessary. Nickel and cobalt are noticeably magnetic as are also certain alloys of all of the previously mentioned metals, salts of iron and of other metals, paper, porcelain, and oxygen. It is believed that all substances are magnetic to a certain degree.

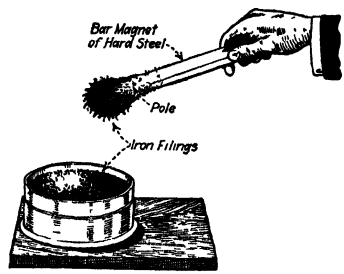


FIG. 25.—Magnet attracting iron filings.

46. Poles of a Magnet.—If a bar magnet is rolled in iron filings they will cling in clusters or tufts to its ends (Figs. 25 and 26,*I*). There will be few or no filings near the center of the bar. Obviously, the attractive property of the magnet is concentrated near its ends. Those portions of a magnet where the attractive power is greatest are called its poles. The poles of a horseshoe magnet are shown in Fig. 26,*II*. A line joining the poles is called the *magnetic axis*. *The equator* is a line through the zone

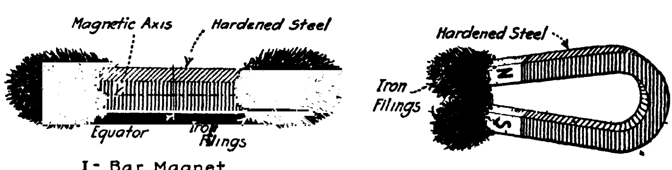


FIG. 26.—Showing poles of a magnet.

of no attraction, at right angles to the axis. Poles are formed only where lines of force (Art. 48) leave or enter a magnet.

47. Like Poles Repel and Unlike Poles Attract; this can be shown as suggested in Fig. 27 or in numerous other ways. If any two magnets—either natural or artificial—are used instead of the two shown in the illustration, the action will be the same. This truth can be nicely demonstrated with a magnetized needle on a floating cork like that of Fig. 28,*I*.

48. North and South Poles.—If an unmagnetized darning needle be laid on a cork floating on water (Fig. 28,*I*) and then be successively pushed around with the finger and allowed to come to rest, it will, in each case, assume no particular final position in preference to others. Now, if the needle be *magnetized*

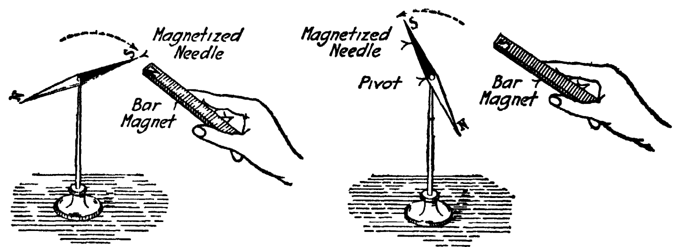


FIG. 27.—Magnetic attraction and repulsion.

by stroking it with a magnet, it will always come to rest in the same position, pointing almost due north and south. Furthermore, if the needle end that points north be marked by tying a thread around it, it will be found that this end always points to the north. The north-pointing end of this needle magnet is

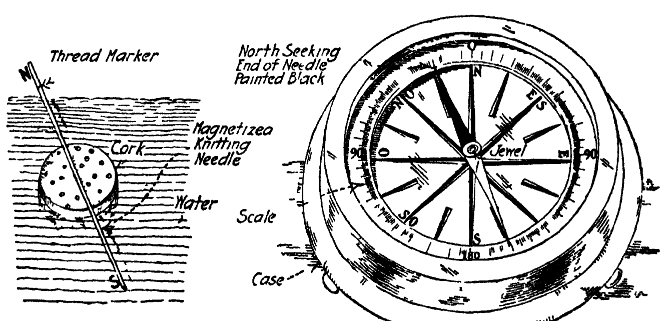


FIG. 28 —The compass.

called its *north pole* or *north-seeking pole.* The other end is its *south* or *south-seeking pole.* Obviously, the magnet on the cork constitutes a compass (Art. 50). It is also apparent that every magnet has at least two poles (for the one exception see Art. 62). The north pole is the one at which the *lines of force* (Art. 56) are

assumed to leave a magnet and the south pole is the one at which they enter it.

49. The Neutralizing Effect of Unlike Poles.—If a permanent bar magnet is laid on a similar magnet so that their like poles are together (Fig. 29,*I*), the magnetic strength of the combination will be greater (practically twice as great, assuming both magnets to be of equal strength) as the strength of either of the magnets alone. On the other hand, if the two magnets of equal strength are laid together so that their unlike poles will lie together (Fig. 29,*II*) then the unlike poles will neutralize one another and the combination will have practically no external field—that is, no strength.

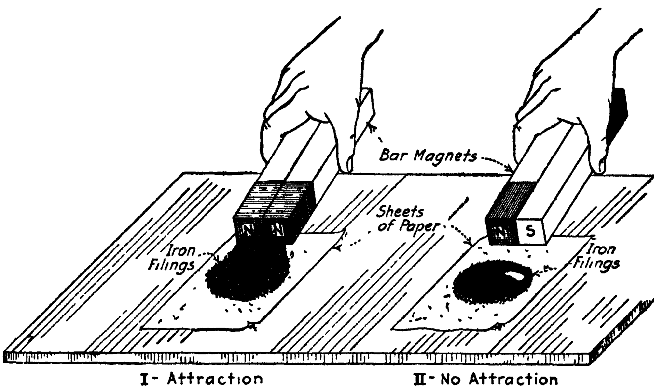


FIG. 29.—Illustrating the neutralizing effect of unlike poles.

EXAMPLE.—The above facts can be readily verified, as suggested in Fig. 29, by testing the attractive strength of a two-magnet combination on a pile of iron filings. With like poles—N to N and S to S—together, the filings will be forcibly attracted. With unlike poles together—a north and a south pole—the combination will offer little or no attraction for the filings.

50. The Compass (Fig. 28,*II*).—It follows from Art. 48 that a compass is merely a nicely balanced and pivoted magnet, contained in a case to exclude disturbing draughts of air, and provided with a suitable scale indicating the N (north), S (south) and intermediate points. *The mariner's compass* is one that is very sensitive and arranged for nautical observations. Its scale or card is divided into the 32 "*points of the compass*" and is attached to and swings with the magnetized needle. Frequently several magnetized bars are arranged side by side, as such a compound magnet (Art. 81) has been found the most reliable. The

N point on the card always points toward the north. A compass needle does not always point *exactly* north (Art. 53).

51. The Earth is a Magnet.—This was discovered by Gilbert. A compass needle points north for this reason. The earth's magnetic poles coincide almost, but not exactly, with its geographical poles. This can be demonstrated with a *dip needle* (Fig. 30,*I*). One can, as shown, be made by pushing a magnetized knitting needle through a cork so that the cork will be at the center of the needle and inserting two shorter pieces a knitting needle or wire for an axis. Tumblers can be used for bearings. The needle—which should be remagnetized after insertion in the cork (Art. 83)—will assume a slanting position. Its north end will drop down due to the earth's attraction. At a point (about 1,000 miles away from the geographical north pole) in Boothia Felix, west of Baffin's Bay, a dip needle becomes nearly vertical. This point is the magnetic north pole of the

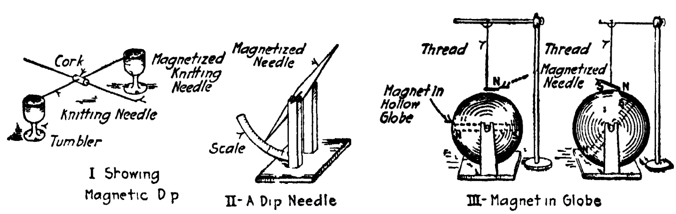


FIG. 30.—Showing that the earth is a magnet.

earth. At the earth's equator a dip needle is horizontal. At the south pole the needle is again almost vertical, with its south pole downward. A dip needle is shown in Fig. 30,*II*. A bar magnet fixed within a globe (*III*), having a magnetized needle suspended over it, will reproduce on a miniature scale the effect of the earth's magnetism on a dip needle.

52. Why the Earth Is a Magnet is not known definitely. It has been suggested that its magnetism is due to the currents of electrified air that ascend at regions adjacent to the equator, then travel, part northward, part southward, to descend at the poles. These movements of electrically charged air have the effect of real electric currents. The general direction of the earth currents within the earth's surface is from the poles to the equator. Fig. 31 indicates the magnetic circuit (Art. 57) of the earth.

53. The Declination of the Compass Needle at any location is the angle between the *magnetic meridian* and the geographic

meridian at that location. The magnetic meridian at a location is the direction of a magnetic needle at that location. Declination is due to the fact that the magnetic and geographic poles of the earth do not coincide (Art. 51). Declination varies at different locations on the earth's surface and gradually changes from year to year. There is an irregular *line of no declination* that circles the earth, passing above the north and south magnetic poles. At locations on this line a compass needle points true north and south. Accurate charts which indicate the declina-

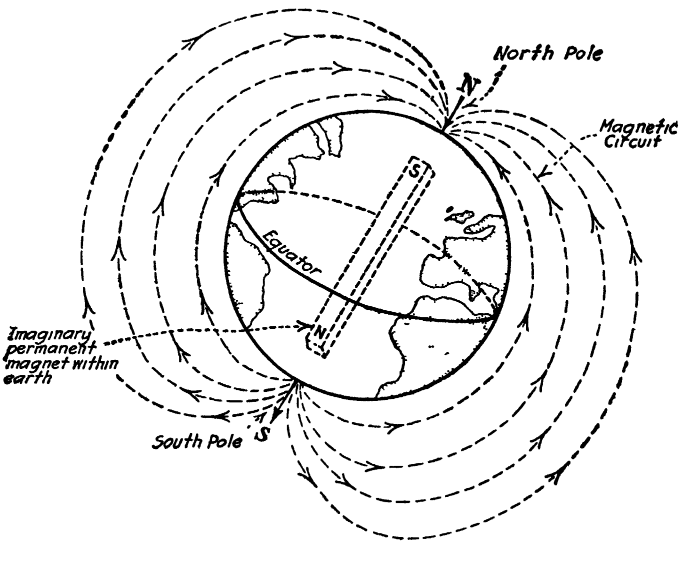


FIG. 31.—The earth's magnetic circuit.

tion at different points of the earth's surface are prepared for and used by navigators.

54. Magnetic Transparency.—A magnet will attract (Figs. 32,*I* and *II*) through glass, wood, mica—in fact through anything except iron. If a plate of iron be substituted for the plate of glass in *I* the number of tacks attracted will be less. Notice at *II* how the magnet, hermetically sealed within the glass tube, attracts filings outside of it. Magnetism acts through all except magnetic (Art. 45) substances. *Magnetic screens* or *shields* (Fig. 32,*III*) are made of very soft iron to enclose certain delicate

instruments and watch-and-clock movements to protect them from external magnetic forces. A compass at A within the shield would not be affected by the magnetism of N. There is no *in-*

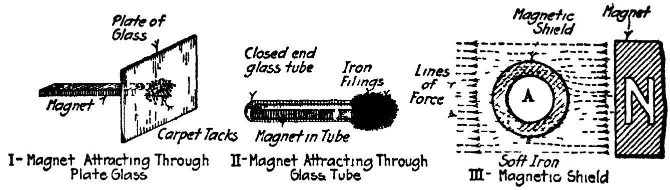


FIG. 32.—Illustrating magnetic transparency.

sulator for magnetism. Iron is the very best "conductor" of magnetism; it is for this reason (because of its great permeance, Art. 63) that it makes such good and effective magnetic shields.

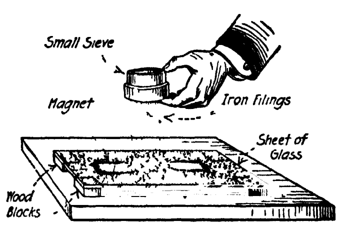


FIG. 33.—Making a magnetic spectrum.

55. A Magnetic Spectrum or Magnetic Figure (Figs. 33 and 33A) can be made by sifting iron filings from a small sieve or muslin bag on a sheet of glass, pasteboard or paper under which a magnet or magnets have been placed. The filings will arrange themselves in the directions of the *lines of force* (Art. 56) emanating from the magnet or magnets and thereby produce an accurate representation of the *field of force* (Art. 61) about the magnet.

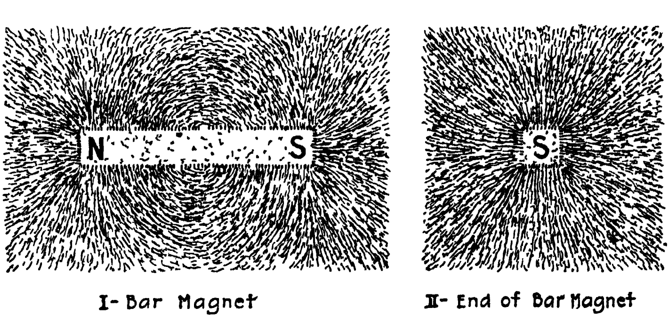


FIG. 33A.—Magnetic spectrums.

The sheet on which the filings are sifted should be gently tapped as the filings fall on it. If the sheet has been previously coated on one side with paraffin, and then allowed to cool, the spectrum

can be made permanent by carefully heating it, with the filings on the coated side, over a stove or by passing a hot soldering iron under it. By this process the filings are imbedded in the paraffin. Blue prints can be made from such positives.

56. Lines of Force or Lines of Magnetic Induction (see also Arts. 226, 59 and 60) are the imaginary lines along which the

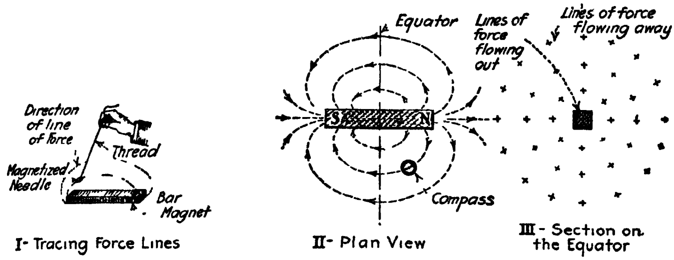


FIG. 34.—Line of force.

attractive or repulsive force of a magnet acts. They map out the lines of magnetic strain. The lines of force, or stream of magnetism, is assumed to leave or flow from the north pole of every magnet, to curve around in the outer medium and to enter

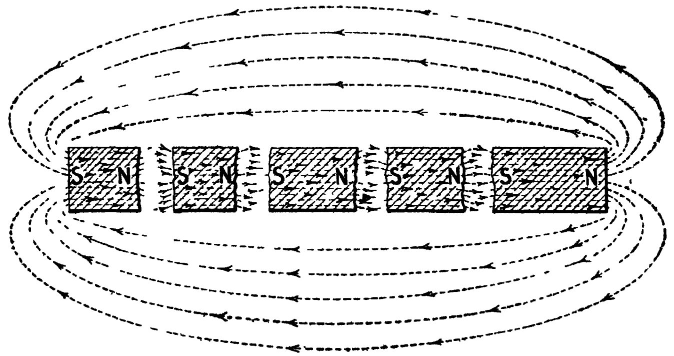


FIG. 35.—Magnetic circuit of lines of force through a bar magnet which has been broken into sections.

the south pole (Fig. 34). Each line of force completes an unbroken, continuous path, or circuit, that is, each is a closed line. The complete course or loop taken by the lines of force—within and without the magnet—comprises the *magnetic circuit* (Art. 57) of that magnet. If a magnet be broken into small sections (Fig. 35) the lines of force will "flow" from section to

section through the intervening air. This proves that each line forms a closed loop. Lines of force never cross one another. Each line of force can be thought of as resembling an invisible, stretched rubber band. There is a rubber-band-like tension along every line tending to shorten it. Each line also exerts a sideways push in all directions tending to crowd adjacent lines away from it. When for any reason lines are distorted they tend to recover from the distortion or to react against it.

57. A Magnetic Circuit is then, the route or path followed by the magnetic lines of force of a magnet. In practice, usually, the greater part of a magnetic circuit is through magnetic materials—iron and steel—but there may be air gaps in a magnetic circuit. See Art. 62, "Ring Circuits." It will be of great assistance in understanding the phenomena and laws of magnetism if the student will think of the lines of force of any magnet as constituting a stream or flux (Art. 255) flowing around

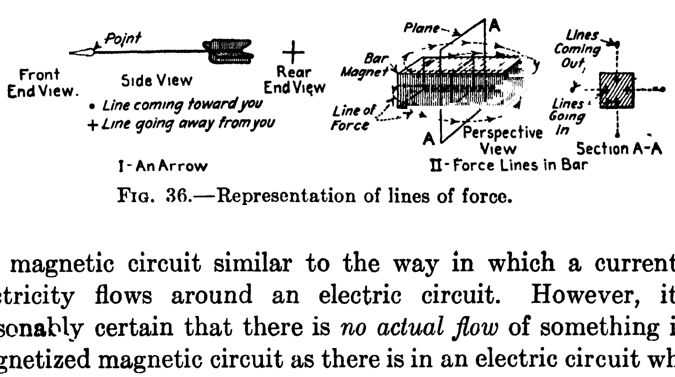


FIG. 36.—Representation of lines of force.

the magnetic circuit similar to the way in which a current of electricity flows around an electric circuit. However, it is reasonably certain that there is *no actual flow* of something in a magnetized magnetic circuit as there is in an electric circuit which is carrying a current—but this does not prevent one from thinking in terms of something flowing. There are many similarities between electric and magnetic circuits, as will be demonstrated—but the two are not, by any means, the same. See Art. 219 for "The Laws of the Magnetic Circuit."

58. The Directions of Lines of Force Can Be Traced by a magnetic spectrum (Fig. 33) or by moving a suspended magnetized needle or compass along a force line (Fig. 34,*I*). The magnetized needle, at any location, will assume the direction of the magnetic force (or line of force) at that location. An infinite number of lines of force can be thus traced from one pole of a magnet to the other.

59. Representation of Lines of Force (see Fig. 36).—An arrow (*I*) or a V-shaped arrow-head thus: >, can be drawn on lines of force to show their directions when one is looking "side on" at the lines. The lines always flow *from the north to the south pole outside of the magnet* and from the south to the north pole inside of the magnet. When looking at force lines end on, a dot (as at *Section AA* in *II*) represents a line when it is flowing toward one. A cross represents it when it is flowing away. This method has been adopted because the head end of an arrow (*I*) looks like a dot and the rear end of an arrow like a cross.

60. Further Explanation of the Term, "Line of Force."—This conception (for it is such, as one can not see or feel lines of force—because they are imaginary lines) is extremely useful in two rather distinct ways:

(*a*) FOR INDICATING THE DIRECTION OF ACTION AND THE EXTENT OF THE FIELD (ART. 61) OF A STREAM OF MAGNETISM. We can make a magnetic spectrum, or instead, plot loops like those of Fig. 34 of the magnetism about a magnet. The spectrum or the plot will then, in a general way, indicate the directions of action and the extent of the magnetic field. We can say that the lines of our reproductions portray or stand for the "lines of force" in the field.

(*b*) AS A UNIT FOR MEASURING THE MAGNETISM IN A MAGNETIC FIELD. It became evident many years ago that some method or unit should be adopted for the measurement of the amount of magnetism in a magnetic field. One of the methods proposed was: that it be assumed that magnetic fields are composed of lines—to be called lines of force—and that every line represent a certain amount of magnetism. Then the total amount of magnetism in a field would be proportional to the total number of these imaginary lines composing it. Hence by computing the number of these imaginary lines of force in a certain field the amount of flux or magnetism in it could be determined. This method—among others—was adopted. It was then agreed by the scientists that a magnetic field containing a certain amount of magnetism (Art. 226) would be considered as comprising 1 line of force. A field of twice the amount is then referred to as a field of 2 lines; a field having a thousand times the amount would have 1,000 lines. It is evident then, that, on this basis we can ascertain the amount of magnetism, that is the flux, of a magnetic field by computing the number of these imaginary, unit lines of force in the field. A definite quantitative definition of a *line of force* is given in following Art. 226.

NOTE.—A line of force is an actual line in the sense that it is a line along which a force due to magnetism acts. Otherwise, it is an imaginary line.

61. A Magnetic Field or a Field of Force is the region adjacent to, but outside of, a magnet which is permeated by the magnet's lines of force and within which magnetic substances (Art. 45)

or conductors conveying electric currents are perceptibly influenced. The extent of a field and the directions of the lines of force composing it can be studied by the magnetic spectrum method (Art. 55) or with a magnetized needle as in Fig. 34. When the iron filings are sprinkled to form a spectrum, each becomes a minute magnet by induction (Art. 64) and they therefore arrange themselves along the directions of the lines of force. Several accompanying illustrations showing magnetic spectra and lines-of-force diagrams illustrate the idea of a magnetic field. *A uniform magnetic field* is one throughout which the lines of force are (or may be thought of as being) equidistantly spaced.

NOTE.—Theoretically the external magnetic field due to any magnet extends from and all around the magnet for an infinite distance but the field becomes weaker and weaker as the distance from the magnet increases. Practically, the field developed by any magnet is strong enough to be perceptible only for a distance of a relatively few feet—or inches—from the magnet. A magnetic field has been likened to a *magnetic atmosphere.*

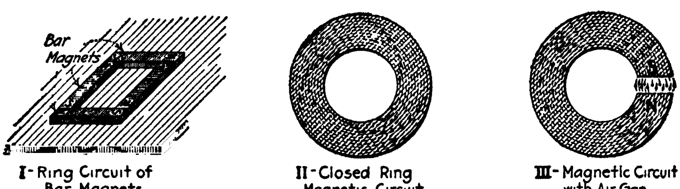


FIG. 37.—Ring magnetic circuits.

62. A Ring Magnet (Fig. 37) has no poles and no external magnetic field. If four bar magnets are arranged to form a closed rectangle (*I*) there will be no appreciable *external field*—that is, no field outside of the iron or steel composing the magnets. Such a group comprising a closed magnetic field constitutes one form of a ring magnet. A steel ring like that of Fig. 37,*II* may be very strongly magnetized and yet have no poles because its lines of force nowhere leave (there is no tendency for them to leave) the iron or steel comprising the magnetic circuit. Where it is desirable that there be little or no external field, ring magnetic circuits are used—as in transformers and in certain electrical instruments. If a piece be cut out of the ring, as at *III*, two powerful poles will be formed at the cut. Then we have the elementary form of the magnetic circuit that is widely utilized for useful permanent magnets (see Fig. 38,*I*, *II*, *V*, *VI*, and *VII*) and for the magnetic circuits of generators and motors.

63. Permeance is a term which relates to the ease or readiness with which a material will "conduct" magnetic flux. It may be considered as a property of certain magnetic materials such as—iron and steel—whereby they offer much less opposition to the flow of streams of magnetism (lines of force) than do other materials. By virtue of this property, if a piece of iron be placed in a magnetic field, the lines of force will tend to flow through the iron, and a great majority or nearly all of them will, Fig. 39,*I*. This creates a condition known as *field distortion*. Not only will the lines tend to concentrate in the iron A, but the number of lines of force in the magnetic stream will be greater with the iron (A) in the field than with the iron (A) out of it. The presence of

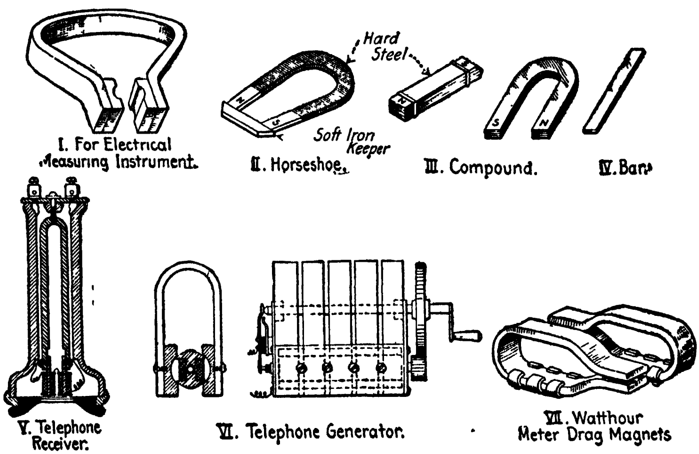


FIG. 38.—Types of permanent magnets.

the iron decreases the opposition to the flow of lines in the magnetic circuit (Art. 57), hence the number of lines of force is proportionally increased by placing the iron in the magnetic circuit. See Art. 238, in which numerical values are given, for more information regarding permeance and also permeability, which is specific permeance. Magnetic induction, Art. 64, is also a result of the great permeance of iron. The symbol for permeance is $\mathcal{P}$.

64. Magnetic Induction.—Magnetism can be imparted by a magnet to a magnetic substance without the two being in actual contact, as in Fig. 39,*I*, where the soft iron bar B is magnetized through the influence or induction of bar magnet A. Magnetism produced in magnetic substances by the influence of a magnet

is said to be *induced*. When magnetism is developed in a body by the induction of a magnet, at least two poles are produced in the body. The two poles of the body and magnet that are nearest together will be of unlike kind (one north and one south) and the poles that are furtherst apart will, in general, be of unlike kind also (see Fig. 39,*II*). Note that there is a similarity be-

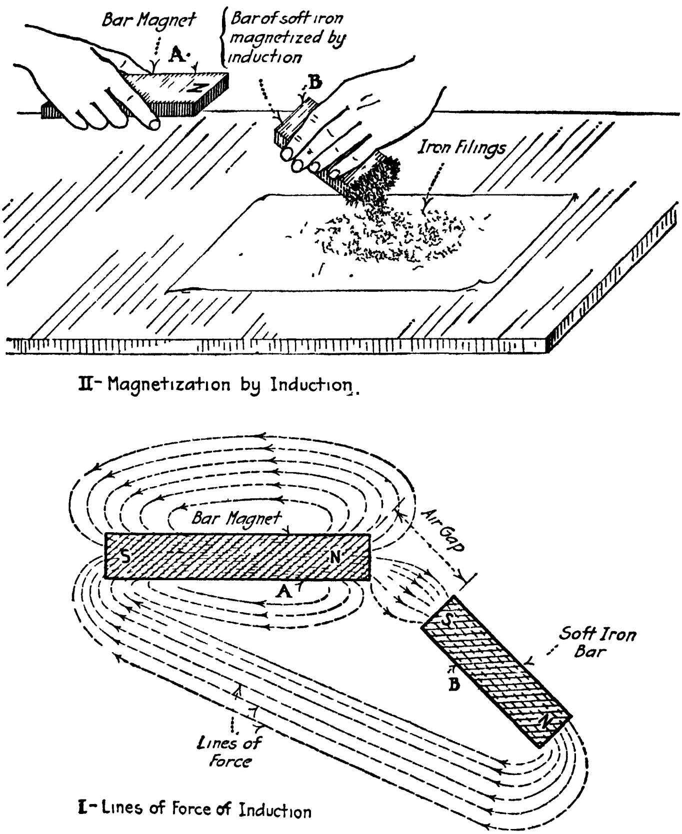


FIG. 39.—The principle of induction.

tween static and magnetic induction. Magnetic induction may also be defined as the production in a magnetic substance of many more lines of force than would be produced in air by the same magnetizing force. In Fig. 39,*II*, for example, when the soft iron bar *B* is introduced in the field the number of lines of force in *B* is increased (because of its permeance, Art. 63) enor-

mously above the number that did thread through the air. Hence the intensity of the field around the iron *B* is then sufficient that filings or other iron objects may be attracted. If magnetism is produced in a soft iron bar by induction, the bar will lose its magnetism when it is removed from the field of the inducing magnet. If magnetism is produced inductively in hard steel the steel will permanently retain a portion even when it is removed from the field.

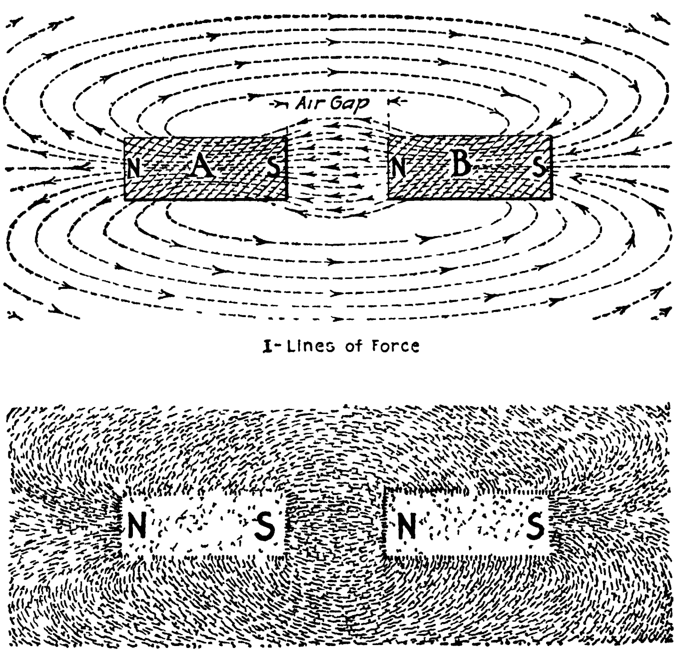
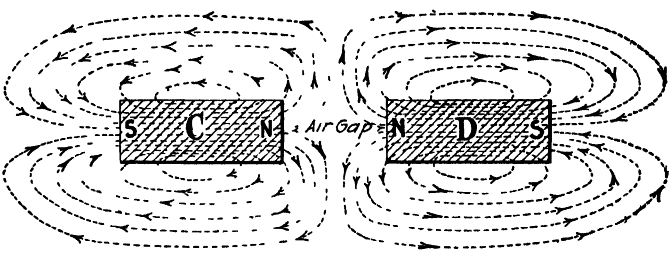


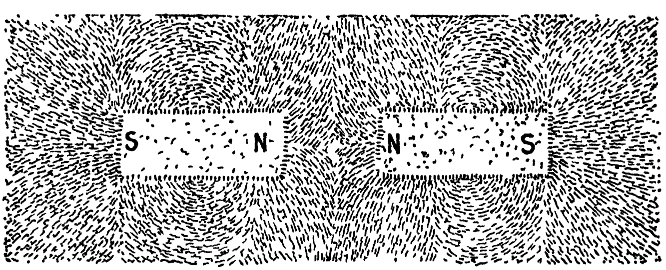
FIG. 40.—Magnetic attraction.

65. Explanation of Attraction and Repulsion.—Art. 48 states that a north pole is formed where lines of force leave a body of magnetic material; that a south pole is produced where lines enter one. Therefore, when a north and a south pole are placed near together (Fig. 40) the lines of force from magnet *A* unite with those of *B* to constitute a stream of lines across the air gap and around the magnetic circuit. There is always a rubber-band-like tension along lines of force (Art. 56). Hence the tension in the lines creates a tendency across the air gap to pull *A*

and B together. This is the explanation of magnetic attraction. When lines of force are induced in a magnetic body by a magnet as in Fig. 39,*II*, the lines pass from A, at north pole N, and enter B at S, creating there a south pole. Then the tension in the force lines causes N and S to attract one another across the air gap. This explains the *attraction due to magnetic induction.* The magnet first magnetizes the body by induction, then the unlike poles attract one another.



I- Lines of Force



II-Magnetic Spectrum

FIG. 41.—Magnetic repulsion.

NOTE.—When like poles—a north and a north pole—are placed near each other as at Fig. 41,*I*, the lines repel each other and the side ways pushing tendency that is a property of lines of force (Art. 56) creates a tendency for the magnets to push apart. This accounts for magnetic repulsion.

66. Magnetization Due to the Earth's Induction.—Iron columns and beams in buildings, stoves and iron members of machinery that have remained stationary in one position for some time acquire magnetic properties. This can be demonstrated by bringing a compass needle near them.

EXAMPLE.—This can also be shown as suggested in Fig. 42. The annealed iron bar will show no magnetism if supported horizontally in an east-and-

west line. Now hold it in the direction of the magnetic meridian and dipping down in the direction indicated by a dip needle (Art. 51). The north end should be about 70 degrees below the horizontal. Strike the bar a sharp rap with a hammer. If it is now tested for polarity with a compass its north end will be found north-seeking and the south end, south-seeking. If now it is turned end for end and again rapped, the polarity will be reversed. The effects above described are due to the inductive action of the earth's magnetism. Natural magnets—lodestones—are doubtless produced in a similar manner.

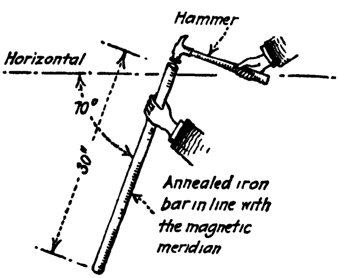


FIG. 42.—Magnetizing a rod by the earth's induction.

67. Consequent Poles are produced if a bar of magnetic metal be so magnetized at certain parts but not at others that the intermediate poles oppose one another as in Fig. 42A. Consequent

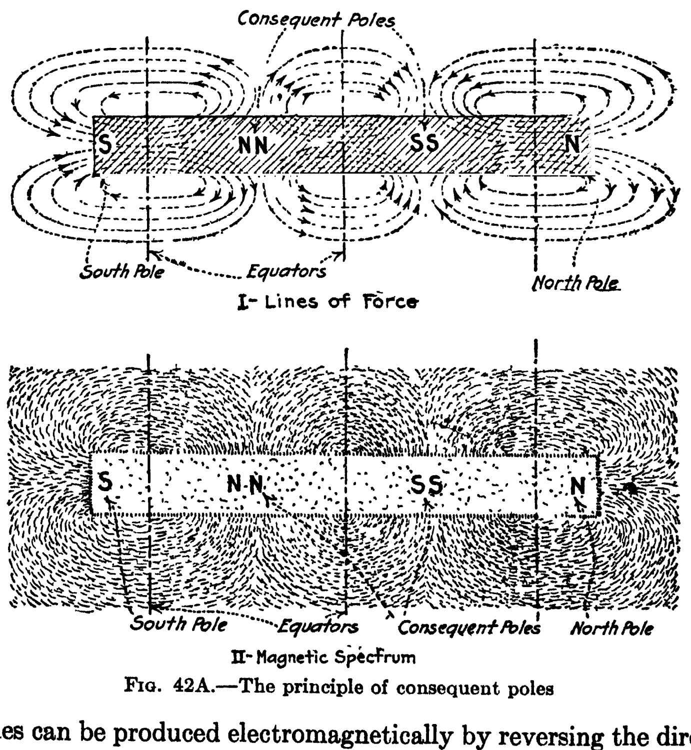


FIG. 42A.—The principle of consequent poles

poles can be produced electromagnetically by reversing the direction of the winding along a bar. It is evident from the illustra-

tion that such a magnet can have two or any greater number of poles.

68. The Explanation of Magnetization.—No explanation that offers an absolute proof has thus far been proposed. Probably, the *atoms* (Art. 12) of a magnetizable body are minute magnets —whether the body as a whole is magnetized or not. Before it is magnetized, their poles point almost every-which-way (Fig. 43,*I*). However, their arrangement into ring magnets (Art. 62) is such that they produce no external field. Increasing the magnetization causes their axes to become more nearly parallel as at *II*. When the body is fully magnetized or saturated (Art. 247) all of the north poles of the molecules point toward the north pole of the body (*III*) and their south poles point toward the south pole of the body. The molecules do not, however, line up quite as regularly as shown at *III*, because if they did the lines of force would pass out only through the ends of the bar. Magnetic spectra show us that some lines pass out of a bar magnet from its sides as well as from its ends. See Art. 218,A for an explanation as to how, on the basis of the electron theory of magnetism, the moving electrons in the atoms are assumed to produce magnetic poles.

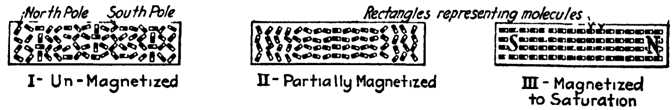


FIG. 43.—Illustrating the so-called "molecular theory" of magnetism.

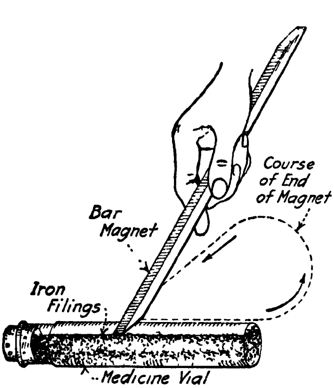


FIG. 44.—Illustrating the theory of magnetization.

NOTE.—It appears that the molecules of steel are harder to turn than those of iron, that is, steel is more difficult to magnetize. But when steel is magnetized it retains its magnetism better than does iron (see Art. 63, "*Permeance.*")

EXAMPLE.—This principle of *molecular magnetization* can be demonstrated with a medicine vial (Fig. 44) almost full of steel filings. If the bottle be thoroughly shaken and tested for polarity no magnetic effect can be observed. If now the filings be *polarized* by stroking the vial with a strong bar magnet and then tested, the bottlefull will have a north pole at one end

and a south pole at the other. If it be again shaken the polarity will be lost. If a knitting needle be magnetized and broken into bits, the smallest piece that it is possible to obtain will be a magnet—like the pieces of the broken bar of Fig. 35. Heating, jarring or any action that tends to disturb the molecular arrangement within a body may deprive it of its properties as a permanent magnet (Art. 80).

69. Uniform Magnetization or a uniform magnetic density is produced when every square-inch cross-section of a magnetized substance has exactly the same number of lines of force passing through it. When the number of lines of force is different through different square-inch cross-sections the magnetization is *non-uniform.*

NOTE.—Although it is seldom that an absolutely uniform magnetic density can be produced in a substance, it is often assumed, to facilitate computations that the magnetic density throughout a body is uniform.

70. The Flux of a Magnetic Field (this subject is discussed further in Art. 225 in the following section on "The Magnetic Circuit") is the total number of lines of force comprising the field or, in other words, the amount or quantity of magnetism in that field. *Total induction* is another term sometimes used that means the same thing. Flux is usually represented by the Greek letter ϕ (pronounced phi).

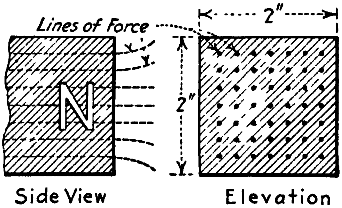


FIG. 45.—An imaginary magnetic flux of 49 lines.

EXAMPLE.—In Fig. 45 is represented, for explanation, the field of an imaginary magnet. It has a flux of 49 of these imaginary lines of force, as determined by counting them in the *Elevation* picture. Therefore, for this imaginary field:

$$\phi = 49 \text{ lines.}$$

71. The Maxwell Is One Unit of Magnetic Flux.—A maxwell is the amount of magnetism passing through every square centimeter of a field of unit intensity. This means that 1 maxwell = 1 line of force. The unit "maxwell" is seldom used in practical work it being more convenient to say "a flux of 10,000 lines" than "a flux of 10,000 maxwells." Further information relating to this situation is given in Art. 226 the following section on "The Magnetic Circuit."

NOTE.—Flux in a magnetic circuit is in many ways analogous to current in an electric circuit. See Art. 226. It should not be assumed that

magnetic flux and electric current are the same thing because they are not. They are entirely different phenomena. However, the laws that govern the development of flux in a magnetic circuit are precisely analogous to the laws governing the development of current in an electric circuit. The formulas for computing magnetic flux are very similar to those for computing electric current. The formulas for the two kinds of circuits are in many cases of identical form. But different letters denoting different, but analogous, quantities are used in the formulas for the two circuits.

72. The Field Intensity or Magnetic Intensity (also called magnetomotive gradient) at some designated point in a magnetic field is a measure of the ability of the field, at that point, to produce flux or lines of force. *Field intensity* is the cause; *flux density* (Arts. 73 and 246) is the effect. Note particularly that the term *field intensity* must relate to some specified point or location.

NOTE.—Field intensity at a certain location is also, stating the situation in another way, the magnetomotive force (Arts. 245 and 261) per unit length of path, at that location or point. Thus, field intensity is *magnetomotive force gradient* and may be expressed in ampere-turns (Art. 261) per inch length of path.

NOTE.—The term *field intensity* and its symbol H, relate only to the ability of a magnetic field to generate flux. A different term (Art. 73) is used to designate the density (flux lines or lines of force per square inch) of magnetic flux in magnetic circuits. Terms which are sometimes used interchangeably with "field intensity" are: *magnetizing force* (not magnetomotive force), *magnetic intensity, strength of field, intensity of magnetic field, field strength* and *field density.*

73. Flux Density is the number of lines of force per unit area passing through any substance through a plane at right angles to the direction of the flux lines. In practical work in the United States flux density is measured in *lines per square inch* and is usually designated by the capital letter B. It follows, therefore, that there is a certain definite relation between *flux* (Art. 70), *flux density* and *area,* thus:

(1) $$\textit{flux density} = \frac{\textit{flux}}{\textit{area}} \qquad \text{(lines per sq. in.)}$$

that is,

(2) $$\mathrm{B} = \frac{\phi}{A} \qquad \text{(lines per sq. in.)}$$

hence,

(3) $$\phi = A \times \mathrm{B} \qquad \text{(lines)}$$

and

(4) $$A = \frac{\phi}{\mathrm{B}} \qquad \text{(sq. in.)}$$

Wherein B = flux density, in lines per square inch, ϕ = the flux, or total number of lines of force, in the area of magnetic circuit under consideration. A = the area, in square inches, of the plane or surface under consideration, taken at right angles to the direction of the lines of force.

NOTE.—Terms which are sometimes used interchangeably with "flux density" are: magnetic induction, magnetic density and magnetism.

EXAMPLE.—What is the flux density in bars A and C of Fig. 46,*I*, assuming that the total flux is 80,000 lines and that it distributes itself uniformly in A and in C? SOLUTION.—Consider C first; it has an area of 2 in. $\times$ 2½ in. = 5 sq. in. Now substitute in the formula (2): $B = \phi \div A = 80{,}000 \div 5 =$ 16,000 *lines per sq. in.* Therefore, the flux density in C is 16,000 *lines of force per sq. in.*

Now consider A; its area is 1 in. $\times$ 1 in. = 1 sq. in. Substitute in the formula (2): $B = \phi \div A = 80{,}000 \div 1 = 80{,}000$ *lines per sq. in.* There-

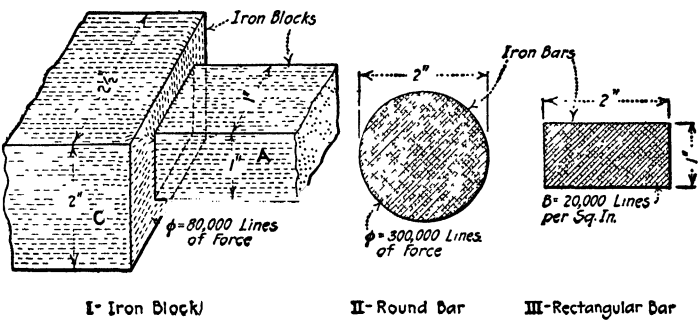


FIG. 46.—Magnetic flux in iron.

fore, the flux density in A is 80,000 lines per sq. in. Note that although there is the same flux of 80,000 lines in A and in C, the flux density in C is less than that in A because of the greater area of C.

EXAMPLE.—What is the flux density in the round bar of Fig. 46,*II* which carries a total flux of 300,000 lines? SOLUTION.—The area of a 2-in. round bar is 2 $\times$ 2 $\times$ 0.785 = 3.14 sq. in. Now substitute in the formula (2): $B = \phi \div A = 300{,}000 \div 3.14 = 95{,}600$ *lines per sq. in.* Hence the flux density, B, in this bar is 95,600 lines per sq. in.

EXAMPLE.—The flux density in the rectangular iron bar of Fig. 46,*III* is 20,000 lines per sq. in. What is the flux in the bar. SOLUTION.—The area of the bar = 2 in. $\times$ 1 in. = 2 sq. in. Substitute in the formula (3): $\phi = A \times B = 2 \times 20{,}000 = 40{,}000$ *lines.* Therefore, the total number of lines of force or the flux through this bar is 40,000 lines.

EXAMPLE.—In Fig. 47, where the flux (or total number of lines of force) is 49, what is the flux density at: (1) plane *DD*, where the field is ½ in. square; (2) at plane *FF*, where the field is 2 in. square; and (3) at plane *EE*, where the field is 1 in. square? SOLUTION.—(1) The area of the field at *DD* is ½

in. × ½ in. = ¼ or 0.25 in.; (2) the area of the field at *FF* is 2 *in.* × 2 *in.* = 4 sq. in.; (3) the area of the field at *EE* is 1 in. × 1 in. = 1 sq. in. Now substitute in the formula (2); $B = \phi \div A$:

(1) $B = \frac{49}{0.25} = 196$ *lines per sq. in.* = flux density at *DD*.

(2) $B = \frac{49}{4} = 12.3$ *lines per sq. in.* = flux density at *FF*.

(3) $B = \frac{49}{1} = 49$ *lines per sq. in.* = flux density at *EE*.

The values of this imaginary example show how flux densities may vary. In (2) above we get a fraction of a line (12.3) in our result for flux density. This should not confuse if it be remembered that a *line of force* is, as used here, a unit of measurement (Art. 60). Hence we can use a fraction of a line of force in figuring, just as we can use a fraction of an inch or of a pound.

EXAMPLE.—In a certain field the flux density is 400 lines per sq. in. and the area of the field, on a plane at right angles to it, is 48 sq. in. What is the

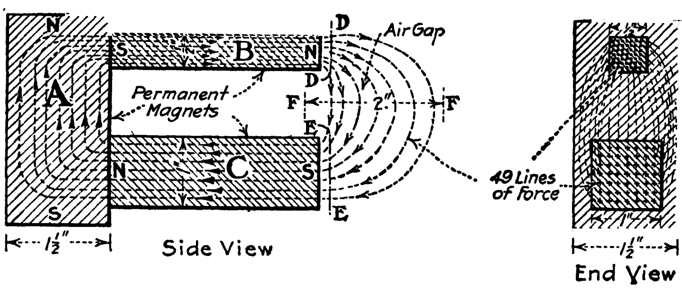


FIG. 47.—A simple magnetic circuit.

total flux? SOLUTION.—Substitute in the formula (3): $\phi = A \times B = 48 \times 400 = 19{,}200$ *lines* = total flux in field.

EXAMPLE.—A field of 36 sq. in. has a flux of 28,800 lines. What is its flux density? SOLUTION.—Substitute in the formula (2): $B = \phi \div A = 28{,}800 \div 36 = 800$ *lines per sq. in.* = flux density.

NOTE.—FLUX DENSITY in a magnetic circuit is analogous to current density (Art. 123A) in an electric circuit. For example, the flux density in the magnetic circuit of Fig. 46,*III* is 20,000 lines per sq. in. while in an electric circuit of the correct proportions the current density might be 20,000 amp. per sq. in. The flux density at different planes, cut through a magnetic circuit of varying cross-section, may be different regardless of the fact that the total flux is constant. Likewise, the current density in different parts of an electric circuit may be different at parts of the circuit.

74. The Gauss Is a Unit of Flux Density which is sometimes used. A flux density of 1 gauss is equivalent to 1 line of force per sq. cm. A field having a density of 10 gausses may be called a field of 10 *maxwells per sq. cm.* or simply "a field of 10 gausses."

A maxwell is equivalent to 1 line of force. The unit "gauss" is not much used in this country, outside of physical laboratories and text-books, it being much more convenient and quite as accurate in practical work to refer to *lines of force per square inch* or "lines per square inch."

75. Magnetic Force is the push with which two magnets repel each other or it is the pull with which they attract each other. Force can be measured in pounds or in any other unit of weight. Obviously, a magnetic force will be exerted between a magnet, and a magnetic substance that has become a temporary magnet by induction (Art. 64), the same as between two permanent magnets. Magnetic force is mutual (Fig. 39,*II*); magnet *A* attracts bar *B* by the same amount as *B* attracts *A*. Magnetic force, sometimes called *magnetic strength*, should not be confused with *lifting power* (Art. 85) which is an entirely different thing.

76. The Laws of Magnetic Force Are:

1. *Like magnetic poles repel one another; unlike magnetic poles attract one another.*
2. *The force exerted between two magnetic poles varies inversely as the square of the distance between them.*

The first law above is discussed in Art. 77. The second law is not strictly true in practice because it assumes that each of the poles is a mere point—a dot. It is, however, closely true when the magnets are not too close to one another and are long in proportion to their sectional areas. This second law can be stated as a formula thus:

$$F : f :: d^2 : D^2 \qquad (5)$$

or

$$\frac{F}{f} = \frac{d^2}{D^2} \qquad (6)$$

or

$$F = \frac{f \times d^2}{D^2} \qquad (7)$$

or

$$f = \frac{F \times D^2}{d^2} \qquad (8)$$

Wherein F = force exerted between two magnet poles in their first position. f = force between them in second position.

D = distance between poles in first position. d = distance between poles in second position.

EXAMPLE.—Assume bar magnets to be arranged as in Fig. 48 so that they are free to move on frictionless glass rollers and so that the pull between them can be measured by delicate spring balances. When the magnet ends are 4 in. apart the mutual pull between the magnets is 20 oz. (Fig. 48,*I*). How many ounces will it be with the magnet end 2 in. apart (Fig. 48,*II*)? SOLUTION.—Substitute in the formula (8): $f = (F \times D^2) \div d^2 = [20 \times (4 \times 4) \div (2 \times 2)] = 320 \div 4 = 80$ oz.

Therefore, with the magnets 2 in. apart the pull or force would be approximately 80 oz. It probably would not be exactly thus in practice because of certain errors, inherent in the formula and method, for which it is not feasible to correct.

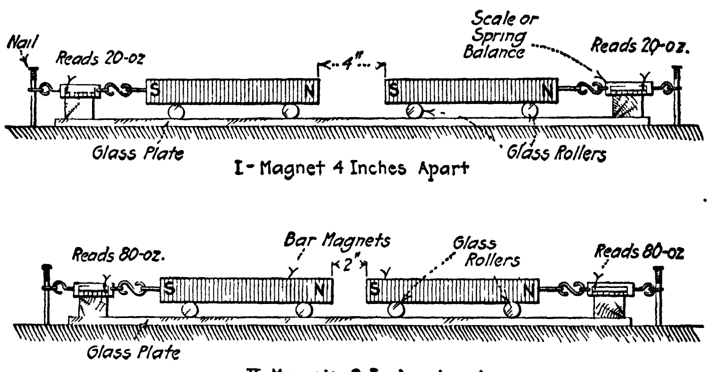


FIG. 48.—Illustrating law of force between two magnets.

77. Why Magnetic Force Varies as the Square of the Distance.—This law (Art. 76) is not strictly true in practice because it is based on the *assumption* that the lines of force from a pole face of a magnet radiate (Fig. 49) from a point or dot which lies within the magnet and which is considered the true location of the pole of the magnet. Actually the lines of force do not converge to a point within a magnet. The magnet face shown is 1 in. square—has an area of 1 sq. in. It is obvious that there are fewer of the lines of force passing through a square-inch area *ABCD*, located at some distance from the pole face, than there are through the square inch *EFGH*, of the face. This means that, since flux density determines the number of lines of force per square inch, the flux density at *ABCD* is much less than at *EFGH*. The further away from the pole one goes, the less will be the flux density. Where the lines radiate from a point, the number of

lines through a given area will vary inversely as the square of the distance that the area is *from the point*—not from the pole face. With a long magnet the lines of force will radiate from its poles almost as shown in Fig. 49 and with such a magnet this "*inversely as the square of the distance*" law will hold closely but not exactly true. However, with a short thick magnet the field of lines of force that is developed will be fairly uniform (Art. 61) for quite a distance from the pole face and then the inverse square law can not be applied with great accuracy.

78. Diamagnetism is an apparent property of certain materials (among them are copper, antimony, bismuth, phosphorus and some liquids) whereby they seem to be feebly repelled from the poles of a strong magnet. Such materials are called *diamagnetic*

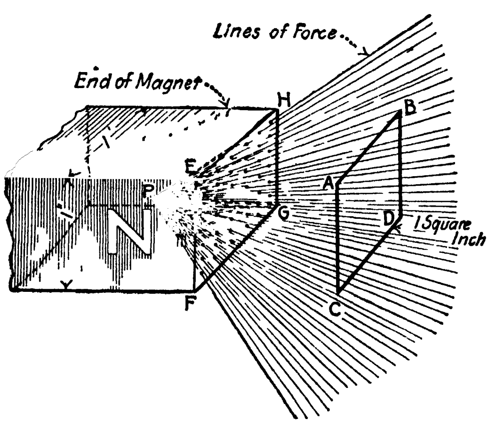


FIG. 49.—Showing the principle of the inverse-square law.

substances. The explanation of this property is, probably, that these materials have less permeance (Art. 63) than have air or magnetic substances. That is, a diamagnetic substance is one that does not conduct magnetism or lines of force as well as does air.

79. A Paramagnetic Substance is one that has greater permeance (Art. 63) than air or, in other words, conducts lines of force better than does air.

80. A Permanent Magnet Can Be Made by stroking the hard steel bar (Fig. 50,*I*) which it is desired to magnetize with another strong permanent magnet. The magnetization thus attained is relatively weak. Strong magnetization can be effected only by placing the bar in a strong, electromagnetic field. This can be

done by causing continuous current (Art. 217) to circulate around the bar (Fig. 50, *II*) or by placing it in the field of a generator (*III*) or of a specially designed magnet (*IV*).

NOTE.—*To make a permanent magnet:* bend the steel into the required form; it can be bent cold if the form required and its section permits it. To harden, heat to a cherry red and plunge into cold running water. Agitate violently while it is cooling. The tongs with which the magnet is to be immersed should be so designed that they will hold it in shape to prevent distortion. Holes should be bored wherever feasible through the jaws of the tongs so that cold water will reach as much of the magnet steel as possible. Where the utmost permanency of magnetization is desirable, ageing (Art. 82) is essential. Artificially aged tungsten-steel magnets used in the best measuring instruments maintain indefinitely an almost perfectly constant magnetization. The most expensive steels do not necessarily make the best permanent magnets.

81. Forms of Permanent Magnets are shown in Fig. 38. When the steel is bent into a closed or nearly closed shape as at *I* and

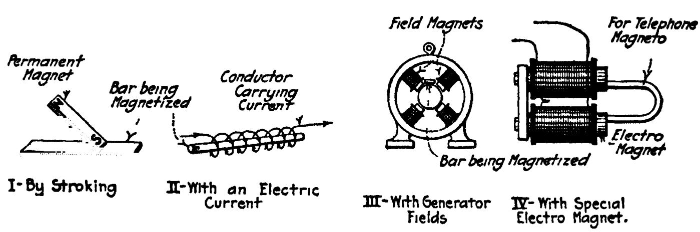


FIG. 50.—Methods of magnetizing permanent magnets.

II the magnet will retain its magnetization much better than if it is in a bar or unclosed form. Thin magnets are stronger in proportion to their weights than are thick ones. It follows that a *compound magnet* (*III*) composed of several thin bars clamped together is more effective than a solid one of the same weight.

82. The Ageing of Magnet Steel whereby it will retain practically fixed magnetization indefinitely, can be accomplished thus: Bend the steel to its ultimate form and temper it to maximum hardness. Heat treat (heat) it in steam for about 40 hr. at 212 deg. F. Magnetize the steel as strongly as possible and give it another similar heat treatment for 8 hr. The reheating effects some de-magnetization but the magnetization that remains will be about permanent for all practical purposes.

83. Rough Treatment Weakens Magnets.—If a magnetized darning needle or any other magnet be heated red hot and al-

lowed to cool it will be found to have lost its magnetism. If a magnetized darning needle be tested for strength by counting the number of tacks it will lift and then vibrated against a table it will be found that it has lost much of its strength. If a magnet is dropped or jarred, it will lose strength. Each time the keeper of an ordinary horseshoe magnet is removed and replaced some of its strength is lost. Obviously, any delicate instrument containing a permanent magnet must not be jarred or subjected to extreme changes in temperature. Any condition that tends to alter the internal molecular or atomic structure (Art. 3.) of a permanent magnet should be avoided.

NOTE.—Where the form of a permanent magnet permits it, a soft iron *armature* or *keeper* should be placed across the poles of the magnet when it is not in use. When this is done, there will be an iron magnetic circuit for the

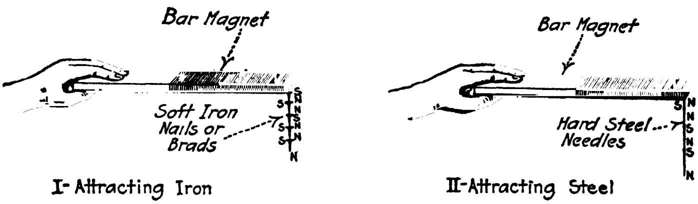


FIG. 51.—Illustrating retentivity.

lines of force to follow, which will tend to maintain the strength of the magnet constant.

84. Retentivity Is Ability to Retain Magnetism. See Art. 45. —Hard steel will retain magnetism very much better than will soft steel. The harder the steel the greater the retentivity. *Pure* soft iron will not retain it at all. This principle can be demonstrated as suggested in Fig. 51. Hang as many soft nails as it will support by its attraction on the end of a bar magnet. Each nail has now become (by induction, Art. 64) a little magnet for the time being. Detach all of the nails. When tested, none will be found to have much magnetism. If the same experiment is tried with needles, which are always of hard steel, each needle will be found to retain considerable magnetism. It follows that hard steel is the only substance suitable for strong permanent magnets. Cast iron and impure wrought iron retain some magnetism. Form affects retentivity. Cubes, short rods and balls will not retain nearly so well as will long rods bent into closed or nearly closed circuits. The keeper on a horseshoe magnet (Fig.

38,*II*) promotes its retentivity. Magnets of practically unvarying strength can be made (Art. 82).

85. The Lifting Power of a Permanent Magnet, sometimes called its *portative force* is difficult to calculate accurately. It depends on: (1) *the quality of the steel,* (2) *its shape,* (3) *the shape and condition of the surface of the object attracted and of the attracting surface* and (4) *the size or weight of the magnet,* it being assumed that the magnet is magnetized as intensely as can be.

NOTE.—A horseshoe magnet will usually lift three or four times as great a weight as will a bar magnet of the same weight. Chamfering the ends or poles of a magnet increases its lifting power (Art. 293). Small magnets will lift more in proportion to their weights than will large ones. A lifting power of 40 lb. per sq in. of pole surface is a splendid performance for a steel magnet. A good horseshoe magnet weighing 1 lb. should lift 25 lb.

SECTION 3

FUNDAMENTAL IDEAS CONCERNING ELECTRICITY

86. The Exact Nature of Electricity has not been definitely determined. While the electron theory (Art. 4) offers explanations for many things which could not be consistently explained on any basis before the development of this theory, it does not tell precisely what electricity is. Electric phenomena are, probably, due to the movement of electrons. But it is not known what an electron is. As previously suggested there is a close relation* between electric light and heat waves (they all travel at the same rate, viz., 186,000 miles per sec.). Also, it is a matter of common observation that electrical energy can be converted into heat and into light.

87. Theories of Electricity. The Electron Theory.—Various theories have in the past been proposed to account for electrical phenomena. None was altogether satisfactory. However, the now generally accepted electron theory offers, as above suggested, rational explanations for nearly all electrical actions. The electron theory is certainly in the right direction—even if it is, as now understood, not absolutely correct in all respects. For the purposes of this book it has seemed preferable to sometimes consider electricity as analogous to a fluid, as explained in Art. 90, rather than endeavor to base explanations wholly on the electron theory. But interpretations on the basis of the electron theory will also, where feasible, be made. The fluid analogy concept does not conflict with the electron theory and it—at least so the author believes—offers a more effective medium for the simple explanation of certain electrical phenomena.

88. It is Not Necessary for One to Know Exactly What Electricity Is in order to be able to utilize electrical phenomena to serve his ends. It is obvious that it is not necessary for one to know exactly what water is when he uses the energy of falling water to develop power. It is of no great moment in this case that water is composed of two gases. It *is*, however, necessary

* See the author's PRACTICAL ELECTRIC ILLUMINATION for a much more extended discussion of this situation

to know *how water acts* and how it may be used in the development of power. The hydraulic engineer must know how to utilize water practically, to make it do work for him but he does not have to know the ultimate constitution of water.

Likewise, in the development of the ideas of practical electricity in this book, the question of what electricity is will not be discussed at great length. But the descriptions of how it acts and how it can be directed and controlled so as to do useful

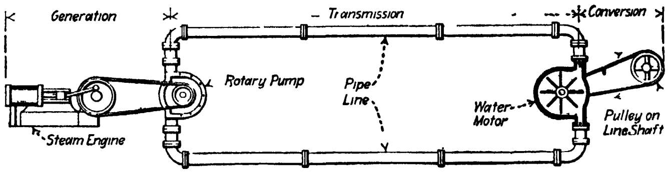


FIG. 52.—Transmitting energy with a current of water.

work—ring bells, furnish illumination, transmit energy and do similar things—will be treated at some length.

89. Electricity May Best Be Thought of as a Conveyor of Energy.—As a weightless medium which can carry energy just as can water (Fig. 52) or air. The laws which govern the flow of electricity in closed circuits are in general similar to those that determine the flow of water or air in water or air circuits. Electricity is not energy (Art. 169) any more than the water flowing

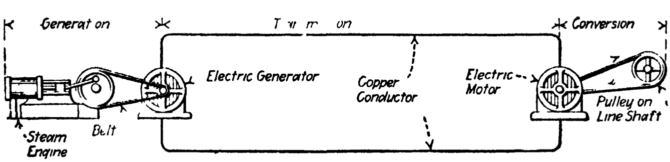


FIG. 53.—Transmitting energy with a current of electricity.

under pressure in the pipe line of Fig. 52 is energy. Water is matter; see Art. 2. The water flowing in the pipe, Fig. 52, is a medium for transmitting energy and so is the electricity flowing in the conductor of Fig. 53 a means of transmitting energy.

NOTE.—The energy developed by the steam engine, Fig. 52, is transmitted to the rotary pump by means of the belt. The rotary pump forces the water around through the pipe circuit to turn a water motor, which, by means of another belt, drives a line shaft. Thus all of these have been mediums in the transmission of energy from the engine to the line shaft. (1) *a belt,* (2) *a pump,* (3) *a current of water,* (4) *a water motor* and (5) *another belt.* In Fig. 53

an electric generator or dynamo is substituted for the rotary pump, electricity conductors (copper wires) for the pipe line and an electric motor for the water motor. In Fig. 53, electricity instead of water is the medium by means of which energy is transmitted over the long distance; otherwise, the two transmission systems are somewhat similar. In either Fig. 52 or Fig. 53 a long belt might have been arranged between the engine and the line shaft pulley and it would transmit the energy as do water or electricity—though possibly not as efficiently. (Obviously, belt transmission over any great distance is not feasible.) These illustrations have been given to show that electricity is merely a medium for the transmission of energy and that it is *not* energy.

90. As to What Electricity Is Like, it will, in the discussions which follow, be of great assistance if electricity is likened to something with which everyone is familiar and which can be readily comprehended. This will enable the reader to acquire a definite physical conception of things. For the purposes of this book it can, then, be stated that electricity is a something permeating everything. This follows from the idea (Art. 3) that everything—all matter—is composed of electrons, which are particles of electricity. *Electricity acts as if it were a weightless invisible, non-compressible fluid permeating all space—saturating everything.* Note particularly that electricity is *not* a fluid—this fact is known definitely. However, it may, for our purposes of explanation, be considered as acting like the imaginary special kind of fluid described above.

NOTE.*—We appear to be getting back to Franklin's single-fluid theory of electricity. The electric current is believed to be a movement of electrons through a conductor, from the negative to the positive instead of flowing from the positive to the negative as was formerly supposed. Many scientists believe that negative electricity is the only kind. "Positive" electricity arises from a lack of electrons. For example a positively charged atom of the metal helium is merely a helium atom that has temporarily lost two of its electrons. In theory the electron is not a particle negatively charged but is in itself a negative charge. This is equivalent to saying that matter is wholly electrical.

The electron enters into the structure of the atom, but is the weight of the atom due entirely to the mass of the electron? It is claimed that the electron has no weight in itself. Its apparent weight is due to the adhering æther (Art. 35) which it drags along as it shoots through space. It is like stirring a bucket of fluid with a cane, a small quantity of the fluid will temporarily adhere to the cane and be carried along by it.

91. The Construction of Matter Which Must Be Assumed if Electricity Is Considered as Being Analogous to a Fluid Filling

* J. A Culler, GENERAL PHYSICS.

All Space is suggested in Figs. 54 to 58. On this basis it must be assumed that every kind of matter is composed of a honeycomblike structure comprising minute cells (Fig. 54). The cells are all filled with the imaginary fluid which is analogous to electricity. The walls of the cells are composed of an elastic material. But cell walls in different kinds of materials are not the same. There is a difference between the material of the cell walls of conductors (Art. 93) and that of non-conductors or insulators (Art. 92) as will now be explained:

92. In the Imaginary Cell Walls of Insulating Substances (non-conductors or dielectrics, Art. 131) the cell-wall material is assumed to be (Fig. 54,*I*) an elastic semi-porous membrane,

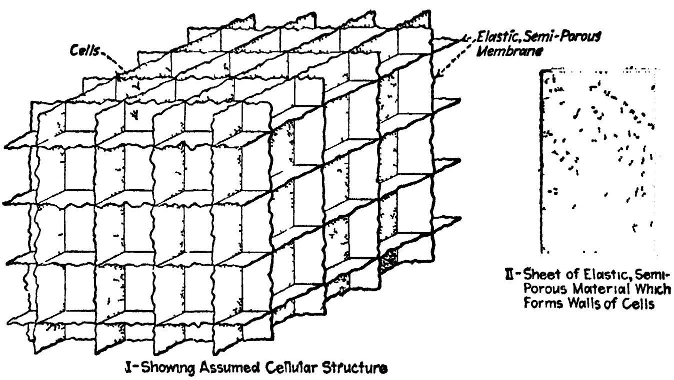


FIG 54.—Illustrating the hypothetical or imaginary structure of an insulating or dielectric material (cellular structure, walls of cells are elastic and of a material through which the electricity—"electric fluid"— can slowly soak).

something like semi-porous sheet rubber—if there could be such a thing. Thus, a block of any insulating substance (such as air, hard rubber, glass or fiber) may be thought of as being composed wholly of thousands of little cells all having semi-porous, elastic walls, each of the cells being entirely filled with an imaginary electric fluid. When an electric pressure (voltage, Art. 117) is impressed on such a block of insulating material the cell walls will be caused to stretch in the direction of the electric pressure. But there will be practically no flow of the electricity fluid through the block because the membrane-like cell walls will, largely, prevent such flow. There will be a displacement of the fluid in the direction of the pressure, because the walls are elastic but practically no actual flow through the walls.

However, since no substance is a perfect insulator of electricity (Art. 124) it must be assumed that the cell walls in the so-called insulating materials are semi-porous. The cell walls in the materials which are the best insulators would be almost impervious to the flow of the fluid; they would have little porosity. Hence, when the fluid in them was subjected to an electric pressure, practically no current of electricity fluid would flow through the walls, but there would be some flow. Poor insulating materials would have cell walls which would be quite porous—such walls might be thought of as being composed of an elastic membrane perforated with many minute holes. Thus, ranging between the very porous elastic membranes composing the cell

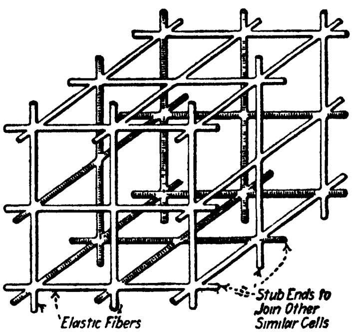


FIG. 55.—One imaginary cell from a "very good conductor."

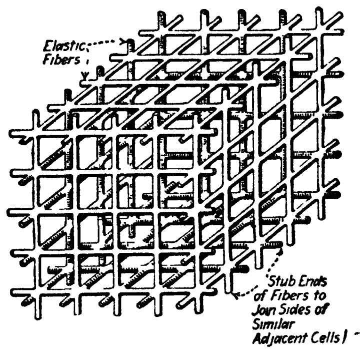


FIG. 56.—One imaginary cell from a "good conductor." (Walls of cell composed of elastic fibers woven or joined into an open tissue.)

walls of poor insulators and the almost impervious membranes composing the walls of the best insulators, there would be wall membranes of every degree of porosity.

NOTE.—If the electric pressure, which is impressed on a block of these imaginary cells, composing an insulating material, were so great that the cell walls would be stretched excessively, then they would rupture and there would be an actual flow of the electric fluid. There would be a disruptive discharge. This is what occurs when there is a lightning discharge through the atmosphere. The air, an insulating material, is broken down by an excessive electric pressure and the lightning-discharge current flows.

92A. Insulating Substances on the Basis of the Electron Theory, are believed to be those substances in which the electrons are held tightly bound in the atoms. Thus electric conduction currents (Art. 104) can not flow readily in an insulating

substance—because its elections can not move from atom to atom. Remember that there can be an electric current only when electrons are moved. However the electrons in the atoms of insulating substances can, probably, be moved or shifted

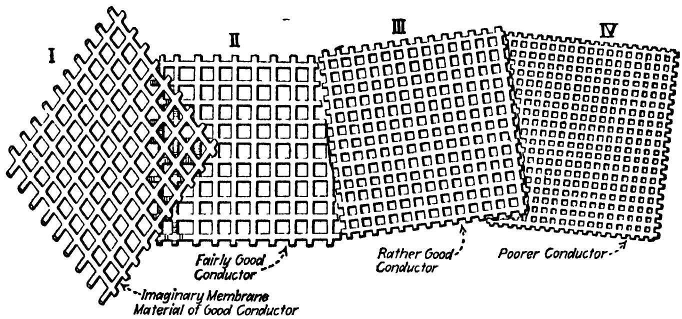


FIG. 57.—Membranes which may be imagined as composing the walls of the cells of which conducting materials may be assumed to be composed.

within the atoms themselves through very short distances, which accounts for the fact that displacement currents (Art. 105) can exist in insulating substances.

93. The Imaginary Cell Walls of Conducting Materials, such as the metals, would be composed of elastic fabrics resembling

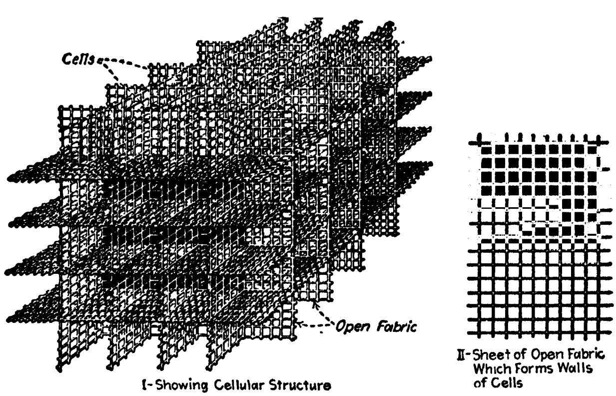


FIG. 58.—Illustrating the hypothetical or imaginary structure of fairly good conducting material.

netting as shown in Fig. 58. While these fabric walls would not prevent the flow of the electric "fluid" they would tend to restrict it. In the materials which are the best conductors, copper or silver for example, the mesh would be very coarse, possibly as

suggested in Fig. 55. In the materials which are the poorest conductors the mesh would be considerably finer as suggested in Fig. 57. Thus, the ability of the material to restrict the flow of the electricity fluid through it—its electrical resistance—would be determined by the closeness of the weave of the cell-wall material. A material having in its cell walls a fabric of very coarse weave (Fig. 55) would have a low—very little—electrical resistance. A material having in its cell walls a fabric of exceedingly fine weave would have a high resistance. In fact, the weave might be so fine that the fabric would be merely a semi-porous membrane, which would put the material into the insulator class described in the preceding article. Obviously, there may be fabrics of many varying degrees of fineness of weave (Fig. 57). Thus, there are materials of many degrees of resistance.

93A. Conducting Substances on the Basis of the Electron Theory are believed to be those substances in which the electrons are held rather loosely in their atoms and can therefore be moved, with relative ease, from atom to atom by the application of an electric pressure or voltage. Thus conduction currents (Art. 104A) flow readily in these substances.

94. Electricity Can Not Be Generated by a dynamo, a battery or by any other device. Contrivances such as batteries (Art. 326) and generators (Art. 509), should be regarded merely as arrangements whereby electricity which is already in existence (electricity fills all space, Art. 90) may be forced to move. All matter—the earth, the envelope of air surrounding it and all the other things on it and in it—may be regarded as an enormous reservoir of electricity. The electricity in these things can, under suitable conditions, be made to move. And when electricity moves there is an electric current. The real function then of a generator or a battery is to furnish a pressure, electromotive force or voltage (Art. 102) which will cause electricity to shift or move.

NOTE.—An electric generator may be thought of as an electricity force pump (Fig. 53) which forces electricity to circulate around in a circuit just as a hydraulic force pump may force water to circulate around in a hydraulic circuit.

95. Generation, Transmission and Conversion of Electrical Energy.—While electricity can not be generated, it is entirely proper to speak of the generation of electrical energy, which subject is discussed at some length under Art. 178. The study of

practical electricity may be conveniently thought of as being divided into the three sub-subjects noted in the heading to this article because where electrical energy is used it is always: (*a*) *generated* by some means or other, (*b*) *transmitted* to the location where it is to be utilized, and it is then (*c*) *converted* into heat, light, mechanical power or into some other agent.

Example.—Even in one of the simplest electric circuits, a vibrating-bell circuit (Fig. 59), the electrical energy is (*a*) *generated* by the dry cell, (*b*) *transmitted* over the small-wire circuit and (*c*) *converted* into sound by the electric bell. Figs. 52 and 53 also show examples of this generation-transmission-and-conversion idea.

96. Potential means electrical level as the word "level" is used in hydraulics and is conveniently substituted for the indefinite term "electrical condition." (Potential may be thought of as analogous to: (1) *pressure* of gases, (2) *head* or *level* of liquids

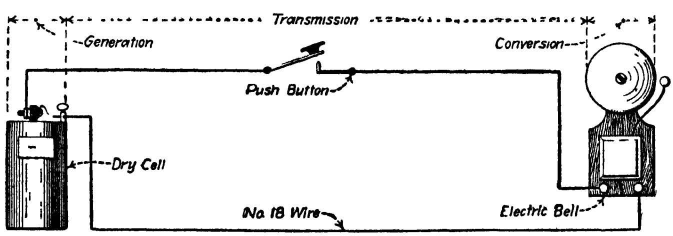


Fig. 59.—Generation and transmission of electrical energy to an electric bell.

and (3) *temperature* of heat.) Electricity always flows—or tends to flow—from points of higher potential to points of lower potential. Hydraulic pressure, or head, due to differences of water level, causes water to flow. Pressures (voltages, Art. 102) due to differences in electrical potential cause electricity to flow. Differences in electrical potentials can be measured in volts (Art. 120). If the difference of potential (or voltage, Art. 120) between two points is great there will be a great tendency to produce a flow of current between them. If the potential difference is small, there will be little tendency. If their potentials are equal, there will be no tendency.

In dealing with heat it is necessary to adopt some starting or reference point for the measurement of temperatures. It has been universally agreed that in the centigrade system we will call the temperature of freezing water "zero" temperature—for no other reasons except that it is convenient and that the thing

started that way. All temperatures hotter than that of freezing water are referred to as "above zero." All temperatures colder than that of freezing water are referred to as "below zero." Some other arbitrary standard, as for instance the temperature of boiling water, might have been adopted and called zero and other temperatures measured as above or below that. However, the freezing-water standard *was* adopted, every one understands it, it is convenient; hence we continue to use it. Likewise, in dealing with hydraulic levels or electrical potentials, it is also necessary to indicate or assume some reference level or potential.

EXAMPLE.—The surface of the water in the reservoir (Fig. 60) is 200 ft. above the sea level and this difference in level is equivalent to 87 lb. per sq. in. pressure as shown by pressure gage *B*. The sea level is the reference level

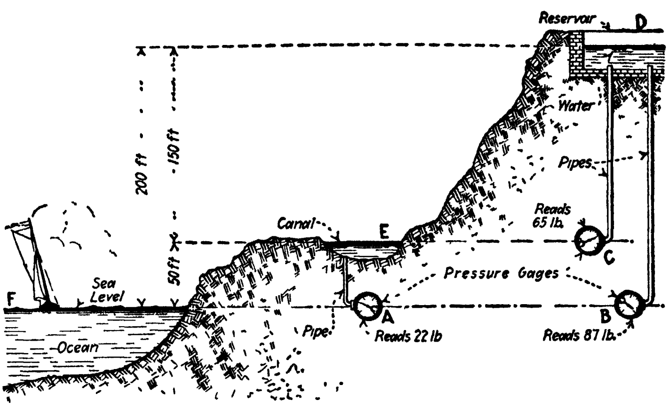


FIG. 60—Illustrating the meaning of the term "potential."

in this case. Or the reservoir level may be 150 ft. above the level of the water in the canal, which is equivalent to 65 lb. per sq. in. pressure as shown by gage *C*. The level of the canal is 50 ft. above that of the sea and the equivalent pressure is 22 lb. per sq. in. on gage *A*. It is apparent that any level must be referred to as being above or below some reference level.

Returning to electricity: *Level* is analogous to potential; pounds *per sq. in.* is analogous to *volts* (Art. 120). As a convenient standard reference potential, zero is usually taken as the potential of the earth's surface. Therefore, the potential of a body or of some point may be 87 volts above that of the earth or it may be but 65 volts above the potential of another body that has a potential 22 volts above that of the earth. The potential of the surface of the earth has, then, been arbitrarily

taken as zero potential and it is the standard reference potential unless otherwise stated.

NOTE.—Since the earth is, in general, a good conductor, all portions of it must be at the same potential Mud, water, rocks, mountains, valleys and the like may affect this somewhat, so that all points on the earth's surface may not be at *exactly* the same potential. This difference of potential at different points does not interfere with its convenience as a standard. For a thorough earth or ground connection connect to some extensive underground piping system.

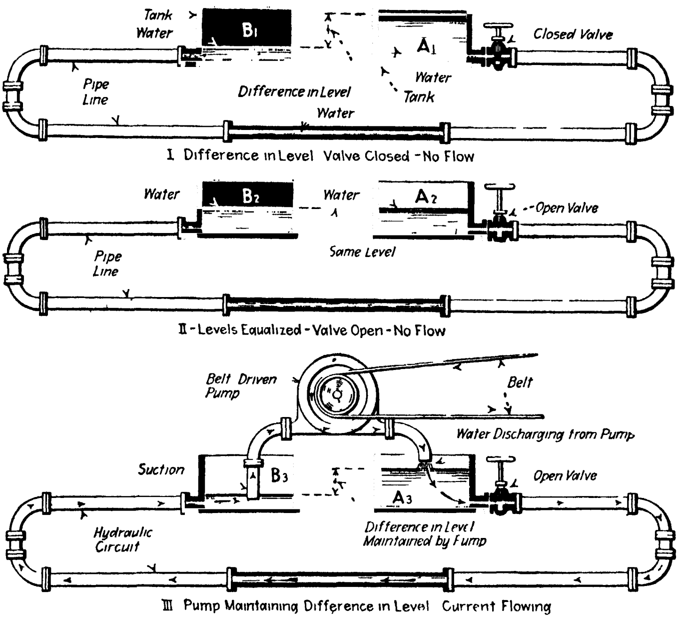


FIG. 60A.—Production of a current of water in a hydraulic circuit.

97. Whenever Two Points at Different Electric Potentials Are Connected by a Conductor, a transference of electricity occurs through the conductor, producing an electric current. But if by any suitable arrangement (Fig. 60A shows an analogy) such as a generator or a battery the difference of potential be maintained constant, the flow of electricity—the current—is continuous.

97A. The Terms Positive Polarity and Negative Polarity or Positive Terminal and Negative Terminal are best defined on the basis of the concept of potential. An electric current always flows from a point of higher potential to a point of lower potential

(Art. 96). It follows from this that the positive pole or terminal, P_D (that is, the binding post to which the line wire is connected), of an electrical-energy-delivering device (D, Fig. 60B), is the terminal from which the current flows from that device to the external circuit and its negative terminal, N_D, is the one into which the current flows from the external circuit. For energy-receiving or energy-consuming devices, the positive terminal, P_R, is the one into which the current flows from the external circuit and the negative terminal, N_R, is the one from which the current flows to the external circuit.

NOTE.—Since, Art. 36C, the electron current actually flows from negative to positive, the positive terminal of a delivering device is the terminal of that device into which the electrons really flow, while the negative terminal of a delivering device is the terminal out of which the electrons flow.

NOTE.—Refer to Arts. 112 and 552 for the definition of the positive direction and the negative direction of current. See Art. 203 for the method of

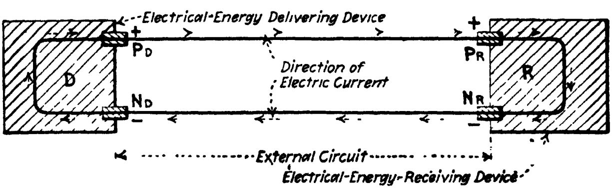


FIG. 60B.—Illustrating the meaning of positive and negative terminals ("+" denotes positive terminals and " − ", negative terminals).

determining polarities. In Art. 334 find an explanation of positive and negative polarities as the terms are applied to cells and batteries. In Art. 521 the meaning of positive and negative direction of rotation is explained.

98. Atmospheric Electricity.—The atmosphere (that is, the minute particles of gases, vapors and other matter composing it) surrounding the earth is always electrically "charged" that is it is always in a state of electric stress. This condition is, possibly, due to some extent to the evaporation from the oceans and other bodies of water. However, it has been shown that the earth (which is negatively charged) is continually discharging into the atmosphere, which is an exceedingly poor conductor. This rate of discharge, though infinitesimal for a square foot or even a square mile of the earth's surface, has been computed to be a constant current of over 1,000 amp. (Art. 122) for the entire earth's surface. No satisfactory explanation as to the source of this supply of negative electricity has been advanced. In fact, the phenomenon of atmospheric electricity has never

been explained. Thunder and lighting storms are, in some way or other, associated with atmospheric electricity.

NOTE —Observers have noted in certain cases a difference of potential of as much as 20 volts between points a foot apart vertically in the atmosphere. A difference of potential as great as 200 volts per ft has been recorded Usually, in fine weather, the atmosphere is positively charged in relation to the earth's surface During rainy weather it is usually negative but may be positive

99. The Aurora Borealis (Northern Lights) and the Aurora Australis (Southern Lights) are phenomena which are observed (Fig 61) almost nightly within the Arctic and Antarctic circles, that is, near the two poles of the earth. Further away from the poles they are seen occasionally. The northern lights and the southern

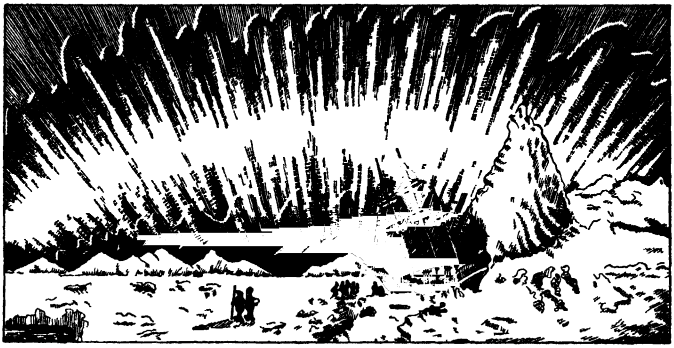


FIG 61 —The Aurora Borealis or Northern Lights

lights appear to be the same phenomena except as to location. Frequently the Aurora appears as a number of dim streaks or streams of pale light. There may be a tinge of red or other colors in the streams. The streaks radiate into a fan-like form as shown in the illustration. Often a trembling, flowing motion is observable in the streaks. The Aurora sometimes extends over the entire sky. When the Aurora appears, compass needles and telegraph lines are affected—sometimes over extensive areas. It appears that the Aurora often does not affect the atmospheric electrical conditions over any portions of the globe except those near the poles.

THE EXPLANATION OF THE AURORA ON THE BASIS OF THE ELECTRON THEORY is this It is believed that electrons are constantly being thrown off by the sun. Electrons are discharged by all incandescent bodies. This

stream of electrons really constitutes a super-enormous stream of cathode rays (Art. 6). Cathode rays are visible in a partial vacuum—like that which surrounds the atmosphere which envelops the earth—but they are not visible in the atmosphere. The Aurora, it is likely, is due to streams of these cathode rays entering the poles of the earth. Cathode-ray streams are deflected by a magnetic field. This is, probably, the reason why the rays appear only near the poles of the earth. The earth is a magnet (Art. 51) and its magnetic field, so it has been suggested, prevents the cathode streams from approaching the earth except near its poles.

SECTION 4

CURRENTS OF ELECTRICITY

100. Electricity in Motion Is Called an Electric Current.—Electric currents are of immense commercial importance some of their useful properties being (see also Fig. 4):

(*a*) Their ability to transmit power, almost instantaneously, along metallic conductors of great length.

(*b*) Their ability to magnetize iron and steel when conducted around the metal; iron temporarily and steel permanently. See Art. 204, "*Electromagnetism.*" In Fig. 62,*I* a bar of iron is shown that is so magnetized by a current of electricity in the wire wrapped around it that it supports a weight.

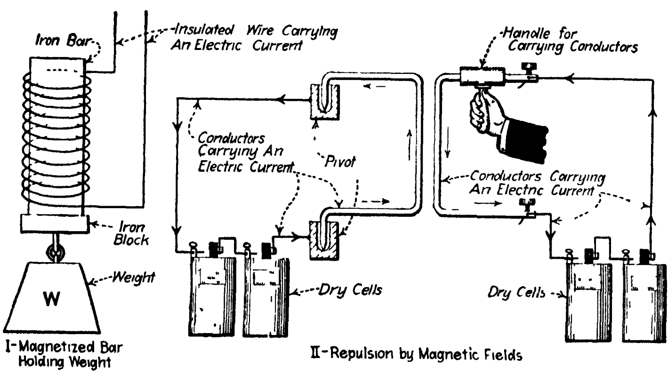


FIG. 62.—Some effects of current electricity.

(*c*) Its ability to create *magnetic fields* (Art. 61) which may, according to the direction of current flow, either attract or repel other magnetic fields. If two conductors carrying current are arranged as shown in Fig. 62,*II*, magnetic fields will be created about both conductors by the current of electricity. If the loop with the handle be brought near to the pivoted loop, the pivoted loop will deflect or rotate. It is repelled by the interaction of the magnetic fields produced by the currents flowing through the two loops. This experiment indicates, in an elementary way, the principle of operation of the electric motor.

(*d*) Its ability to generate an *induced current* in a neighboring circuit by its own variation. If the key shown in Fig. 63,*I* be pressed, an instantaneous current of electricity will be induced in loop *B* and its presence will be indicated by a momentary flutter of the delicate measuring instrument shown.

When the key is released, another instantaneous electrical current will be induced in B but it will be in an opposite direction from the first. See Art. 454.

(e) Its ability to heat conductors of high resistance (Art. 155) to incandescence. Electricity flowing in the filament of carbon or of tungsten in an incandescent lamp heats it as shown in Fig. 63,*II* and the white-hot filament gives forth light.

(f) Its ability to transfer metal by electro-chemical action, from one of two metallic plates in a conducting solution, to the other plate. In Fig. 63,*III*, electricity flowing from A to B will "eat" metal away from A and deposit it on B. This illustrates the principle of electroplating (Art. 384), of electrotyping (Art. 385), of electrolysis of underground pipes (Art. 383) and of other electro-chemical phenomena.

101. An Electric Current is electricity (electrons) in motion. A current of water in a pipe is water in motion in that pipe.

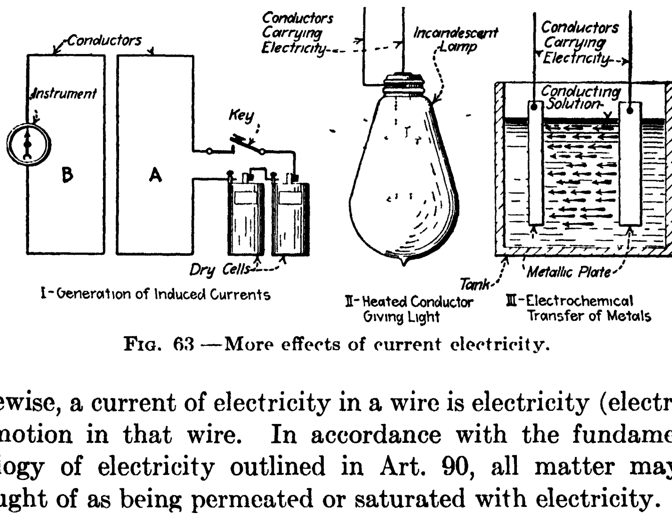


FIG. 63.—More effects of current electricity.

Likewise, a current of electricity in a wire is electricity (electrons) in motion in that wire. In accordance with the fundamental analogy of electricity outlined in Art. 90, all matter may be thought of as being permeated or saturated with electricity. An electric current is established when this electricity is caused to move.

102. An Electromotive Force (E.m.f.) or Voltage is the name which has been given to that force which causes electricity to move—which drives or impels electricity. E.m.fs. and the unit, the volt, in which they are measured are discussed in detail in Art. 120. For the present note that if an e.m.f. is applied to a closed or complete conducting circuit, it (the e.m.f.) will force the electricity already in the metal or material of that circuit to circulate or flow around in the circuit. An analogous phenomenon occurs if a hydraulic pump, which is developing a hydraulic

pressure, is connected into a hydraulic or pipe circuit already full of water. The pump will cause the water to circulate or flow around in the circuit. The result is a current of water. If the pressure exerted by the pump is discontinued, the current of water will cease to flow around in the pipe circuit. When the source of e.m.f. (electromotive force) is disconnected from an electric circuit, the electricity which was flowing therein will then cease to move. That is, there will then no longer be an electric current. It must now be obvious to the reader how the term e.m.f. originated.

NOTE.—Think of an e.m.f. as something intangible—merely a force which pushes the electrons along, something like the force exerted by a pump. You see the effects of the force and know that it is there but you cannot see the force.

103. All Electric Currents May Be Classified into Three General Divisions.—An electric current was defined in Art. 101 as electricity in motion. Whenever electricity is moved then this movement constitutes an electric current. There are, as will be shown in following articles, three well-defined types or classes of electricity movement. It follows, therefore, that there may be three different sorts of electric currents. They are:

(a) *Conduction currents.*
(b) *Displacement currents.*
(c) *Convection currents.*

These three different sorts may be further classified into subclasses as described in Arts. 110 to 116. The three different kinds of currents, *a*, *b* and *c*, are tabulated above in the order of their probable importance to readers of this book. Each is treated in a following article. The intensity of any electric current can be measured in amperes, Art. 122.

104. A Conduction Current is an electric current which will flow in a conductor, which forms a closed circuit, if some source of e.m.f. is inserted in that circuit. As long as the e.m.f. is impressed on the circuit the conduction current will flow. When the e.m.f. is removed, the current will cease. A water current in a pipe circuit, Fig. 52, may be likened to an electric conduction current, in an electric conductor, Fig. 53. Hence, any current in a conductor is a conduction current. It follows that most of the electric currents with which the practical man deals are conduction currents since most of the currents in which he is interested are conveyed by wires.

104A. The Explanation of a Conduction Current on the Basis of the Electron Theory is, probably, something like that diagrammed in Fig. 63A. Where *AB* is supposed to be a conductor substance composed of atoms (which of course are enlarged many, many thousand times in the illustration), each atom comprising electrons revolving about the center of the atom. Now assume that an e.m.f. or voltage is impressed across *A,B*. There will, then, be a tendency for the transference of electrons—an electron current (Art. 36C)—from *A* to *B*.

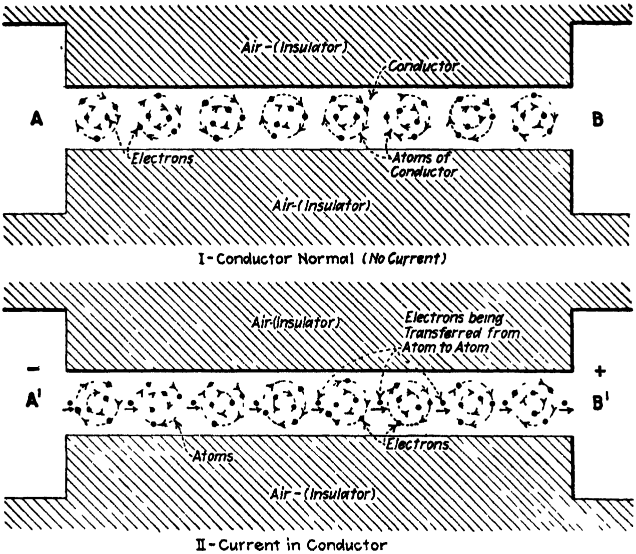


FIG. 63A.—Diagrammatic representation of the phenomena of an electron current in a conductor.

EXPLANATION.—This transference probably occurs somewhat as shown at Fig. 63A,*II*, wherein electrons are forced from atom to atom, from *A* to object *B*. Fig. 63B, an enlarged diagram, illustrates in more detail, the probable process of the transference of electrons between atoms. To appreciate how this transfer occurs, imagine a row of boys (Fig. 63C) standing between two boxes of baseballs, *D* and *E*. Assume that each boy has one baseball in one of his hands and in addition one in each of his four pockets. That is, each boy is holding five balls. By stretching our imaginations, we can liken each of the boys with the five balls he is holding, to an atom of a conductor. The five balls correspond to the five constituent electrons of an atom.

Now to start a current of baseballs along through this series of boys, boy *G* would, with his left hand, place one of his balls in box *E* and at the same

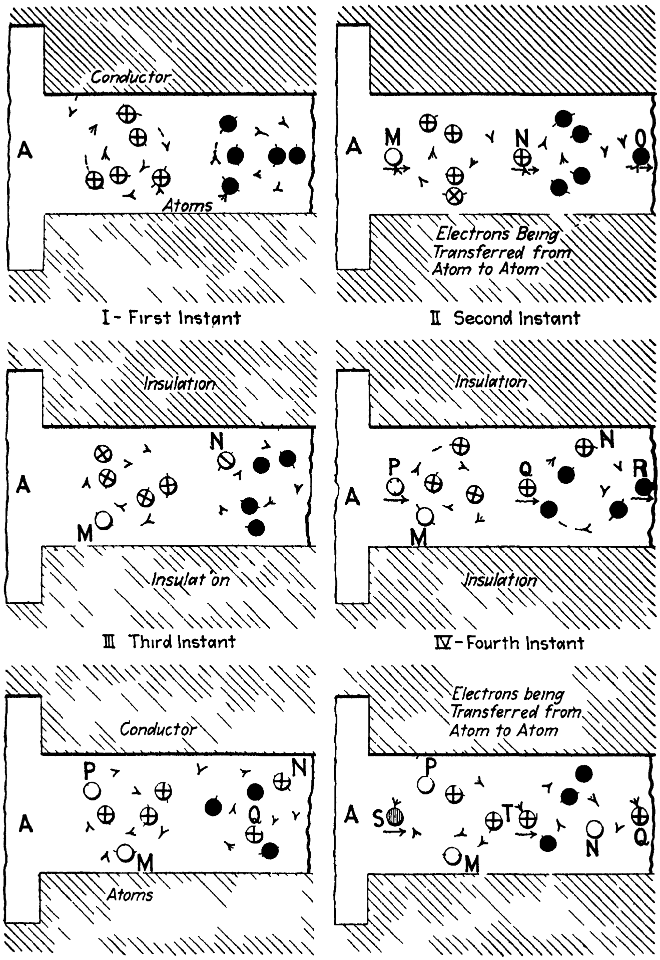


FIG. 63B.—Showing how the electrons pass from atom to atom in "flowing" through a conductor. (In *I* no current is flowing. Current starts in *II* and electrons *M*, *N* and *O* are forced to transfer between atoms. The electrons of the different atoms are shown by different symbols in the picture so that it will be easier to understand just how the transfer—current flow—takes place. Actually, so it is believed, all electrons are exactly alike. In *III* electrons *M* and *N* have assumed temporarily positions in the atoms. But an instant later other electrons *P*, *Q* and *R* are forced along between the atoms. This process continues as long as the current flows.)

time receive with his right hand a ball from his neighbor on the right, each of the other boys would simultaneously hand to his neighbor on his left one of the balls and at the same time F would take another ball from box D. Thus, each boy always has five balls. So long as the boys continued handing baseballs along the line, there would be a current of the spheres flowing from D to E.

Thus while current flows in the conductor, the process is somewhat similar to that suggested in Figs. 63A, and 63B. The electrons are, as long as the

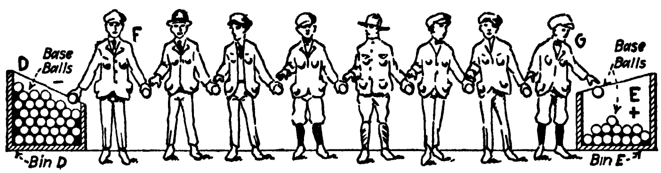


FIG. 63C.—Boys in a row passing base balls from one to another (analogous to the movement of electrons from atom to atom when a current of electricity—electrons—flows in a conductor)

current flows, being transferred between the atoms in the conductor. When an atom receives an electron from its neighbor on one side, it at the same time gives up an electron to its neighbor on the other side. Simultaneously other atoms in the conductor are doing the same thing, and they can do this, probably, only because *the structure of conductor materials is such that the atoms can part readily with their constituent electrons.*

When the string of boys in Fig 63C passed the baseballs to one another, the flow of baseball current from D to E was due to the muscular efforts of each

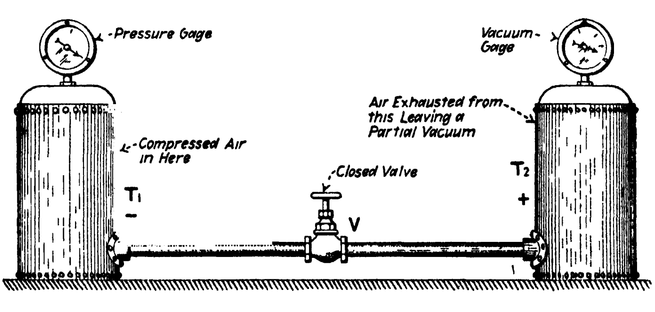


FIG. 63D.—Pneumatic analogy to a negative and a positive electrification.

boy in the string. But in the case of the electric current of Fig. 63A,*II* the transfer of electrons is not due to the efforts of the atoms. The electrons are forced along the conductor because of the electromotive force applied across AB. They are forced from A^1 to B^1 in somewhat the same way that air would be forced from a tank containing compressed air (Fig. 63D) into one containing a vacuum. It could be stated that the tank T_1 is negatively charged with air because the pressure of the air in it is in excess of the pressure of the surrounding atmosphere, likewise it could be stated that

tank T_2 is positively charged because the air in it is at a lesser pressure than that of the surrounding atmosphere.

104B. The Speed or Velocity of a Conduction Current is, probably, much lower than is usually imagined. The moving electrons which constitute the current "move very slowly,* perhaps only several inches per minute." The speed of a current should not be confused with the velocity of propagation of electric impulses through space, which is equal to the velocity of light or 186,000 miles per second.

NOTE.—It requires but a small fraction of a second for a signal transmitted electrically from one end of an electric circuit to reach the other end. This fraction of a second is the time interval required for getting the electrons in that circuit into motion. An analogous situation is this: With the hydraulic circuit of Fig. 93,*I*, there would be but a short time interval, after the pump was started, before the water motor would start although the pump might be rotating and the water in the circuit moving very slowly.

105. A Displacement Current is a current which flows momentarily in a dielectric or insulating material when an e.m.f. is impressed across that material or when an e.m.f. impressed on the material is changed in intensity. When an e.m.f. is impressed across a block of an insulating substance a *charging or capacity* current will flow momentarily in the direction of the e.m.f. Referring to the cell-structure analogy of Fig. 54, an impressed electric force will force the electricity fluid in the cells to shift until the elastic reaction of the cell walls prevents any further shifting. The electricity fluid in the cells is thus displaced or moved—and this constitutes a displacement current. So long as the impressed e.m.f. is continued, without variation in intensity, across the block of insulating material, the cell walls will remain in the stressed position. But the displacement current flows only when the electricity fluid is shifting or being displaced. When there is no displacement of the fluid there is no displacement current. But, now, if the impressed e.m.f. is *removed*, the walls will, by virtue of their elasticity, return to their original unstressed positions. Again, there would be a shifting of electricity fluid but this time its direction would be the reverse of that occurring when the e.m.f. was *impressed* on the block of material. Hence, again, there would be a displacement current—but in the opposite direction. A consideration of the situation will render it obvious that whenever there is a *change* in

* Comstock and Troland, THE NATURE OF MATTER AND ELECTRICITY, *D. Van Nostrand*, page 24.

the intensity of impressed e.m.f. a displacement current will flow.

EXAMPLE.—The momentary current which flows when a permittor or condenser (Art. 753) is charged or discharged is a displacement current.

NOTE.—For an alternating e.m.f. (Art. 429) there may be a corresponding alternating displacement current. For a continuous (Art. 108) e.m.f., the displacement current will be zero when the stress in the "stretched" dielectric balances the electric force to which the strain is due.

106. Convection Currents are those currents which are due to the movement of electricity—electrically charged particles (ions) or electrons—through electrolytes (Art. 335) and gases. An example of a convection current is the flow of charged particles in mercury vapor lamps and rectifiers.

107. A Direct Current is a unidirectional current, that is, one that always flows in the same direction. Such a current may

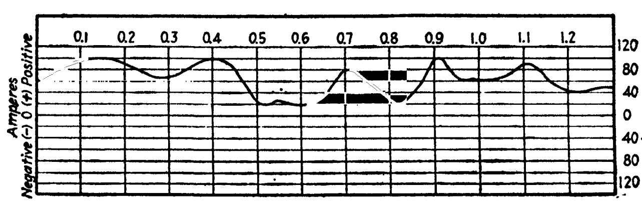


FIG. 64.—Graph of a continuous direct current.

vary in intensity (amperes) but must always flow the same way. A direct current may be continuous or pulsating or constant. Read Art. 112, "Positive and Negative Directions."

EXAMPLES.—Figs. 64, 65 and 66 all show graphs of direct currents.

108. A Continuous Current is a steady, or non-pulsating direct current—a current that always flows in one direction. The direct currents used for commerical lighting and power service are direct currents. Their ampere values may not be constant but they always flow in the same direction and they are non-pulsating. Obviously, a "continuous" e.m.f. is necessary for the production of a continuous current.

EXAMPLE.—Fig. 64 illustrates the graph of a continuous current. The current values change from second to second making the current graph irregular but there are no regular pulsations in it as there are in the graph of Fig. 65. In Fig. 64, at different times the current is as high as 100 amp. and at others it is as low as 20 amp., but it is always in the same direction.

109. A Constant Current is one that continues to flow for some time with unvarying strength. A constant current may be either alternating or direct. A direct, constant current is also of course a continuous current.

EXAMPLE.—Fig. 66 shows the graph of a direct, constant current in which the current value is constantly 80 amp. Constant currents are used commercially for series arc and incandescent lighting service—both direct and alternating current.

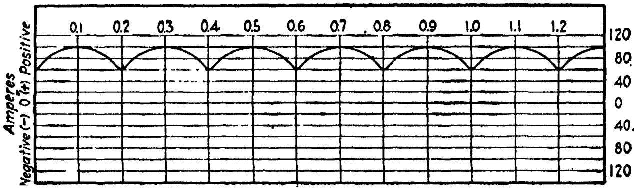


FIG. 65.—Graph of a direct, pulsating current.

110. A Pulsating Current is a regularly varying or continuous current. Pulsating currents are rarely employed in practical electrical work.

EXAMPLE.—Probably the most familiar example of a pulsating current is that that flows through an electric vibrating bell circuit (Art. 285). Each time the armature is attracted to the magnet the circuit is broken and the current ceases to flow. When the armature flies back the circuit is completed and the current flows again. This produces a true pulsating current

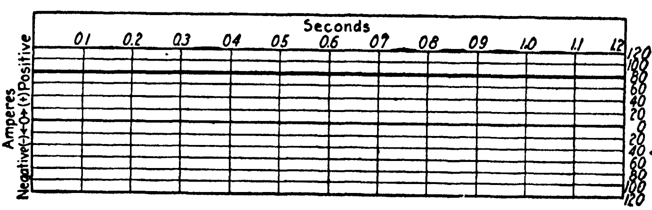


FIG. 66.—Graph for a direct constant current.

in the electric bell circuit. Fig. 65 shows the graph of a pulsating current which attains a value of 100 amp. every 0.2 sec., and decreases to 60 amp. every 0.2 sec. If we start at 3 o'clock the current in the circuit would at that instant be 60 amp., at 0.1 sec. after 3 the current would be 100 amp., at 0.2 sec. after 3 the current would again be 60 amp., and so on.

111. An Oscillatory or Oscillating Current is one which is periodically alternating in direction and of decreasing amplitude. Oscillating currents are seldom encountered by the central-

station man during normal operation. They may occur at times of disturbances—when circuits are opened and closed and during lightning storms. Lightning-discharge currents have been generally thought to be oscillatory in character but there is now a difference of opinion in regard to this point.

Example.—Fig. 67 shows the graph of an oscillating current. Suppose we imagine that the graph starts at 2 o'clock. At 0.1 sec. after 2 a current of

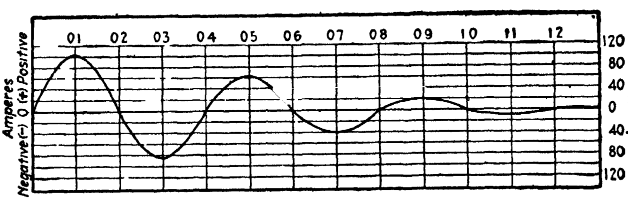


Fig. 67 —Graph of an oscillating current.

100 amp. is flowing in the circuit in a + or positive direction. At 0.2 sec. after 2 the current has decreased to zero and no current is flowing. At 0.3 sec. after 2 the current is reversed and is flowing in a – or negative direction and its value is 80 amp. At 0 4 sec. after 2 the current is again zero. At 0.5 sec. after 2 it is flowing in a + or positive direction and its value is 60 amp. Thus the current continues to reverse in direction and decrease in value until about 1 3 sec. after 2 it has died down to zero—that is there is no current flowing. This is an example of an oscillating current.

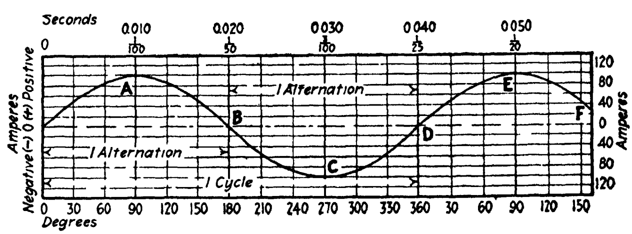


Fig. 68.—Graph of a 25-cycle alternating current.

112. Meaning of "Positive (+) Direction" and "Negative (–) Direction."—When we say that "*a current is flowing in a positive,* + (*or negative,* –) *direction*" this has nothing to do with positive and negative polarities. Any direction can be assumed as positive, then the opposite direction will be negative. If the positive direction is assumed to be from left to right in a conductor, then the negative direction in that conductor is from right to left. Likewise, in plotting graphs like that of Fig. 68, it is usually assumed that the values of the currents that are flow-

ing in a positive direction (assumed) are plotted above the zero (0) line while values of currents that are flowing in the opposite or negative direction are plotted below the zero line. (See also second note under Art. 97A.)

EXAMPLE.—If we assume that a current flowing in the circuit of Fig. 69 from A through the motor to B is flowing in a positive or + direction, then a current flowing in the opposite direction in that circuit, or from B through the motor to A, is flowing in a negative or − direction.

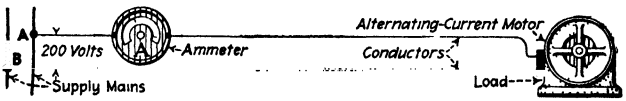


FIG. 69.—Ammeter in an alternating-current circuit.

113. An Alternating Current is one that reverses its direction at regular intervals. (Alternating currents will be only defined and briefly discussed here; they are treated at length in Sec. 41 of this book.) Or to be more specific: it is usually taken as a current which reverses in direction at regular intervals, increasing from zero to its maximum strength and decreasing to zero with the current flowing in one direction and then, with the current flowing in the opposite direction, similarly increasing to a maxi-

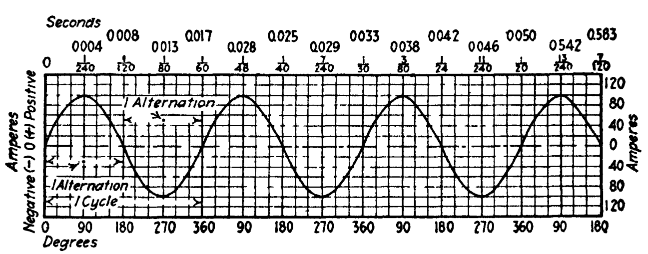


FIG. 70.—Graph of a 60-cycle sine-wave form alternating current.

mum and again decreasing to zero. The current continues to rapidly vary thus in direction and in strength (60 times a second for a 60-cycle, Art. 683, circuit) as long as the circuit remains closed. Figs. 68 and 70 show respectively the curves of a 25-cycle (Art. 683) and a 60-cycle, alternating current, both plotted to the same scale.

EXAMPLE.—Fig. 71,*I* and *II* show hydraulic analogies of direct- and alternating-current circuits. The hydraulic circuit at *I* corresponds to a direct-current electric circuit. As the centrifugal pump, which corresponds to a direct-current generator, operates, it creates a continuous pressure

(voltage) which is always in the same direction and which, therefore, forces the current of water around the circuit—always in the same direction. The hydraulic circuit of *II* corresponds to an alternating-current, electric circuit. The valveless, reciprocating pump is analogous to an alternating-current generator. As the belt uniformly turns the pump, its piston is driven regularly back and forth. This creates an alternating pressure which causes the current of water to flow around the circuit alternately, first in one direction

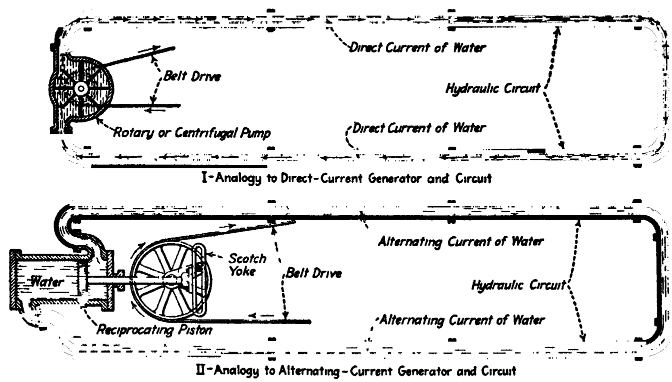


FIG. 71 —Hydraulic analogies to direct- and alternating-current generators and circuits

and then in the other. This current of water, which is flowing first one way and then the other is an analogy to an alternating current of electricity.

EXAMPLE.—A simple demonstration of the alternating character of an alternating current can be made as shown in Fig. 72 with an incandescent lamp (preferably one with a looped filament) and a bar magnet. If the bar magnet is held, as at *I*, near a lamp through which direct current is flowing, the filament will be repelled from or attracted to the magnet, depending on

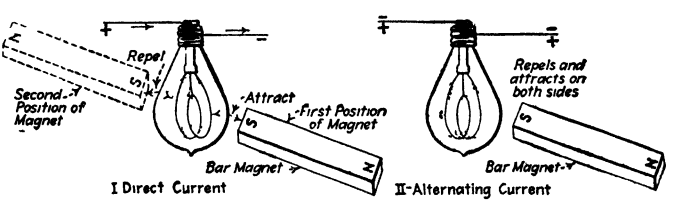


FIG. 72 —Showing magnetic effect of an alternating current.

whether the *N* or the *S* pole of the magnet is presented to the lamp. The current through the lamp filament generates lines of force thus creating a magnetic pole (Art. 46) which is attracted to the pole of the bar magnet. If the same pole of the bar magnet is presented at the opposite side of the filament, the movement of the filament will be the reverse of what it was in the first position (Art. 47). Now, if the magnet be brought near a filament carrying an alternating current, as at *II*, the filament will then merely vi-

brate as the alternating current produces at the filament magnetic poles of alternating polarity.

114. Graphs of Alternating Currents are shown in Figs. 68 and 70. The curves of currents encountered in practice are frequently more or less irregular as shown in Fig. 73,*II*, *III* and *IV*. The curves of the currents produced by old-fashioned revolving-armature generators were often quite irregular. However, modern alternating-current generators produce curves that are almost true *sine curves* (Art. 116) like those of Figs. 68 and 70.

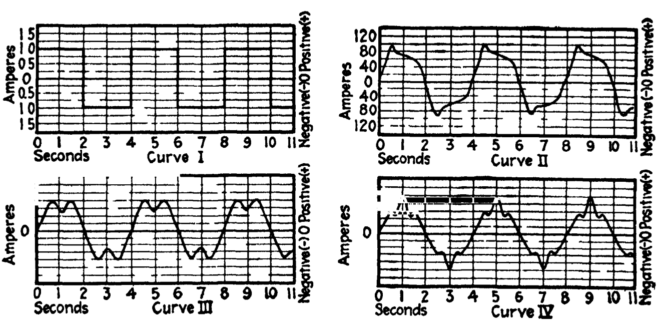


FIG 73 —Graphs of alternating currents.

115. Further Explanation of an Alternating Current.—It is so essential that the reader have a good conception of what an alternating current really is that the following supplementary explanation is included:

EXPLANATION.—Consider the circuit of Fig. 74 in which the incandescent lamp is fed by a battery through a commutating or reversing switch (four-way snap or flush switches are forms of reversing switches). Current from a battery always flows from the + to the − binding post of the battery With the switch turned as at *I* the current flows in, say a positive direction, from *A* around through the lamp to *B*. Then, with the switch turned as at *II* the current through the lamp is reversed and now flows in a negative direction, that is from *B* through the lamp to *A*. Each time the switch is shifted, the direction of the current through the lamp is reversed. If the switch be shifted at regular intervals, the current will flow through the lamp first in one

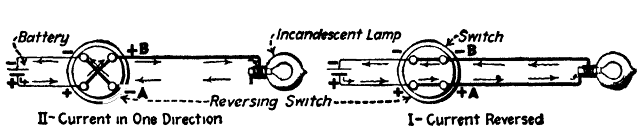


FIG. 74 —Illustrating an alternating current

or a positive direction for a period and then in the other or a negative direction for a period.

Hence so long as the switch is being shifted at regular intervals an alternating current will flow through the lamp. If the battery is circulating a current of 1 amp. around the circuit and the switch is turned every 2 sec., the graph of the resulting alternating current through the lamp would be that of Fig. 73,*I*. The current would flow: first in one (a positive) direction from *A* to *B* for 2 sec.; then in the opposite (negative) direction, from *B* to *A*, for the next 2 sec.; from *A* to *B* for the next 2 sec.; from *B* to *A*, for the next 2 sec. and so on. Note, that for an instant at the end of each 2-sec. interval, while the switch is being shifted, the current changes from a maximum value of 1 amp. in the positive direction to its maximum value of 1 amp. in the negative direction. During this instant the current is 0 (zero), that is, no current flows during this instant. While the graph of Fig. 73,*I* is that of a true alternating current, commercial alternating currents always have sine-wave forms, Art. 177.

116. Explanation of a Sine-curve or Sine-wave-form Alternating Current.—A sine curve (Figs. 68 and 70) is one the contour of which follows a certain definite law, which, however, can not be discussed here. See Art. 517 for details. The currents from modern alternating-current generators have almost true sine-wave forms. This is desirable because a true sine curve is the ideal form (Art. 525) and because calculations relating to them may be readily made whereas calculations relating to irregular curves like those of Fig. 73,*II*, *III* and *IV*, can not be easily made. With a sine-wave-form alternating current, the current increases and decreases gradually as shown in Figs. 68 and 70 and not abruptly as suggested in the alternating-current curve of Fig. 73,*I*.

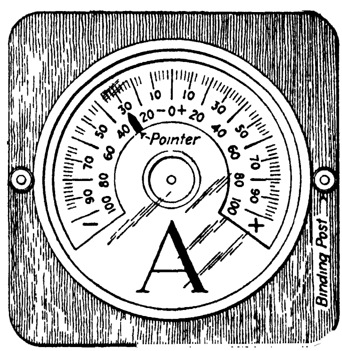


FIG. 75.—Direct-current ammeter with zero (0) mark at center of scale.

EXAMPLE.—Imagine an ideal ammeter (an instrument for measuring electric current intensity in amperes), like that of Fig. 75, which is so sensitive that it will immediately indicate, in amperes, the minutest variation in current and which will also indicate whether the current flowing through it is in a positive or in a negative direction. Also, imagine that one's eyes are sufficiently sharp and quick to enable him to read this instrument at fractional intervals of a second. (Obviously, there can not actually be an instrument so sensitive nor eyes so keen.) Assume this super-sensitive instrument connected into an (60-cycle, Art. 683) alternating-current circuit to a motor,

Fig. 69. Now record the readings at each $\frac{1}{480}$-sec. interval, commencing at any time, say at 3 o'clock. The readings of the varying current in this alternating-current circuit might then be these:

Time	Ammeter reading	Time	Ammeter reading
3 o'clock..........	0.0 amp.	$\frac{10}{480}$ sec. after 3	+100.0 amp.
$\frac{1}{480}$ sec. after 3...	+ 70.7 amp.	$\frac{11}{480}$ sec. after 3	+ 70.7 amp.
$\frac{2}{480}$ sec. after 3...	+100.0 amp.	$\frac{12}{480}$ sec. after 3	0.0 amp.
$\frac{3}{480}$ sec. after 3...	+ 70.7 amp.	$\frac{13}{480}$ sec. after 3	− 70.7 amp.
$\frac{4}{480}$ sec. after 3...	0.0 amp.	$\frac{14}{480}$ sec. after 3	−100.0 amp.
$\frac{5}{480}$ sec. after 3...	− 70.7 amp.	$\frac{15}{480}$ sec. after 3	− 70.7 amp.
$\frac{6}{480}$ sec. after 3...	−100.0 amp.	$\frac{16}{480}$ sec. after 3	0.0 amp.
$\frac{7}{480}$ sec. after 3...	− 70.7 amp.	$\frac{17}{480}$ sec. after 3	+ 70.7 amp.
$\frac{8}{480}$ sec. after 3...	0.0 amp.	$\frac{18}{480}$ sec. after 3	+100.0 amp.
$\frac{9}{480}$ sec. after 3...	+ 70.7 amp.	and so on as long as the current continues to flow.	

If the above values are plotted, they will give the alternating-current sine graph for the 60-cycle, circuit shown in Fig. 70. This graph shows that the maximum current in the circuit is 100 amp. also the graph and the above table shows how the current in the circuit varies from instant to instant.

NOTE.—As shown in Fig. 70, and explained in example under Art. 528, it is possible to express time, as it relates to an alternating current, in degrees as well as in seconds.

SECTION 5

ELECTROMOTIVE FORCE, CURRENT RESISTANCE AND OHM'S LAW

117. The Term Electromotive Force, abbreviated e.m.f., and sometimes called voltage, electric pressure, or difference of potential, is used to designate the "push" that moves or tends to move electrons from one place to another—that causes electricity to flow. Note that voltage or e.m.f. is not electricity; it is merely the pressure that causes electricity to flow. As explained in Art. 123, there may be great electrical pressure but if the circuit is not closed there can be no flow or current of electricity.

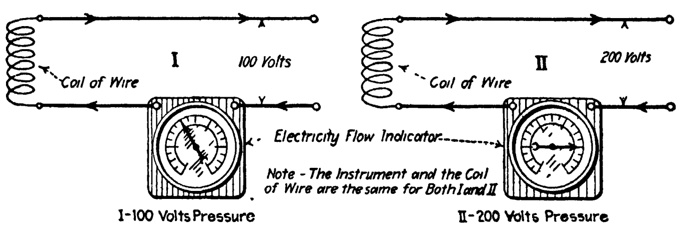


FIG. 76.—Illustrating the effect of increasing electric pressure or voltage.

118. Hydraulic Analogy of E.m.f.—The flow of water through a pipe, that is the number of gallons per second, is determined largely by the hydraulic pressure—pounds per square inch—that is forcing the water through the pipe. A similar electric pressure or e.m.f., measured in volts, causes electricity to flow. A volt (Art. 120) has, when speaking of electricity, somewhat the same meaning as has "*a pound per square inch*" when speaking of hydraulics. A greater hydraulic pressure is required to force a given amount of water through a small pipe than through a large one in a given time. Similarly, a higher voltage is required to force a given amount of electricity through a small wire than through a large one in a given time. If the voltage impressed on a circuit is increased, the current will be correspondingly increased as shown in Fig. 76 wherein doubling the voltage has doubled the current.

119. E.m.fs. May Be Developed in Three Different Ways, viz.: (*a*) *By contact of unlike substances,* either by the application of heat or by chemical action. Heat applied to the junction of two dissimilar metals (Fig. 77,*I*) will (Art. 317) generate an e.m.f., however it will be relatively small; hence the method is not commercial. If a piece of carbon and a piece of zinc (*II*) are immersed in a solution of sal-ammoniac, an electric cell results which will generate an e.m.f. If the key is closed, an electric current will flow and the bell will ring. (*b*) *By magnetic flux:* If the conductor (*III*) be moved up and down between the magnet poles so as to cut across the lines of force (Art. 418), an e.m.f. will be generated. This illustrates the principle of the dynamo and the principle of the cheapest way to generate an e.m.f. if large amounts of electrical energy are required. (*c*) *By dielectric*

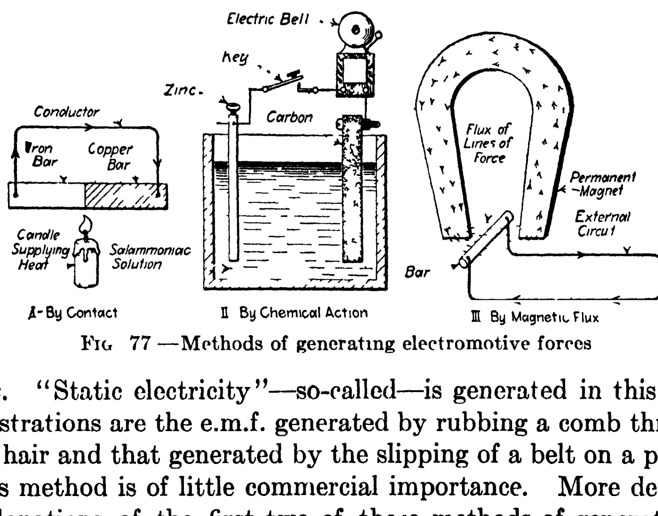


FIG. 77.—Methods of generating electromotive forces

flux. "Static electricity"—so-called—is generated in this way. Illustrations are the e.m.f. generated by rubbing a comb through the hair and that generated by the slipping of a belt on a pulley. This method is of little commercial importance. More detailed explanations of the first two of these methods of generating a difference of potential or an e.m.f. are given in other sections.

120. The Volt Is the Practical Unit of E.m.f.—It is that difference in electrical pressure that will maintain a current of 1 amp. (Art. 122) through a resistance of 1 ohm (Art. 126). A *millivolt* is $\frac{1}{1,000}$ (one-thousandth) of a volt; a *kilovolt* is 1,000 (one thousand) volts.

EXAMPLES.—Ordinary, interior, incandescent-lamp circuits usually operate at a pressure of 110 volts, although 220 volts is sometimes used. Direct-current, street railway voltages in towns and cities are usually about 550 volts. The voltage of an ordinary door-bell (Leclanche) cell is about $1\frac{1}{2}$ volts.

The gravity cell used for telegraphy develops about 1 volt. Electroplating generators usually develop about 2 or 3 volts while those for electrotyping develop from 5 to 10 volts.

121. The Distinction Between Voltage and Potential should be clearly understood. The term "*potential*" (Art. 96) is analogous with the hydraulic term *level* and the terms "*electromotive force*" and "*voltage*" are analogous with the hydraulic terms "*difference-in-level*" and "*pressure*." It is not correct to say "the potential across the incandescent lamp is 110 volts." Instead one should say "the potential difference" or "voltage" across the incandescent lamp is 110 volts.

NOTE.—Voltage or pressure between two points in a circuit is sometimes spoken of as *difference in potential* or *drop of potential* between the two points. Just as in water pipes, where a difference in level produces a pressure and the pressure produces a flow when the faucet is opened, so a difference of potential produces e.m.f. and the e.m.f. impels a flow of electricity as soon as the circuit is connected so that the electricity can move. E.m.f. may be expressed as a voltage or difference of potential or *vice versa.*

121A. Drop of Potential means drop or difference in electrical level. The meaning of the word "potential" is explained in Art. 96 which should be reviewed. The terms "drop of potential," "potential difference," "loss of potential," "volts loss," and "volts drop," while they may have different shades of meanings all indicate about the same thing and can, usually, be used interchangeably with the terms just noted. "Volts drop" and "voltage loss" are sometimes used to mean "drop in potential." But it is doubtful whether their usage in this sense is altogether correct.

The pressure due to differences of water level causes water to flow and the pressures due to differences of electric level causes electricity to flow. Water is, in Fig. 77A, arranged to flow from the level *L* in the stand-pipe and discharged into a tank at level *G*. The water in the horizontal pipe, *PQ*, will be at a constantly decreasing level (pressure) as it approaches the tank. This is indicated by the height to which the water rises in the pressure pipes and by the pressure gages. The reference level in Fig. 77A is the level of the water in the tank *G* which is taken as 0 (zero) level. This means 0 (zero) pressure. Point *L*, as referred to point *G*, is at a pressure of 100 lb. per sq. in. This gives a difference of pressure of $(100 - 0 = 100)$ 100 lb. per sq. in. between *L* and *G*. It is this difference in pressure that causes the water to

flow. Obviously, there can be no flow between two points of the same level or pressure. Point C is at a pressure of 67 lb. per sq. in. Point G is at a pressure of 0 lb. The difference between them is $(67 - 0 = 67)$ 67 lb. per sq. in. Similarly, the pressure

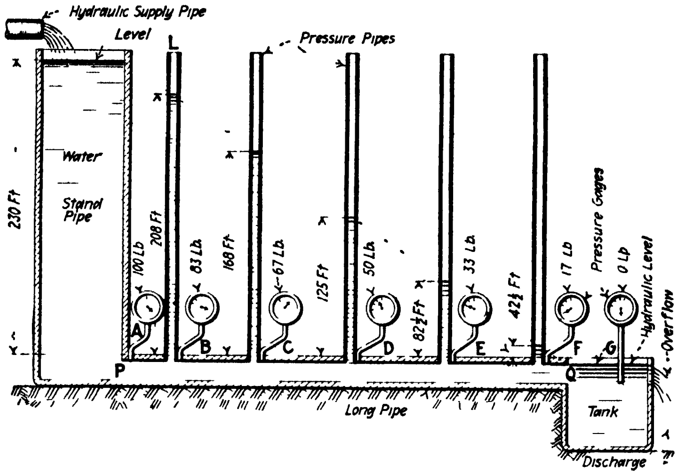


FIG 77A —Illustrating drop in hydraulic pressure or potential

difference between A and D is $(100 - 50 = 50)$ 50 lb. per sq. in. The pressure difference between B and D is $(83 - 50 = 33)$ 33 lb. per sq. in. It is due to this difference in pressure of 33 lb.

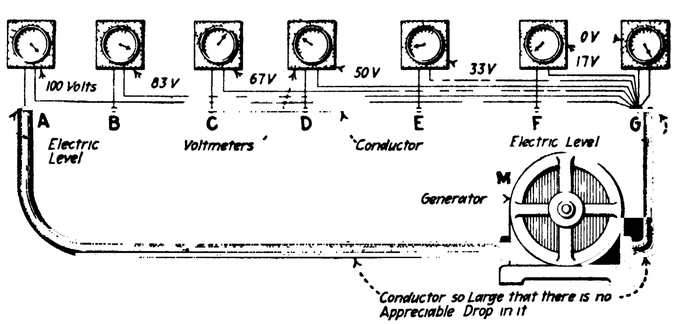


FIG 77B —Illustrating drop of electric pressure or potential in a circuit.

per sq. in. between B and D that water flows between these two points.

Now, if an electrical conductor, AG, be arranged as shown in Fig. 77B, and electricity be forced through it by generator M,

points A, B, C, D, etc., along the conductor will be at constantly decreasing potentials as referred to the reference point G, which is taken as a point of 0 potential. Voltmeters connected between the points on the conductor and the reference point show this. Point A is at a potential of 100 volts as referred to G. Therefore,

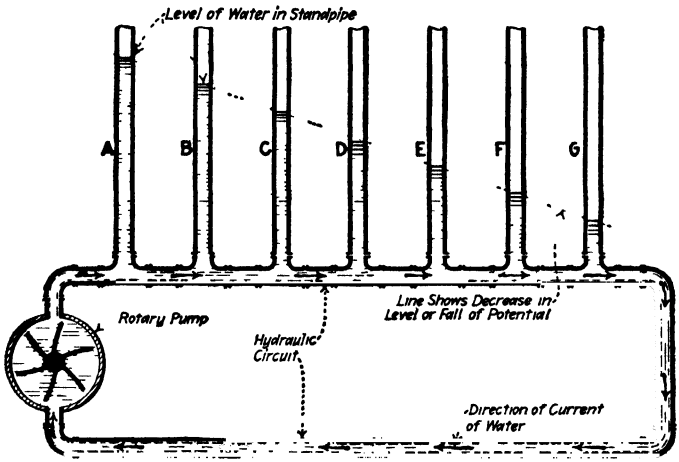


FIG. 77C—Hydraulic analogy to fall of potential in an electric circuit.

the difference of potential or drop in potential which causes electricity to flow from A to G is: $100 - 0 = 100$ *volts.* The potential difference between C and G is $67 - 0 = 67$ *volts.* Similarly the potential difference between A and D is $100 - 50$

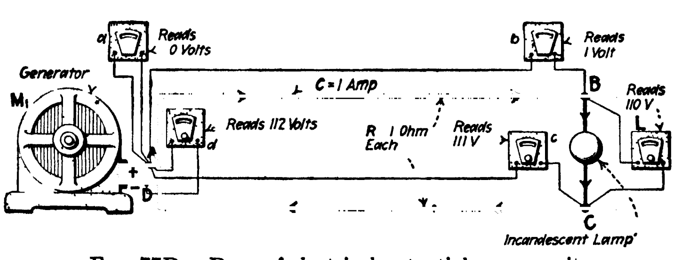


FIG. 77D.—Drop of electrical potential in a circuit.

$= 50$ *volts* and the potential difference between B and D is $83 - 50 = 33$ *volts.* It is these "potential differences" or "drops of potential" between the points that causes the flow of electricity. Fig. 77C illustrates the drop of potential in a hydraulic circuit.

Fig. 77D illustrates the drop of potential in an electric circuit.

The voltmeters, with the exception of L, indicate the potentials at the various points on the circuit as referred to the potential of the + terminal of the generator M_1, which, in this example, is assumed to be at 0 potential. (The small letters, identifying the voltmeters, correspond with the large letters shown at the points the potentials of which the voltmeters indicate.

Voltmeter L (Fig. 77D) indicates the potential difference across the incandescent lamp. The potential at the point B is 1 volt and the difference or drop in potential between A and B is $1 - 0 = 1$ *volt.* The potential at point C is 111 volts and the potential difference between C and B is: $111 - 1 = 110$ *volts,* which is the same difference or drop in potential that voltmeter L indicates. The potential at point D is 112 volts and the potential difference between D and C is: $112 - 111 = 1$ *volt.* Similarly the potential difference between D and A is: $112 - 0 = 112$ *volts,* which is the e.m.f. imposed on the circuit by the generator M_1.

122. The Ampere Is the Practical Unit of Electric Current Flow.—If a pressure of 1 volt be impressed on a closed circuit having a resistance of 1 ohm, then 1 ampere (amp.) will flow through the circuit. Currents of water through pipes are measured by the amount of water that flows through the pipe in a second. Thus we say: "1 gal. per sec.," "10 gal. per sec.," and the like. In a similar manner, flow or currents of electricity are measured by the amount of electricity that flows through a conductor in a second. Thus we may say: "1 coulomb per sec.," "10 coulombs per sec." and the like. Now a coulomb is a certain quantity of electricity (Art. 10) just as a gallon is a certain quantity of water. A term "*ampere*" has been applied to a rate of flow of a *coulomb per second.* Hence a current of 1 ampere or 1 amp. is a current flowing at the rate of a coulomb per second. It so happens that in practical work we are nearly always interested in the rate of flow of electricity (amperes) and seldom in the amount of electricity that flows (coulombs).

NOTE.—Art. 10, that a coulomb of electricity comprises, so it has been estimated, about five million million million electrons.

EXAMPLES.—(1) If electricity is flowing through a conductor at the rate of 10 coulombs per sec., the current in the conductor is 10 amp. (2) If 40 coulombs of electricity flow through a conductor in 10 sec., the average current in the conductor during that time is $40 \div 10 = 4$ amp.

EXAMPLES.—An ordinary 16-c.p. carbon filament incandescent lamp requires about ½ amp. A 16-c.p. tungsten incandescent lamp requires about ⅕ amp. Street arc lamps require from 4 to 10 amp. depending on

their size. The current in a telegraph wire is about $\frac{3}{100}$ (0.03) to $\frac{6}{100}$ (0.06) amp.

NOTE.*—Whatever the reasons which lead originally to the choice of the magnitudes of the *ampere* and the *ohm,* these units can now be considered as two arbitrary fundamental units established by an international agreement. They may be considered as arbitrary units in the same way as the *foot* and the *pound* are arbitrary units. Their values can be reproduced to a fraction of a per cent., according to detailed specifications adapted by practically all civilized nations. Since their values have been established by international agreement, they are called *international electrical units.* These two units (the ampere and the ohm) together with the *foot* (or inch), the *second,* and the *degree* centigrade permit the determination of the values of all other electric and magnetic quantities.

The unit "ohm" is represented by a column of mercury of specified dimensions. The "ampere" by a silver voltameter. The "volt" can then

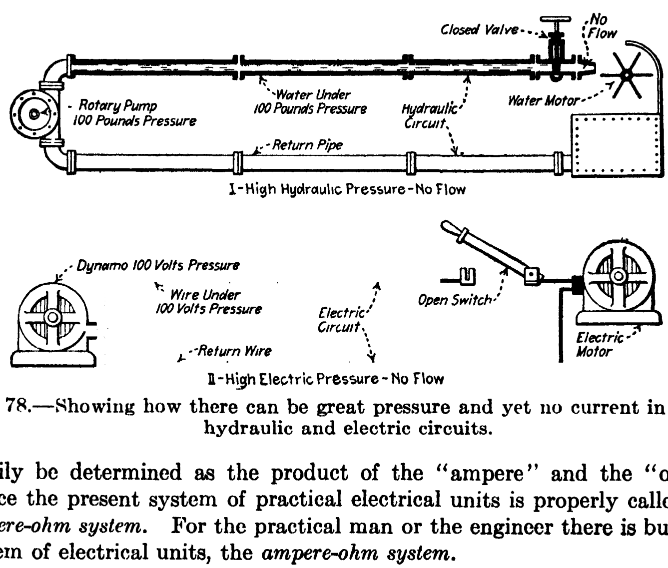


FIG. 78.—Showing how there can be great pressure and yet no current in both hydraulic and electric circuits.

readily be determined as the product of the "ampere" and the "ohm." Hence the present system of practical electrical units is properly called *the ampere-ohm system.* For the practical man or the engineer there is but one system of electrical units, the *ampere-ohm system.*

123. The Difference Between Amperes and Volts.—Amperes relate to the rate at which electricity is flowing while volts refer to the push or pressure that causes the electricity to flow. In both hydraulic and electric circuits there may be great pressure or voltage and yet no flow or current. Consider Fig. 78,*I*, where flow of water is prevented by a closed valve; the pump is maintaining a high hydraulic pressure and yet there can be no flow or current so long as the valve is closed. Likewise, in the electric circuit at *II*, although the dynamo or generator is maintaining

* V. Karapetoff, THE MAGNETIC CIRCUIT and THE ELECTRIC CIRCUIT, McGraw-Hill Book Company, Inc.

a high electric pressure—voltage—there can be no flow of current so long as the switch is open.

Voltage or electrical pressure can never of itself accomplish anything electrical, that is voltage can not, unaided, make a lamp burn or a motor turn. Current (amperes) is always necessary to effect electrical results—the current makes the lamp burn and the motor turn. However, a voltage or difference of potential (Art. 97) is always required for the production of a current. Hence the first step toward effecting any electrical accomplishment is the production of a voltage. Then the voltage will cause a current to flow and the current will produce the result.

NOTE.—The distinction between volts and amperes may be better understood from a study of Fig. 79. In the portion of a hydraulic circuit shown at *I* a current or flow gage *A* has been arranged. The faster the water flows through the pipe, that is, the more water that flows, the greater will be the push against the vane. Evidently the deflection of the vane of *A* will be

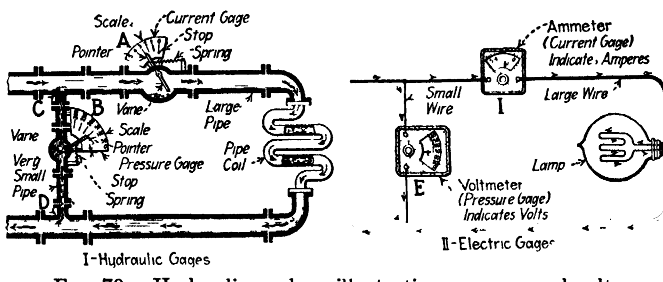


FIG. 79.—Hydraulic analogy illustrating amperes and volts.

proportional to the flow of water or current, analogous to amperes. Hence if the scale of *A* is properly marked, the pointer attached to the vane will indicate at any instant the rate of flow or current of water passing through the pipe at that instant.

Likewise, a gage or indicator can be arranged as at *B* to show the pressure that is causing the water to flow through the pipe coil. The pipe between *C* and *D*, which serves the pressure gage, should be very small as compared with the main pipe so that the amount of water that can flow through this small pipe will be insignificant and can therefore be disregarded practically. However, *some* water will flow between *C* and *D* and the amount that flows will be proportional to the pressure between *C* and *D*, or to the pressure that is forcing water through the pipe coil. A vane and indicator similar to that at *B* can be arranged at *A* to indicate the hydraulic pressure at any instant, which is analogous to volts.

Similarly, in the electric circuit at *II* the *ammeter* or current meter is arranged to indicate the current of electricity that is flowing through the lamp while the *voltmeter* or pressure indicator is arranged to show the push that is causing the electricity to flow.

123A. Current Density may be taken as the *current per unit cross-section of a conductor.* It is usually most conveniently expressed in *amperes per square inch.* Thus, if the electricity flow or current is distributed uniformly through the cross-section of a conductor:

$$U = \frac{I}{A} \qquad \text{(amp. per sq. in.)} \tag{8a}$$

or

$$I = A \times U \qquad \text{(amp.)} \tag{8b}$$

and

$$A = \frac{I}{U} \qquad \text{(sq. in.)} \tag{8c}$$

Wherein U = current density, in amperes per square inch. I = current, in amperes, in the conductor under consideration. A = cross-sectional area, in square inches, of the conductor under consideration. Compare equations of Art. 73 with above.

EXAMPLES.—If there is a current of 1,000 amp. flowing in a busbar having a cross-sectional area of 4 sq. in., the current density in this busbar would be: $U = I \div A = 1{,}000 \div 4 = 250$ *amp. per sq. in.* If the current in a conductor having a cross-sectional area of 0.5 sq. in. is 350 amp., the current density in this conductor would be: $U = I \div A = 350 \div 0.5 = 700$ *amp. per sq. in.*

124. Electrical Resistance is the opposition which is offered by electrical conductors to the flow of current. It is the physical property of a material by virtue of which the material opposes the flow of electric current. It is obvious that the opposition offered by the friction of the flowing water against the insides of the pipes will tend to decrease the current of water in a hydraulic circuit. It follows that the opposition or resistance of conductors will tend to decrease a current of electricity in an electric circuit. It is therefore evident that the magnitude of a current of electricity that will flow through a given circuit will be determined not only by the pressure—voltage—circulating the current, but also by the opposition—resistance—of the conductors. With a specified voltage (a pressure), the greater the resistance the smaller the current—and *vice versa.* No material is a perfect conductor, hence all materials have resistance. However (Art. 129), some materials have much less resistance than others.

125. A Resistor is an object having resistance; specifically, a conductor inserted in a circuit to introduce resistance. *A rheostat* is a resistor so arranged that its effective resistance can, within its range, be varied at will.

126. The Ohm Is the Practical Unit of Resistance.—If a pressure of 1 volt is impressed on a circuit and 1 amp. flows, that circuit has a resistance of 1 ohm (Fig. 80). A *microhm* is $\frac{1}{1,000,000}$ (one-millionth) of an ohm; a *megohm* is 1,000,000 (one million) ohms.

Examples.—A column of mercury 106.3 cm. long and having a cross-sectional area of 1 sq. mm. has a resistance of exactly 1 ohm. In English units, the column would be about 3½ ft. long and its sectional area would be $\frac{15}{10,000}$ sq. in., $0.\dot{0}015$ sq. in.; its diameter would be $\frac{49}{100,000}$ in. A piece of No. 14, B. & S. gage, copper wire 380 ft. long has, roughly, a resistance of 1 ohm. A piece of No. 10 B. & S. gage copper 1,000 ft. long has a resistance of almost precisely 1 ohm. The resistance of 10 ft. of German-silver wire the diameter of a lead pencil is about 1 ohm. A 2½-in. vibrating bell will ordinarily have a resistance somewhere between 1½ and 3 ohms depending on how it is wound; a similar 5-in. bell will have a resistance of about 5 ohms.

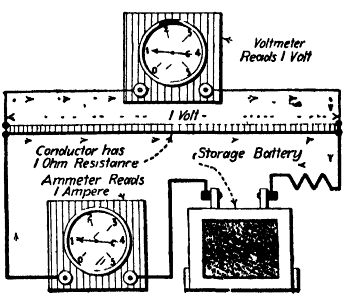


FIG. 80.—1 volt pressure forces 1 ampere of current through a resistance of 1 ohm.

126A. Resistivity is specific resistance. That is, it is the resistance of a block, of the material under consideration, having a specified length and cross-sectional area. Thus, the resistance, in ohms, of a 1-in. cube of a material, from one face to the opposite face may be taken as the resistivity of the material. In practice, it is often convenient to specify resistivities in ohms per circular mil-foot; see the values in Table 143. Resistivities of insulating materials may be specified in megohms per square inch-mil as in Table 132.

127. What Determines Resistance.—Although pipes offer opposition or resistance to the flow of water in much the same way as do conductors to the flow of electricity, there is no unit of resistance to water flow that corresponds with the unit (the ohm) of resistance to electricity flow. The amount of resistance offered to the flow of water through a pipe or to the flow of electricity through a conductor is determined by somewhat analogous properties of the pipe and of the conductor respectively as indicated in Table 128.

NOTE.—With a certain pressure, with both electricity and water flow, the longer the wire or pipe, the less the flow and the smaller the diameter of the

wire or pipe, the less the flow—and *vice versa*. See Art. 133 and following articles for more detailed information concerning resistance.

128. Properties Determining Flow of Currents.

	Of water current through a pipe	Of electricity current through a wire
1	Diameter of pipe	Diameter of wire.
2	Length of pipe	Length of wire.
3	Material of pipe or its internal smoothness.	Material and temperature of wire.

129. The Resistances, That Different Materials Offer to electricity flow, vary greatly. No material is a perfect conductor and no material is a perfect insulator. However, some materials, the metals for instance, have very small resistances and therefore conduct electricity so readily that they are called *conductors* (Art. 93). Other materials such as wood and slate have, at least when moist, relatively high resistances and are therefore called *semi or partial conductors*. Glass, porcelain, paraffin and certain other materials have such high resistances that they are practically non-conducting, hence they are called *insulators*. Tables 143 and 132 show the relative resistances of some materials and Table 157 gives the resistances of different sizes of copper wire. Refer to the author's American Electrician's Handbook for other resistance tables.

130. Conductance is, in a sense, the opposite of resistance. The mho (ohm spelled backward) is the unit of conductance. Resistance represents the opposition that a conductor offers to the passage of electricity while conductance represents the readiness or ease with which a conductor conducts electricity. *Conductance, numerically expressed in mhos is the reciprocal of resistance in ohms.* That is, the conductance of a conductor in mhos is equal to 1 divided by the resistance, in ohms, of the conductor. The relative conductances (as compared with copper) of the common metals are given in Table 143. *Conductivity* is specific conductance and is expressed in mhos per unit volume. Conductivity is the reciprocal of resistivity which is defined in Art. 126A. That is: *Conductivity* = 1 ÷ *Resistivity.*

Examples.—What is the conductance of a wire having a resistance of 2 ohms. Solution.—Its conductance = 1 ÷ *its resistance* = 1 ÷ 2 = ½ or 0.5 *mho*. Likewise, the conductance of a coil having a resistance of 20

ohms = 1 ÷ 20 = $\frac{1}{20}$ or 0.05 *mho*. The conductance of a rod having a resistance of 6.8 ohms = 1 ÷ 6.8 = 0.147 *mho*.

131. Insulating Materials, sometimes called dielectrics, is the name given to that class of substances that are very poor conductors. There is no material that is absolutely opaque to electricity, through which electricity can not be forced, therefore, there is no perfect insulator. However, certain materials are such poor conductors that they are opaque for all practical purposes and hence these are called insulators. These materials have exceedingly high, though measurable, resistances. Table 132 indicates the resistance values of some of these materials.

132. Resistance of Insulating Materials.—(FOWLER'S POCKET BOOK).

Material	Thickness used in dynamo work inches	Resistivity, megohms* per square inch-mil
Asbestos	0.004–0.020	7
Asbestos and muslin, oiled	0.010–0.030	850
Cotton, single covering	0.005–0.012	10
Cotton, single covering, soaked in paraffin	0.006–0.015	11,800,000
Cotton, double covering	0.012–0.020	10
Cotton, double covering, shellacked	0.015–0.025	25
Fiber, red, vulcanized	0.030–0.075	470
Mica	0.001–0.125	33,000
Micanite cloth, flexible	0.008–0.020	440,000
Micanite paper, flexible	0.010–0.025	500,000
Micanite plate, flexible	0.010–0.020	320,000
Oiled cloth	0.005–0.030	650
Oiled paper, double coat	0.006–0.010	1,600
Brown paper	0.005–0.010	2
Paraffined paper	0.002–0.008	11,800,000
Rubber sheet	0.015–0.060	3,000,000
Shellacked cloth	0.006–0.012	30
Silk, single covering	0.001–0.0025	50
Silk, single covering, shellacked	0.0015–0.004	75
Silk, double covering	0.0015–0.005	50
Silk, double covering, shellacked	0.002–0.007	75

* A megohm = 1,000,000 ohms. This column gives resistances in megohms for a square inch of material $\frac{1}{1,000}$ in. in thickness.

NOTE.—THE DIFFERENCE BETWEEN INSULATION RESISTANCE AND DIELECTRIC STRENGTH.*—Properties of dielectrics, that are frequently confused, are *insulation resistance* and *dielectric strength.* No dielectric is a perfect insulator. Some current will flow through it between points of different potential. The current will vary with the difference of potential and inversely with the resistance of the path. The resistance to the flow of this current is insulation resistance. It varies directly with the length of the path and inversely with the area and is measured in ohms or megohms. Dielectric strength is a measure of the ability of the dielectric to withstand puncture; it is not necessarily high in a material having high resistance. It is measured in "volts per millimeter" necessary to puncture the insulation.

133. What Resistance Is Proportional To.—Neglecting temperature effects (Art. 147), electrical resistance is directly proportional to the length of a conductor and is inversely proportional to its cross-sectional area. See Art. 143*A* and following paragraphs for methods of calculating resistance.

134. Ohm's Law.—There is a simple but most important relation between the e.m.f. (volts), the current (amperes) and the resistance (ohms) in any electric circuit. This relation is expressed in Ohm's law, viz.: *The electric current in a conductor equals the electromotive force applied to the conductor divided by the resistance of the conductor.* The law may be simply stated: *current = e.m.f. ÷ resistance.*

Or, stating the same thing in another way:

$$\text{amperes} = \text{volts} \div \text{ohms} \tag{8d}$$

or,

$$\text{volts} = \text{amperes} \times \text{ohms} \tag{8e}$$

or,

$$\text{ohms} = \text{volts} \div \text{amperes}. \tag{8f}$$

This law can also be expressed as a formula (Fig. 81):

$$I = \frac{E}{R} \quad \text{(amp.)} \tag{9}$$

or

$$E = I \times R \quad \text{(volts)} \tag{10}$$

or

$$R = \frac{E}{I} \quad \text{(ohms)} \tag{11}$$

* *Electric Journal.*

Wherein I = current, in amperes. E = the e.m.f., in volts. R = the resistance, in ohms.

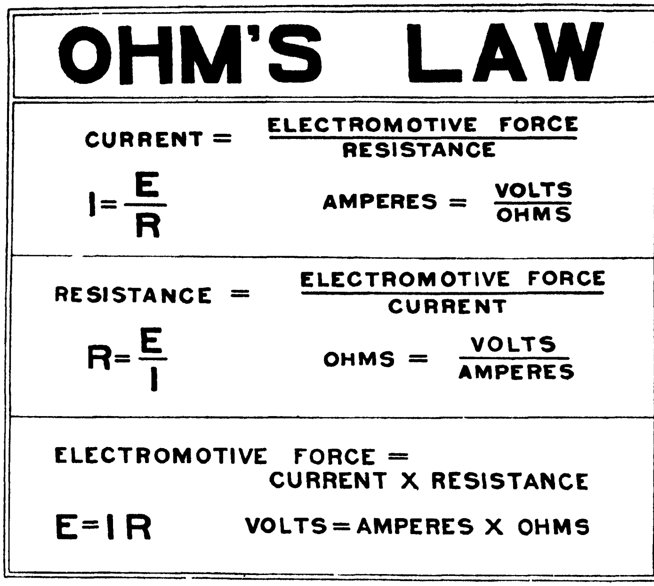


FIG. 81.—The Ohm's law equations. (From chart prepared by The David Ranken Jr. School of Mechanical Trades, St. Louis.)

NOTE.—Ohm's law is merely a specific statement, as applied to the electric circuit, of the very important general law which governs all physical phenomena. This general law is: *The result produced is directly proportional to the effort and inversely proportional to the opposition.* This general law

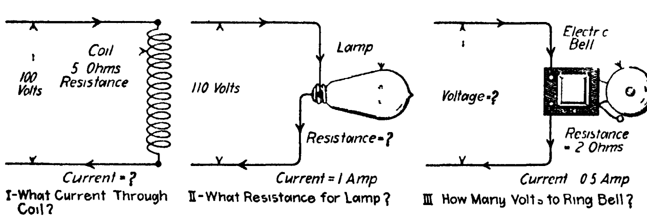


FIG. 82.—Ohm's law examples; parts of circuits.

applies to all circuits: electric, magnetic (Art. 231), hydraulic, pneumatic, heat, etc.

EXAMPLE (Fig. 82,*I*).—What current will flow through a coil having a resistance of 5 ohms if the impressed e.m.f. is 100 volts? SOLUTION.—Substituting in the formula (9): $I = E \div R = 100 \div 5 = 20$ *amp.*

EXAMPLE (Fig. 82,*II*).—What is the resistance of an incandescent lamp

through which 1 amp. flows when 110 volts is impressed on it? SOLUTION.—Substituting in the formula (11): $R = E \div I = 110 \div 1 = 110$ *ohms*.

EXAMPLE (Fig. 82, *III*).—How many volts will be required to force 0.5 (½) amp. through an electric bell having a resistance of 2 ohms? SOLUTION.—Substituting in the formula (10): $E = I \times R = 0.5 \times 2 = 1$ *volt*.

See following articles for further examples.

135. The Above Simple Form of Ohm's Law Applies Only to Direct-current Circuits and to alternating-current circuits having no *inductance* (Art. 47) or permittance (Art. 471). Where used for alternating-current circuits the law should be modified as suggested in the sections of this book relating to alternating currents.

136. In Applying Ohm's Law it is easy to make mistakes unless certain precautions are observed. The law is applicable to an entire circuit or to only a portion of a circuit. WHEN APPLIED TO AN ENTIRE CIRCUIT: *The current (amperes) in the entire circuit equals the e.m.f. (volts) across the entire circuit divided by the resistance (ohms) of the entire circuit.* WHEN APPLIED TO A PORTION

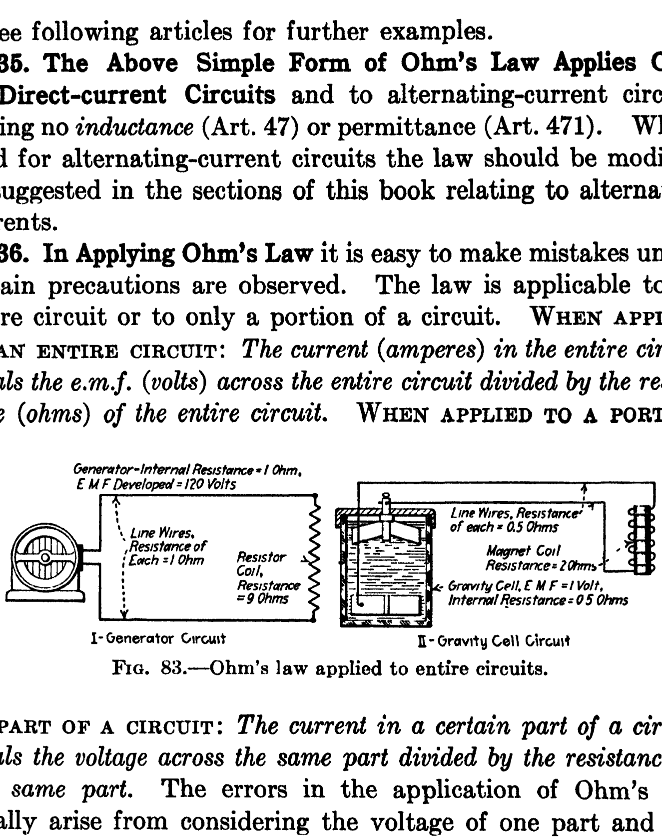


FIG. 83.—Ohm's law applied to entire circuits.

OR PART OF A CIRCUIT: *The current in a certain part of a circuit equals the voltage across the same part divided by the resistance of that same part.* The errors in the application of Ohm's law usually arise from considering the voltage of one part and the resistance and amperage of a different part—or *vice versa*.

EXAMPLE.—What will be the current in the circuit of Fig. 83, *I*? SOLUTION.—An entire circuit is shown. It is composed of a dynamo, line wires and a resistance coil. The e.m.f. developed by the dynamo (do not confuse this with the e.m.f. impressed by the dynamo on the line) is 120 volts. The resistance of the entire circuit is the sum of the resistances of dynamo, line wires and resistance coil. Substituting in the formula (*a*): $I = E \div R = 120 \div (1 + 1 + 9 + 1) = 120 \div 12 = 10$ *amp*.

EXAMPLE.—What current will flow in the circuit of Fig. 83, *II*? SOLUTION.—This again is an entire circuit. Substituting in the formula (*a*): $I = E \div R = 1 \div (0.5 + 0.5 + 2 + 0.5) = 1 \div 3.5 = 0.28$ *amp*.

EXAMPLE.—With 10 amp. flowing what will be the voltage or drop across each of the line wires of Fig. 84. SOLUTION.—Each has a resistance of 0.1 ohm, hence, substituting in formula (10): $E = I \times R = 10 \times 0.1 = 1$ *volt.*

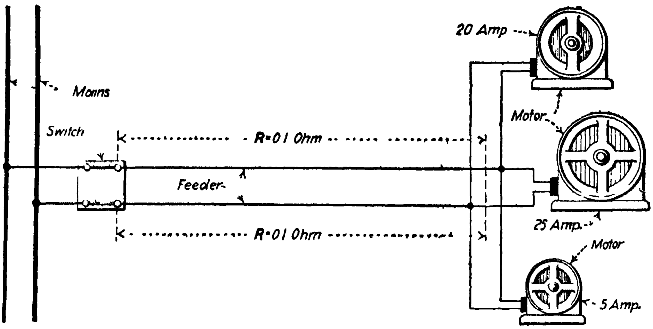


FIG. 84.—Calculating volts drop in motor feeder with Ohm's law.

The drop in both line wires or in the circuit between the switch and the motors would be 2 volts.

EXAMPLE.—What is the resistance of the incandescent lamp of Fig. 85,*I*? It is tapped to a 110-volt circuit and the ammeter reads 0.5 amp. The

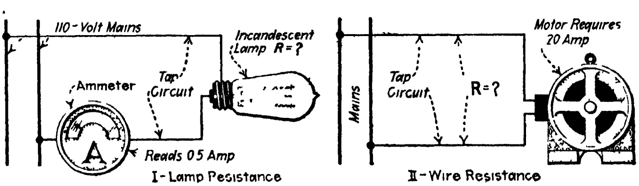


FIG. 85.—Applying Ohm's law to portion of circuits.

branch wires are so short that their resistance can be neglected. SOLUTION.—Substitute in the formula (11): $R = E \div I = 110 \div 0.5 = 220$ *ohms.*

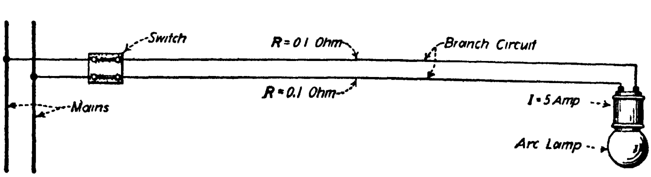


FIG. 86.—Ohm's law applied to a portion of a circuit.

EXAMPLE.—The motor of Fig. 85,*II* takes 20 amp. and the drop in voltage in the branch wires should not exceed 5 volts. What is the greatest resistance that can be permitted in the branch conductors. SOLUTION.—Substi-

tute in the formula (11): $R = E \div I = 5 \div 20 = 0.25$ *ohm.* This (0.25 ohm) is the resistance of both wires. Each would have a resistance of 0.125 ohm.

EXAMPLE.—The arc lamp, Fig. 86, takes 5 amp. The resistance of each wire is 0.1 ohm. What will be the drop in volts in each branch wire? SOLUTION.—Substitute in the formula (10): $E = R \times I = 0.1 \times 5 =$ 0.5 *volt.* In both branch wires or in the branch circuit the volts lost would be $2 \times 0.5 = 1$ *volt.*

EXAMPLE.—Three motors (Fig. 84) taking respectively: 20 amp., 25 amp., and 5 amp. (these values are those stamped on the name plates of the motors) are located at the end of a feeder having a resistance of 0.1 ohm. on each side. What will be the volts drop in the feeder? SOLUTION.—Substitute in the formula (10); $E = R \times I = (0.1 + 0.1) \times (20 + 25 + 5) = 0.2 \times 50 = 10$ *volts.*

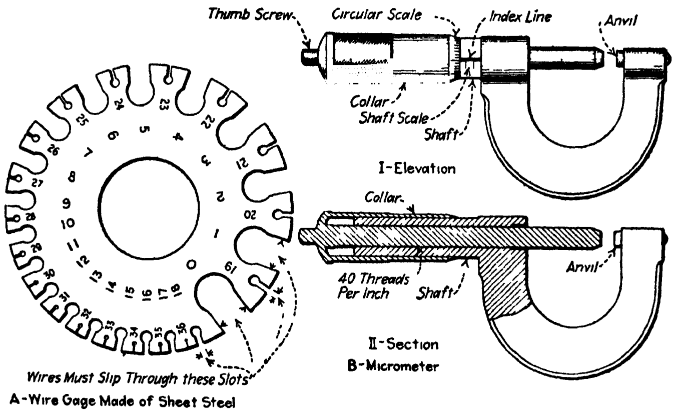


FIG. 87.—Instruments for measuring wire diameters.

137. Wire Gages are arbitrary standards for the measurement of diameters of wire and thicknesses of sheet metal. Many different gages have been proposed. The *Brown & Sharpe Gage* which is the same as the *American Wire Gage* is the standard in the United States for the measurement of copper wire diameters. Wire sizes are referred to by gage numbers, usually the smaller the number the bigger the wire. Wire measuring gages, Fig. 87*A*, are made of hardened steel plate. With the kind shown, the wire being measured is placed in succession in the slots which are located around the periphery of the gage until a slot (not a circular hole) is found in which the wire just fits; its gage number is then indicated opposite the slot. See the AMERICAN ELECTRICIAN'S HANDBOOK for a rather complete

schedule and comparison of all of the wire gages. *Micrometers* (Art. 138) are rapidly superseding sheet-metal wire gages for the determination of wire sizes.

138. Micrometers Are Now Widely Used for Measuring Wire and will doubtless ultimately supersede measuring gages for that purpose. Fig. 87,*B* shows a micrometer that will measure easily and accurately to 1/1,000 in. Where micrometers are used for measuring, the wire diameters are usually expressed in thousandths of an inch. This is a convenient and accurate method. Some concerns have ceased to use wire gages and now specify all wire diameters in thousandths of an inch.

139. A Circular Mil is, by definition, the *area* of a circle 1/1,000 (one one-thousandth) in. in diameter, Fig. 88. (A mil is 1/1,000 —one one-thousandth—in.) The areas of electrical conductors are usually measured and expressed in circular mils because the circular mil is the most convenient unit for this purpose. The areas of circles vary as the squares of their diameters. That is, the area of a circle of a diameter of 0.002 in. (2 mils) is four times the area of a circle having a diameter of 0.001 in. (1 mil). It follows that since, by definition, a circle of a diameter of 0.001 in. is 1 cir. mil, a circle of a diameter of 0.002 in. has four times the area of the 0.001-in. diameter circle and, therefore, has an area of 4 cir. mils. Thus it is evident that the area of any circle can be expressed in circular mils by merely squaring its diameter, which must, however, be expressed in thousandths of an inch. The advantage of expressing cross-sectional areas of round conductors in circular mils is a decided one. To compute the cross-sectional area of a round conductor in square inches a somewhat tedious calculation is necessary. But to compute its area in circular mils, it is only necessary to square the diameter.

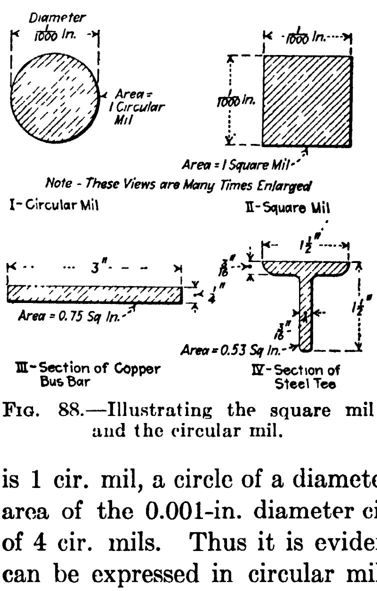


FIG. 88.—Illustrating the square mil and the circular mil.

EXAMPLE.—Since 3/8 = 0.375 (375 mils) the area of a circle 3/8 in. in diameter would be: 375 × 375 = 140,625 *cir. mils*. Likewise, the area of a circle 0.005 in. (5 mils) in diameter would be: 5 × 5 = 25 *cir. mils*.

140. A Square Mil is the area of a square having sides $\frac{1}{1,000}$ (one-thousandth) in. long, Fig. 88. Areas of square or rectangular conductors are sometimes measured in square mils. Areas in square mils are calculated by multiplying together the length and the breadth of the rectangle expressed in thousandths of an inch. In actual area, a circular mil is about eight-tenths as great as a square mil as is evident from Fig. 88.

EXAMPLE.—The area of a rectangle ½ in. wide and 2 in. long would be: $500 \times 2{,}000 = 1{,}000{,}000$ *sq. mils.*

141. Square Mils May Be Reduced to Circular Mils or Vice Versa by using one of the formulas that are given below:

$$\text{(12)} \qquad Sq.\ mils = Cir.\ mils \times 0.7854.$$

or

$$\text{(13)} \qquad Cir.\ mils = \frac{sq.\ mils}{0.7854}.$$

$$\text{(14)} \qquad Cir.\ mils = \frac{sq.\ in.}{0.000{,}000{,}785{,}4}.$$

or

$$\text{(15)} \qquad Sq.\ in. = cir.\ mils \times 0.000{,}000{,}785{,}4.$$

EXAMPLE.—The sectional area of the busbar in Fig. 88,*III*, is in circular mils:

$$\frac{3 \times \frac{1}{4}}{0.000{,}000{,}785{,}4} = \frac{0.75}{0.000{,}000{,}785{,}4} = 955{,}000\ cir.\ mils.$$

EXAMPLE.—The sectional area of the steel tee, shown in Fig. 88,*IV*, is in circular mils:

$$\frac{sq.\ in.}{0.000{,}000{,}785{,}4} = \frac{0.53}{0.000{,}000{,}785{,}4} = 674{,}800\ cir.\ mils.$$

142. A Circular Mil-foot is the unit conductor. A wire having a sectional area of 1 cir. mil and a length of 1 ft. is a circular mil-foot of conductor. The resistance of a circular mil-foot of a metal is sometimes called its *specific resistance* or preferably *resistivity* (Art. 126*a*). Resistances of circular mil-feet of different conductors are given in the accompanying Table 143, and tables showing more complete data appear in the author's AMERICAN ELECTRICIAN'S HANDBOOK.

143. Approximate Resistivities—Specific and Relative—of Conductors.*

Conductor	Resistivity = p (resistance of 1 cir mil-foot in ohms)		Relative conductivity (as compared with copper)	Relative resistivity (as compared with copper)
	0°C or 32°F	23 8°C or 75°F		
Silver, pure annealed.	8 831	9 674	108 60	0 925
Copper, annealed	9 590	10 505	100 00	1 000
Copper, hard-drawn	9 810	10 745	97 80	1 022
Aluminum (97.5 per cent pure)	16 031	17 699	59 80	1 672
Zinc (very pure)	34 595	37 957	27 72	3 608
Iron wire	58 702	65 190	16 20	6 173
Nickel	74 128	85 138	12 94	7 726
Steel (wire)	81 179	90 150	11 60	8 621
Brass	43 310		22 15	4 515
Phosphor-bronze	51 005		18 80	5 319
German silver	127 800		7 50	17 300
Gray cast iron	684 000			

143A. The Computation of the Resistance of Any Conductor may be made by considering the factors which determine resistance. Thus, Art. 127, the resistance of any conductor will vary with: (*a*) *The material of the conductor.* (*b*) *Directly as the length of the conductor.* (*c*) *Inversely as the area of the conductor.* That is, the longer a conductor is, the greater its opposition or resistance will be. The smaller in cross-sectional area the conductor is, the greater its resistance will be. Resistance of conductors to electricity flow is, in these respects, analogous to the resistance which pipe lines offer to water flow. By properly combining these factors into a formula we have:

$$R = \frac{p \times l}{A} \quad \text{(ohms)} \tag{15a}$$

and

$$l = \frac{A \times R}{p} \quad \text{(in.)} \tag{15aa}$$

Wherein R = resistance of the conductor in ohms. p = the resistivity (Art. 126A) of the conductor; if A is expressed in square

* International Textbook Company—ELECTRICAL ENGINEER'S HANDBOOK See the author's AMERICAN ELECTRICIAN'S HANDBOOK for a more complete table.

inches, p should be expressed in ohms per inch cube. A = the cross-sectional area of the conductor which, if p is expressed in ohms per inch cube, should be expressed in square inches, l = length of the conductor, which should be expressed in inches if p is in ohms per inch cube and A is in square inches.

NOTE.—The above formula (15*a*), while it is important because of the truth which it expresses, is seldom used in practice because the form given in Art. 144, in which resistivities in ohms per circular mil-foot are used) is usually more convenient in general application.

143B. To Compute the Conductance (Art. 130) of a conductor a formula which can readily be derived from (15*a*) may be used. Conductance is, by definition, the reciprocal of resistance. Hence, from (15*a*):

$$g = \frac{A}{p \times l} \qquad \text{(mho)} \tag{15b}$$

But also (Art. 130), *resistivity* or $p = 1 \div$ *conductivity*. Then substituting this expression for p in (15*b*) there results:

$$g = \frac{A}{(1 \div \text{conductivity}) \times l} = \frac{\text{conductivity} \times A}{l} \qquad \text{(mho)} \tag{15c}$$

or

$$g = \frac{\gamma \times A}{l} \qquad \text{(mho)} \tag{15d}$$

Wherein g = conductance of conductor in mhos. γ = conductivity of the material, usually expressed in ohms per mho per inch cube. A = area of cross-section of conductor, in square inches if γ is expressed in mhos per inch cube. l = length of conductor, in inches, if γ is expressed in mhos per inch cube.

144. The Practical Method of Computing the Resistance of a Circular Conductor of Any Common Metal follows from formula (15*a*). However, the practical working formula which is given below differs from that of (15*a*) in two details. (1) *It utilizes a resistivity value expressed in ohms per circular mil-foot.* (2) *It utilizes an area value expressed in circular mils.* These working formulas are:

$$R = \frac{p \times l}{d^2} \qquad \text{(ohms)} \tag{16}$$

or

$$l = \frac{R \times d^2}{p} \qquad \text{(ft.)} \tag{17}$$

or

$$d = \sqrt{\frac{p \times l}{R}} \quad \text{(mils)} \tag{18}$$

and

$$p = \frac{d^2 \times R}{l} \quad \text{(ohms per cir. mil-ft.)} \tag{18a}$$

Wherein R = resistance of the round conductor, in ohms. p = resistivity in ohms per circular mil-foot of the metal composing the wire as taken from Table 143. l = length, in feet, of the conductor. d = diameter of the conductor, in mils. d^2 = diameter in mils squared, or what is the same thing, is the sectional area of the conductor, in circular mils. See Art. 147 regarding corrections that must, in refined work, be made for changes in temperature.

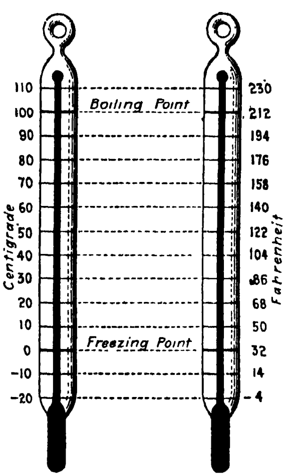


FIG. 89.—Comparison of Centigrade and Fahrenheit thermometer scales.

EXAMPLE.—What will be the resistance of a piece of iron wire, at 75 deg. F. (23.8 deg. C.), that is 3/10 in. (0.300 in.) in diameter and 400 ft. long? SOLUTION.—The resistance per circular mil-foot of this wire, taken from Table 143 is 65.19 ohms. Now substituting in the formula (16): $R = (p \times l) \div d^2$ $(65.19 \times 400) \div (300 \times 300) = 26{,}076 \div 90{,}000 =$ 0.29 *ohm.*

145. To Compute the Resistance of Conductors That Are Not Circular in Cross-section.—First figure their areas in square inches and then reduce this square inch area to circular mils as outlined in Art. 141. Then proceed, using the formula (16) given in Art. 144 to obtain the resistance value.

146. Thermometer Scales.—Thermometers are usually calibrated in accordance with either the Centigrade system or the Fahrenheit system. In the Fahrenheit system, the zero is 32 deg. below the freezing point of water and the boiling point of water is 212 deg. (see Fig. 89). In the Centigrade system the zero is the freezing point of water and the boiling point is 100 deg. The Centigrade system of measuring temperatures is used largely in engineering work and almost exclusively in rating electrical machinery. The Fahrenheit system is the one in common use in the United States. A temperature value in one sys-

tem can be reduced to the corresponding value in the other system directly from a table or by using formulas.*

147. The Resistance of All Pure Metals Increases as They Become Hot (Fig. 90).—The resistances of certain alloys do not increase with the temperature. Table 143 shows that the resistance of a circular mil-foot of annealed copper is about 9.6 ohms at 32 deg. F. and is 10.505 ohms at 75 deg. F. Therefore, where extreme accuracy is essential, it is necessary to note the temperature at which any resistance value is taken when the resistance value is quoted. In practical work with wire, particularly in outside and inside wiring, changes in resistance due to changes in temperature are so small, relatively, that they need not be considered. The ohms increase in resistance, per ohm,

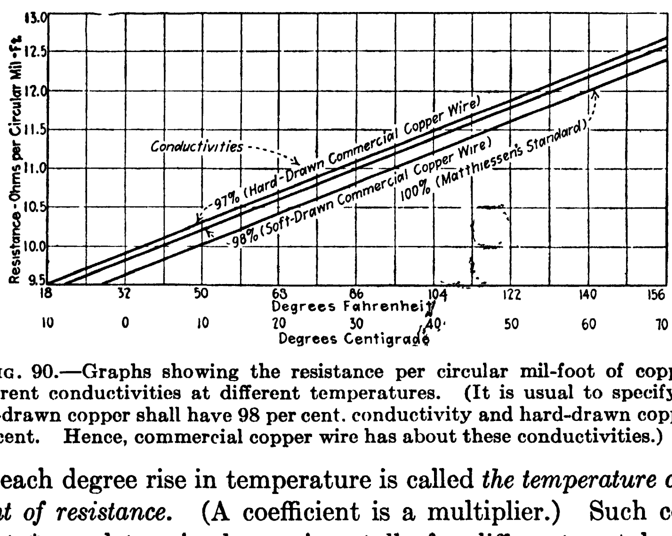


FIG. 90.—Graphs showing the resistance per circular mil-foot of copper of different conductivities at different temperatures. (It is usual to specify that soft-drawn copper shall have 98 per cent. conductivity and hard-drawn copper 97 per cent. Hence, commercial copper wire has about these conductivities.)

for each degree rise in temperature is called *the temperature coefficient of resistance.* (A coefficient is a multiplier.) Such coefficients* are determined experimentally for different metals. An abridged list is shown in 149. For all pure metals the coefficient is practically the same and is 0.004 per *deg. for temperatures in degrees Centigrade* and 0.0023 *per deg. for temperatures in degrees Fahrenheit.* The temperature coefficient of an alloy is generally less than the average of the coefficients of its constituents.

EXAMPLE.—If the resistance of a pure metal wire is 20 ohms at 60 deg. F. what will its approximate resistance be at 90 deg. F.? SOLUTION.—The difference in temperature is 90 − 60 = 30 deg. Now resistance of all pure metals increases about 0.004 ohm per deg. rise for each ohm original resist-

* See the author's AMERICAN ELECTRICIAN'S HANDBOOK.

ance. Therefore for a 30 deg. rise the increase per ohm would be $0.004 \times 30 = 0.12$. Hence the increase for 20 ohms would be $20 \times 0.12 = 2.4$ *ohms*. Therefore, the resistance at 90 deg. F. would be $20 + 2.4 = 22.4$ *ohms*. Art. 250 shows a formula whereby these calculations can be made directly. Note also Art. 153 for an exact method of determining resistances at different temperatures.

NOTE.—The resistance of carbon increases as its temperature decreases and decreases as its temperature increases.

148. Alloys with Zero Temperature Coefficients can be compounded. That is, the resistances of conductors composed of these alloys remain practically constant at all ordinary temperatures. Their temperature coefficients are 0.0. For example, *Manganin*, an alloy of 84 parts copper, 12 parts nickel and 4 parts manganese, all by weight, has a negligible temperature coefficient for practical purposes. Other alloys having similar properties are produced. The resistances of most alloys increase with their temperatures but to a less degree than do the resistances of pure metals.

149. Approximate Temperature Coefficients of Conductors.*

Conductor	(*A*) *a* Average temperature coefficient per degree C. between 0° and 100°C.	(*B*) *a* Average temperature coefficient per degree F. between 32° and 212°F.
Silver, pure annealed	0.004000	0.002220
Copper, annealed	0.004020	0.002230
Copper, hard-drawn	0.004020	0.002230
Aluminum (97.5 per cent. pure)	0.004350	0.002420
Zinc (very pure)	0.004060	0.002260
Iron wire	0.004630	0.002570
Nickel	0.006220	0.003460
Steel (wire)	0.004630	0.002570
Phosphor-bronze	0.000640	0.000356
German silver	0.000400	0.000220
Platinoid	0.000310	0.000172
Manganin	0.000000	0.000000

150. To Find the Resistance of a Conductor at Any Ordinary Temperature, Approximate Method but sufficiently accurate for all ordinary work. See Art. 153 for the exact method. The formulas are:

$$R_h = R_c + a \times R_c(T_h - T_c) \qquad \text{(ohms)} \tag{19}$$

* International Text Book company—ELECTRICAL ENGINEER'S HANDBOOK. See the author's AMERICAN ELECTRICIAN'S HANDBOOK for a more complete table.

or,

$$T_h - T_c = \frac{R_h - R_c}{a + R_c} \qquad \text{(deg. C.)} \tag{20}$$

Wherein R_h = resistance, in ohms, hot. R_c = resistance, in ohms, cold. T_h = temperature of conductor, hot, in degrees. T_c = temperature of conductor, cold, in degrees. a = the average temperature coefficient of the material of the conductor from Table 149.

EXAMPLE.—The resistance of a circular mil-foot of annealed copper is 9.59 ohms at 32 deg. F. What will be its resistance at 75 deg. F.? SOLUTION.—From Table 149 the coefficient is 0.002,23. Substitute in the formula (19): $R_h = R_c + a \times R_c(T_h - T_c) = 9.59 + [0.002,23 \times 9.59\ (75 - 32)] = 9.59 + (0.002,23 \times 9.59 \times 43] = 9.59 + 0.92 = 10.51$ *ohms* at 75 deg. F.

151. Why the Method of Art. 150 Is Not Exact.—The formula therein given assumes that the temperature coefficient of resistance is constant for all temperatures. This assumption is not strictly true because the temperature coefficient for a metal decreases as the temperature increases, as shown for copper in Table 152. The reason for this is that the resistance of any conductor is greater at, for example, 35 deg. C. than it is at 0 deg. C. Hence, the *proportional* increase in resistance for each ohm, for each degree rise in temperature, will be less at 35 deg. C. than for each ohm at 0 deg. C. The values given of a referred to in the formula of Art. 150 and given in Table 149 are average values.

152. To Find the Resistance of a Copper Conductor at Any Ordinary Temperature, Exact Method.—To obtain the exact resistance of a copper conductor at any temperature use this formula:

$$R_2 = R_1\,(1 \pm a \times T) \qquad \text{(ohms)} \tag{21}$$

Wherein R_2 = resistance, in ohms, at second temperature. R_1 = resistance, in ohms, at initial temperature. T = change or difference of temperature, in degrees, Centigrade. a = the temperature coefficient from Table 152 at the initial temperature of the problem.

EXAMPLE.—A copper field coil has a resistance of 20 ohms at 20 deg. C. What will be its resistance at 30 deg. C.? SOLUTION.—The change in temperature T is $30° - 20° = 10°$. The temperature coefficient for an initial temperature of 20 deg. C. is from Table 152, "0.00388." Now substitute in the formula (2): $R_2 = R_1\ (1 \pm a \times T) = 20\ (1 + 0.0038 \times 10) = 20(1 + 0.0388) = 20 \times 1.0388 = 20.78$ ohms at 30 deg. C.

153. Exact Temperature Coefficients for Copper.*—

a = change in resistance per degree Centigrade for each ohm at temperature t.

t Initial temperature, Centigrade	a Temperature coefficient	t Initial temperature, Centigrade	a Temperature coefficient
0	0.00420	26	0.00379
1	0.00418	27	0.00377
2	0.00417	28	0.00376
3	0.00415	29	0.00374
4	0.00413	30	0.00373
5	0.00411	31	0.00372
6	0.00410	32	0.00370
7	0.00408	33	0.00369
8	0.00406	34	0.00368
9	0.00405	35	0.00366
10	0.00403	36	0.00365
11	0.00402	37	0.00364
12	0.00400	38	0.00362
13	0.00398	39	0.00361
14	0.00397	40	0.00360
15	0.00395	41	0.00358
16	0.00394	42	0.00357
17	0 00392	43	0.00356
18	0.00391	44	0.00355
19	0.00389	45	0.00353
20	0.00388	46	0.00352
21	0.00386	47	0.00351
22	0.00385	48	0.00350
23	0 00383	49	0.00348
24	0.00382	50	0.00347
25	0.00381		

154. Temperature Rises in a Conductor Can Be Determined by Measuring the Resistance of the Conductor when cold and when hot by using this formula:

$$T = \frac{R_2 - R_1}{a \times R_1} \qquad \text{(deg. C.)} \tag{22}$$

* STANDARDIZATION RULES, American Institute of Electrical Engineers.

Wherein R_2 = resistance, in ohms, at second temperature. R_1 = resistance, in ohms, at initial temperature. T = change of temperature, in degrees, Centigrade. a = coefficient from Art. 152 at initial temperature.

EXAMPLE.—The resistance of a set of coils measured 20 ohms at a room temperature of 20 deg. C. After carrying current for a few hours the resistance measured 20.78 ohms. What was the average temperature rise in the coil? SOLUTION.—Substitute in the formula (22): $T = (R_2 - R_1) \div (a \times R_1) = (20.78 - 20) \div (0.00388 \times 20) = 0.78 \div 0.0776 = 10$ deg. = *average temperature rise.*

For ordinary commercial estimates a room temperature of 25 deg. C. (77 deg. F.) is often assumed and the above formula then becomes:

$$T = \frac{R_2 - R_{25°}}{0.00388 \times R_{25°}} \quad \text{(deg. C.)} \tag{23}$$

Wherein T = increase in temperature, in degrees, Centigrade. R_2 = resistance, in ohms, hot. $R_{25°}$ = resistance, cold, at room temperature, assumed to be 25 deg. C.

155. Heat Is Developed in Any Conductor through Which Electricity Flows and the temperature of the conductor is raised thereby. The heat represents the loss due to the overcoming of the resistance by the current. Often the amount of heat developed is very small and is not noticeable—but it is present nevertheless. If there is an excessive current in a conductor, heat may be developed more rapidly in the conductor than it can be dissipated—then the conductor will become very hot and may possibly melt. Heat is dissipated by air currents (*convection*) and by *radiation.* It is therefore often desirable to so arrange a conductor which must be kept cool, that cool air can circulate around it and the heat can be readily radiated from it.

Often the principal requirement of electrical conductors is that they be large enough to carry the necessary current without becoming too hot for safety. Tables have been compiled (Art. 156) indicating the safe currents for different size conductors. These should always be consulted before a conductor is selected to carry a given current.

EXAMPLES.—Fuses operate because of the heat developed in them by current; when the current becomes excessive, the fuse wire melts and thereby the circuit is automatically opened in case of overload. Incandescent lamps produce light because their filaments are heated white hot by the passage of current. Electric heating devices operate because the resistors therein are heated by the passage of current.

156. Dimensions, Weights and Resistances of Pure, Solid

* Gage No.	Diam., in.	Area		Safe carrying capacities		Weight	Sp. gr. 8.9
		Cir. mils (d^2) 1 mil = 0.001 in.	Sq. mils ($d^2 \times 0.7854$)	Rubber ins., amp.	Other ins., amp.	Lb. per 1,000 ft.	Lb. per mile
0000	0.460000	211,600.00	166,190.0	225	325	639.33	3,375.7
000	0.409640	167,805.00	131,790.0	175	275	507.01	2,677.0
00	0.364800	133,079.40	104,520.0	150	225	402.09	2,123.0
0	**0.324860**	**105,538.00**	**82,887.0**	**125**	**200**	**318.86**	**1,683.6**
1	0.289300	83,694.20	65,733.0	100	150	252.88	1,335.2
2	0.257630	66,373.00	52,130.0	90	125	200.54	1,058.8
3	0.229420	52,634.00	41,339.0	80	100	159.03	839.68
4	0.204310	41,742.00	32,784 0	70	90	126.12	665.91
5	**0.181940**	**33,102.00**	**25,998.0**	**55**	**80**	**100.01**	**528.05**
6	0.162020	26,250.50	20,617.0	50	70	79.32	418.81

No. 6 and larger conductors, where they are to be used in interior work or are to be
struction, solid wires up to and including No. 00 can be used but for larger conductors

7	0.144280	20,816.00	16,349.0	43	56	62.90	332.11
8	0.128490	16,509.00	12,966.0	35	50	49.88	263.37
9	0.114430	13,094.00	10,284.0	30	40	39.56	208.88
10	**0.101890**	**10,381.00**	**8,153.2**	**25**	**30**	**31.37**	**165.63**
11	0.090742	8,234.00	6,467.0	30	29	24.88	137.37
12	0.080808	6,529.90	5,128.6	20	25	19.73	104.18
13	0.071961	5,178.40	4,067.1	17	22	15.65	82.632
14	0.064048	4,106.70	3,225.4	15	20	12.44	65.674
15	**0.057068**	**3,256.70**	**2,557.8**	**10**	**14**	**9.84**	**51.956**
16	0.050820	2,582.90	2,028.6	6	10	7.81	41.237
17	0.045257	2,048.20	1,608.6	5	9	6.19	32.683
18	0.040303	1,624.30	1,275.7	3	5	4.91	25.925
19	0.035876	1,287.10	1,011.69	......	....	3.88	20.507
20	**0.031961**	**1,021.50**	**802.28**	The above		**3.09**	**16.315**
21	0.028462	810.10	636.25	values are those		2.45	12.936
22	0.025347	642.70	504.78	specified in the		1.94	10.243
23	0.022571	509.45	400.12	National Elec-		1.54	8.1312
24	0.020100	404.01	317.31	trical Code.		1.22	6.4416
25	**0.017900**	**320.40**	**251.64**	In lighting work,		**0.97**	**5.1216**
26	0.015940	254.01	199.50	no wire smaller		0.77	4.0656
27	0.014195	201.50	158.26	than No. 14 is		0.61	3.2208
28	0.012641	159.79	125.50	used, except for		0.48	2.5344
29	0.011257	126.72	99.526	fixtures.		0.38	2.0064
30	**0.010025**	**100.50**	**78.933**	....		**0.30**	**1.5840**
31	0.008928	79.71	62.604	....		0.24	1.2672
32	0.007950	63.20	49.637	....		0.19	1.0032
33	0.007080	50.13	39.372	....		0.15	0.7920
34	0.006304	39.74	31.212	....		0.12	0.6336
35	**0.005614**	**31.52**	**24.756**	....		**0.10**	**0.5280**
36	0.005000	25.00	19.635	....		0.08	0.4224
37	0.004453	19.83	15.567	....		0.06	0.3168
38	0.003965	15.72	12.347	....		0.05	0.2640
39	0.003531	12.47	9.7939	....		0.04	0.2112
40	0.003144	9.89	7.7676	....		0.03	0.1581

* **Calculated on the basis of Dr. Matthiesen's standard, namely, 1 mil of pure copper**

Bare Copper Wire.* (Approximate)
American Wire Gage or Brown & Sharpe's Gage

Length		Resistance at 75 deg. F.			Gage No.
Ft. per lb.	Ft. per ohm	*R*, ohms per 1,000 ft.	Ohms per mile	Ohms per lb.	
1 56	20,383.0	0.04906	0.25903	0.000076736	0000
1.97	16,165.0	0.06186	0.32664	0.00012039	000
2.49	12,820.0	0.07801	0.41187	0.00019423	00
3.14	**10,166.0**	**0.09838**	**0.51937**	**0.00038500**	**0**
3.95	8,062.3	0.12404	0.65490	0.00048994	1
4.99	6,393.7	0.15640	0.82582	0.00078045	2
6.29	5,070.2	0.19723	1.0414	0.0012406	3
7.93	4,021.0	0.24869	1.3131	0.0019721	4
10 00	**3,188.7**	**0.31361**	**1.6558**	**0.0031361**	**5**
12.61	2,528.7	0.39546	2.0881	0.0049868	6
15.90	2,005.2	0.49871	2.6331	0.0079294	7
20.05	1,590.3	0.62881	3.3201	0.012608	8
25.28	1,261.3	0.79281	4.1860	0.020042	9
31.38	**1,000.0**	**1.0000**	**5.2800**	**0.031380**	**10**
40.20	793.18	1.2607	6.6568	0.050682	11
50.69	629 02	1.5898	8.3940	0.080585	12
63.91	498 83	2.0047	10.585	0.12841	13
80 38	395.60	2.5278	13.347	0.20322	14
101.63	**321.02**	**3.1150**	**16.477**	**0.31658**	**15**
128.14	248.81	4 0191	21.221	0.51501	16
161.59	197.30	5.0683	26.761	0.81900	17
203 76	156 47	6.3911	33.745	1.3023	18
257.47	123 99	8.0654	42.585	2.0759	19
324 00	**98.401**	**10.163**	**53.658**	**3.2926**	**20**
408 56	78.067	12.815	67.660	5.2355	21
515.15	61 911	16.152	85.283	8.3208	22
649 66	49.087	20.377	107.59	13.238	23
819 21	38.918	25.695	135.67	21.050	24
1,032.96	**30.864**	**32.400**	**171.07**	**33.466**	**25**
1,302.61	24.469	40.868	215.79	35.235	26
1,642.55	19.410	51.519	272.02	84.644	27
2,071 22	15.393	64.966	343.02	134.56	28
2,611.82	12.207	81.921	432.54	213.96	29
3,293.97	**9.6812**	**103.30**	**545.39**	**340.25**	**30**
4,152 22	7.8573	127.27	671.99	528.45	31
5,236.66	6.0880	164.26	867.27	860.33	32
6,602.71	4.8290	207.08	1,093.4	1,367.3	33
8,328.30	3.8281	261.23	1,379.3	2,175.5	34
10,501.35	**3.0363**	**329.35**	**1,738.9**	**3,458.5**	**35**
13,238.83	2.4082	415.24	2,192.5	5,497.4	36
16,691.06	1.9093	523.76	2,765.5	8,742.1	37
20,854.65	1.5143	660.37	3,486.7	13,772.0	38
26,302.23	1 2012	832.48	4,395.5	21,896.0	39
33,175.94	0.9527	1,049.7	5,542.1	34,823.0	40

drawn into conduits, should be cables so they will be flexible. For outside pole-line con-
cables should be employed because of the greater ease of handling stranded conductors.

wire of 1/16 in. diameter equals 13.59 ohms at 15.5 deg. C. or 59.9 deg. F.

157. Contact Resistance is the resistance developed at the point of contact of two conductors in series. The greater the clamping pressure between the conductors in contact and the greater the area of contact, the less will be the contact resistance. If a contact has a high resistance, excessive heat will be developed thereat. Certain safe *current densities* (amperes per square inch) have been experimentally determined for contacts of different kinds: sliding, screwed, spring and the like (see table in AMERICAN ELECTRICIAN'S HANDBOOK). These safe densities should not be exceeded or excessive heating will result. See Art. 605 for "Brush Resistance" and "Brush Contact Resistance."

157A. Voltage Gradient* is the volts drop per unit length of circuit. It is used most frequently in connection with high-voltage phenomena but does not necessarily relate solely to such. For example, when a "charged" conductor C (Fig. 90A) is separated from ground by a dielectric or insulator, the potential of C above the potential of G may be expressed in volts which may be designated by the letter E. Thus, if a voltmeter V, connected as shown, reads 6,600 volts, then the potential of C would be 6,600 volts above that of the earth, which is always assumed to be zero. The average voltage gradient is obtained by dividing the potential difference by the distance.

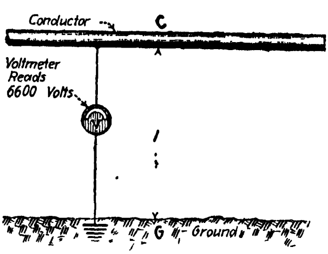


FIG. 90A.—Illustrating voltage gradient

EXAMPLE.—If the distance l in the illustration were 30 ft., then the average voltage gradient for these conditions would be: 6,600 ÷ 30 = 220 *volts per ft.* The dielectric strengths of insulating materials may be measured in *volts per unit length.* The voltage gradient at any point in an insulating material must not exceed the dielectric strength of that material or it will break down or rupture.

157B. The Equations for Voltage Gradient Calculations follow from the above discussion. When the voltage drop is uniformly distributed along a conductor (or an insulator):

$$G = \frac{E}{l} \quad \text{(volts per ft. or per in.)} \tag{23a}$$

or

$$E = G \times e \quad \text{(volts)} \tag{23b}$$

* *Electric Journal.*

and

$$l = \frac{E}{G} \qquad \text{(ft. or in.)} \tag{23c}$$

Wherein G = voltage gradient, in volts per foot or volts per inch length, depending on whether l is measured in feet or in inches. E = e.m.f., in volts, impressed across the length l of the conductor under consideration. l = length, in inches or in feet, of the portion of the conductor under consideration. G, when expressed in volts per inch, really represents the voltage impressed on a 1-in. cube of the conductor or dielectric under consideration, and is sometimes called* "the electric intensity at a point."

157C. Two Other Formulas for Voltage Gradient, which are important because of the truths which they disclose can be derived (as shown below) from formulas which have preceded. These formulas which state what Karapetoff calls "the Ohm's law for the unit conductor" are:

$$G = p \times U \qquad \text{(volts per in.)} \tag{23d}$$

and

$$G = \frac{U}{\gamma} \qquad \text{(volts per in.)} \tag{23e}$$

Wherein G = voltage gradient, in volts per inch length. p = resistivity of the material, in ohms per inch cube. U = current density, in amperes per square inch. γ = conductivity in mhos per inch cube.

DERIVATION OF ABOVE EQUATIONS.—From (23*a*), $G = E/l$. Now from (15*aa*), $l = A \times R/p$; then substituting this expression for l in (23*a*), $G = E \times p/A \times R$. But from (8*c*), $A = I/U$; then substituting this expression for A in the formula just preceding: $G = E \times p \times U/I \times R$. Now from (10), $E = I \times R$; hence $G = I \times R \times p \times U/I \times R$. The "$I \times R$" expressions cancel out leaving $G = p \times U$, which is equation (23*d*) above. Formula (23*e*) may be derived by a similar process.

* Karapetoff, in his THE ELECTRIC CIRCUIT.

SECTION 6

WORK, POWER, ENERGY TORQUE AND EFFICIENCY

158. Work is the overcoming of opposition through a certain distance. Work is measured by the product of the opposition times the space through which it is overcome. Work is also measured by the product of the moving force times the distance through which the force acts in overcoming the opposition. Work can be measured in foot-pounds (ft.-lb.). A *foot-pound* of work is the amount of work done in raising a weight of 1 lb. a distance of 1 ft. Also, a foot-pound is the amount of work done in overcoming a force of 1 lb. through a distance of 1 ft.

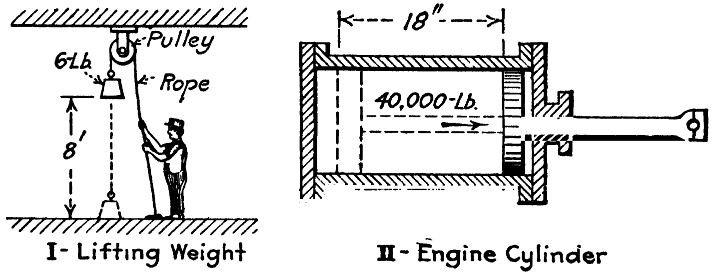


FIG. 91.—Examples of work.

EXAMPLES.—(1) If a weight of 6 lb. is lifted a distance of 8 ft. (Fig. 91,*I*), the work done will be $6 \times 8 = 48$ ft.-lb. (2) If 20 gal. of water are pumped a vertical distance of 32 ft. (1 gal. of water weighs 8 lb.), the work done by the pump will be $20 \times 8 \times 32 = 5{,}120$ ft.-lb. (3) If the piston in a steam engine travels, during a certain interval, $1\frac{1}{2}$ ft. (Fig. 91,*II*), and the total pressure on the piston is 40,000 lb., the work done during that interval would be $1.5 \times 40{,}000 = 60{,}000$ *ft.-lb.*

159. Power is rate of doing work. The faster that work is done, the greater the power that will be required to do it. Energy has to do only with work, while power has to do with work and time—foot-pounds and minutes.

EXAMPLES.—If it requires 10 horse power (*h.p.*) to raise a loaded elevator a certain distance in 2 min., 20 h.p. will be required to raise it the same distance in 1 min.

160. The "horse power" is a unit of power and is about equal to the power of a strong horse to do work for a short interval. Table 163 gives equivalent values for a horse power expressed in other units. Numerically a horse power (*h.p.*) is 33,000 ft.-lb. per min. = 550 ft.-lb. per sec. = 1,980,000 ft.-lb. per hr. Expressed as a formula:

$$(23) \qquad h.p. = \frac{L \times W}{33{,}000 \times t} = \frac{ft.\text{-}lb.\ per\ min.}{33{,}000} \qquad (h.p.)$$

Wherein $h.p.$ = horse power. L = distance, in feet, through which W is raised or overcome. W = weight, in pounds, of the thing lifted or the push or pull in pounds of the force overcome. t = the time, in minutes, required to move or overcome weight W through distance L.

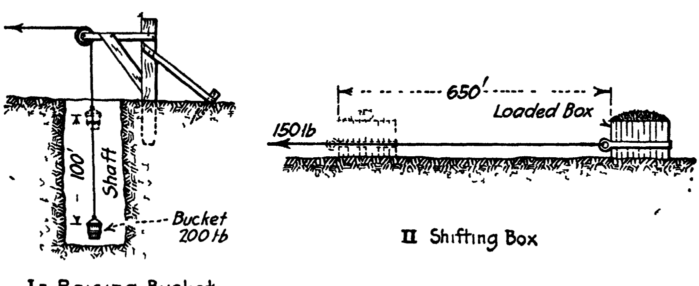


FIG 92 —Examples in calculating horse power.

EXAMPLE.—What horse power is required in raising the load and bucket weighing 200 lb., shown in Fig. 92,*I*, from the bottom to the top of the shaft, a distance of 100 ft., in 2 min.? SOLUTION.—Substitute in the formula (23): $h.p. = (L \times W) \div (33{,}000 \times t) = (100 \times 200) \div (33{,}000 \times 2) = 20{,}000 \div 66{,}000 = 0.3\ h.p.$

EXAMPLE.—What average horse power is required while moving the box loaded with stone, in Fig. 92*II*, from *A* to *B*, 650 ft. in 3 min.? It takes a horizontal pull of 150 lb. to move the box. SOLUTION.—Substitute in the formula (23): $h.p. = (L \times W) \div (33{,}000 \times t) = (650 \times 150) \div (33{,}000 \times 3) = 97{,}500 \div 99{,}000 = 0.98\ h.p.$

161. Electric Power Is Numerically Expressed in Watts or in kilowatts. A kilowatt is 1,000 watts. Electric power is the rate at which energy is being transformed in a circuit. See the following articles. Electric power is numerically expressed by the product of the instantaneous values of e.m.f. and current in a circuit. The *watt* represents the amount of power of a circuit

when the current in that circuit is 1 amp. and the e.m.f. is 1 volt.

162. Hydraulic Analogy of Electrical Power.—It is obvious from the picture of the hydraulic circuit shown in Fig. 93,*I*, that the power output of the water motor will depend on: (1) *the pressure* generated by the rotary pump; and (2) *the volume*, that is the gallons per minute pumped or forced through the water motor. More power will be developed by a 20 gal. per min. flow at 100 lb. per sq. in. pressure than at 50 lb. per sq. in. pressure. Furthermore, more power will be developed with a 100 lb. per sq. in. pressure by a 40 gal. per min. flow than by a 20 gal. per min. flow. The power developed by the water motor depends then on the pressure and on the flow.

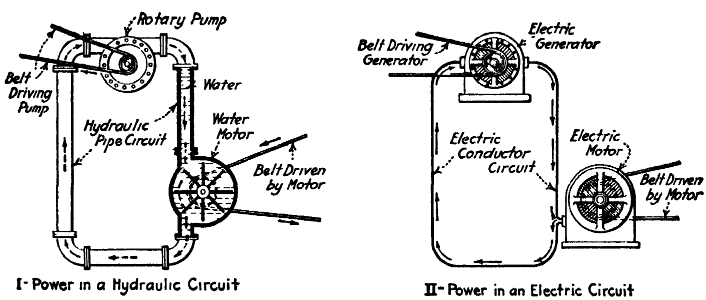


FIG. 93.—Power in circuits.

In the electric circuit shown in Fig. 93,*II*, the conditions are analogous. The greater the electric pressure (volts) impressed on the motor and the greater the flow (coulombs per second—or amperes) through the motor, the greater will be the amount of power developed by it. In the hydraulic circuit at *I*, if the pump generates a hydraulic pressure much greater than that for which the water motor is designed, the water motor will break down and if the generator in *II* generates an electric pressure much greater than that for which the electric motor is designed, the electric motor will break down. In both cases, *I* and *II*, the flow of current is determined by the pressure and if an excessive pressure causes an excessive flow, the motors will probably be injured.

163. Equivalent Values for Power Expressed in Various English and Metric Units.*

	Watt	Kw.	English h.p.	Continental h.p.	Kg.-m. per sec.	Ft.-lb. per sec.	Kg.-cal. per sec.	B.t.u. per sec.
1 watt is equal to.	1.00	0.001000	0.00134	0.00136	0.102	0.737	0.000238	0.000947
1 kw. is equal to..	1000.00	1.000000	1.34000	1.36000	102.000	737.000	0.238000	0.947000
1 English (and American) h.p..	746.00	0.746000	1.00000	1.01500	76.000	550.000	0.178000	0.707000
1 Continental h.p.	735.00	0.735000	0.98500	1.00000	75.000	541.000	0.175000	0.696000
1 kg.-m. per sec..	9.81	0.009810	0.01310	0.01330	1.000	7.230	0.002340	0.009300
1 ft.-lb. per sec...	1.36	0.001360	0.00182	0.00185	0.138	1.000	0.000324	0.001290
1 kg.-cal. per sec.	4200.00	4.200000	5.61000	5.70000	427.000	3090.000	1.000000	3.970000
1 B.t.u. per sec..	1055 00	1.055000	0.41500	0.42200	107.600	778 000	0.252000	1.000000

164. Power in Electrical Direct-current Circuits is equal to the product of volts and amperes. Expressing this rule as a formula:

$$P = I \times E \qquad \text{(watts)} \tag{24}$$

but since (Art. 134) $I = E/R$, it may also be stated that:

$$P = \frac{E}{R} \times E = \frac{E^2}{R} \qquad \text{(watts)} \tag{25}$$

and also since (Art. 134) $E = I \times R$, it may be stated that

$$P = I \times I \times R = I^2 \times R \qquad \text{(watts)} \tag{26}$$

Wherein I = current, in amperes. E = voltage, or e.m.f., in volts. R = resistance, in ohms. P = the power, in watts.

The above three equations are very important. (They may be subject to modification for alternating-current circuits, Art. 783.) In applying these formulas the same cautions (Art. 136) must be observed as with Ohm's law. The values of current, voltage and resistance used in any one problem must all apply to the same circuit or to the same portion of a circuit. Variations of the above three fundamental formulas are:

$$I = \frac{P}{E} \qquad \text{(amp.)} \tag{27}$$

or

$$I = \sqrt{\frac{P}{R}} \qquad \text{(amp.)} \tag{28}$$

$$E = \frac{P}{I} \qquad \text{(volts)} \tag{29}$$

* H. M. Hobart, in GENERAL ELECTRIC REVIEW.

or

$$E = \sqrt{R \times P} \qquad \text{(volts)} \tag{30}$$

$$R = \frac{E^2}{P} \qquad \text{(ohms)} \tag{31}$$

$$R = \frac{P}{I^2} \qquad \text{(ohms)} \tag{32}$$

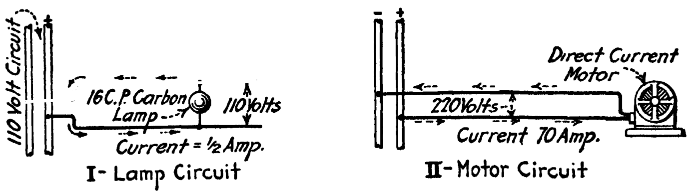


FIG. 94.—Power in direct-current circuit.

EXAMPLE.—How many watts are consumed by the incandescent lamp in Fig. 94,*I*? SOLUTION.—Substitute in the formula (24): $P = I \times E = \frac{1}{2}$ amp. $\times$ 110 volts $= 55$ *watts.*

EXAMPLE.—How many watts are taken by the motor of Fig. 94,*II*? How many kilowatts? How many horse power? SOLUTION.—Substitute in the formula (24): $P = I \times E = 70$ amp. $\times$ 220 volts $= 15{,}400$ *watts.*

$$kw. = \frac{\text{watts}}{1{,}000} = \frac{15{,}400}{1{,}000} = 15.4\ kw.$$

$$\text{h.p.} = \frac{\text{watts}}{746} = \frac{15{,}400}{746} = 20.6\ h.p.$$

EXAMPLE.—In the transmission line of Fig. 95 what will be the power lost in the line wires to the motor? SOLUTION.—Substitute in the formula (26): $P = I^2 \times R = (40 \times 40) \times (0.3 + 0.3) = 1{,}600 \times 0.6 = 960$ *watts.*

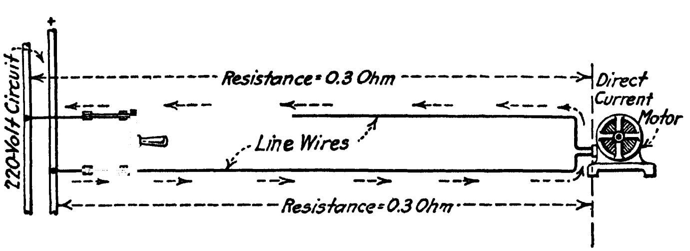


FIG. 95.—Power loss in a transmission line. (Current in line wires is 40 amp.)

165. In Applying the Equations of Art. 164 to Alternating-current Problems, it may be that certain corrections should be made to obtain the correct power value (Art. 782). In general,

the above equations may without great error be applied directly to alternating-current circuits if the connected load is *non-inductive* (Art. 471) or practically non-inductive. Incandescent lamps are practically non-inductive but alternating-current motors, arc lamps and most other devices containing coils of wire through which electricity flows are *inductive*. Where the circuit or load is inductive the above equations may not give a correct result (Art. 783) for alternating currents.

166. Watts, Kilowatts and Horse Power.—Since, as explained in Table 163, 1 h.p. equals 746 watts, it follows that:

$$h.p. = \frac{watts}{746} = watts \times 0.0013\ h.p. \qquad (33) \quad \text{(h.p.)}$$

$$watts = h.p. \times 746 \qquad (34) \quad \text{(watts)}$$

$$h.p. = \frac{kw.}{0.746} = kw. \times 1.34\ h.p. \qquad (35) \quad \text{(h.p.)}$$

$$kw. = h.p. \times 0.746 \qquad (36) \quad \text{(kw.)}$$

For ordinary estimates: to get horse power, multiply kilowatts by $1\frac{1}{3}$; to get kilowatts, multiply horse power by 0.7.

EXAMPLE.—Watts = 2,460, h.p. = ? SOLUTION.—Substitute in the formula (33): $h.p. = watts \div 746 = 2{,}460 \div 746 = 3.3\ h.p.$

EXAMPLE.—A motor takes 30 kw. How many h.p. is it taking? SOLUTION.—Substitute in the formula (35): $h.p. = kw. \div 0.746 = 30 \div 0.746 = 40.2\ h.p.$ or using the other equation (35): $h.p. = kw. \times 1.34 = 30 \times 1.34 = 40.2\ h.p.$

167. The Power Loss in any Conductor Traversed by an Alternating Current or a Direct Current is always, using the equation of Art. 164:

$$P = I^2 \times R \qquad (26) \quad \text{(watts)}$$

or

$$I = \sqrt{\frac{P}{R}} \qquad (28) \quad \text{(amp.)}$$

or

$$R = P \div I^2 \qquad (32) \quad \text{(ohms)}$$

Wherein P = the power lost in the conductor, in watts. I = current, in amperes, in the conductor. R = resistance of the conductor, in ohms. This rule is perfectly general and applies to all direct-current circuits and all alternating-current circuits of

ordinary voltages and frequencies. The watts power loss, P, reappears as heat power and heats the conductors and the things adjacent to them. The heat from the conductors is dissipated into the air and surrounding objects.

EXAMPLE.—What is the power loss in the incandescent lamp of Fig. 96, *I*? SOLUTION.—Substitute in the formula (26): $P = I^2 \times R = (2.2 \times 2.2) \times 98 = 4.84 \times 98 = 474$ *watts*.

This 474 watts appears as heat raising the lamp filament to a white-hot temperature and thereby produces light.

EXAMPLE.—What is the power loss in the inductive winding of Fig. 96,*II* with an alternating current of 3 amp.? SOLUTION.—Substitute in the formula (26): $P = I^2 \times R = (3 \times 3)7 = 9 \times 7 = 63$ *watts*.

168. Rating Motors in Kilowatts and in Horse Power.—The kilowatt and the horse power are both units of power. For many years motors were rated in horse power because most of

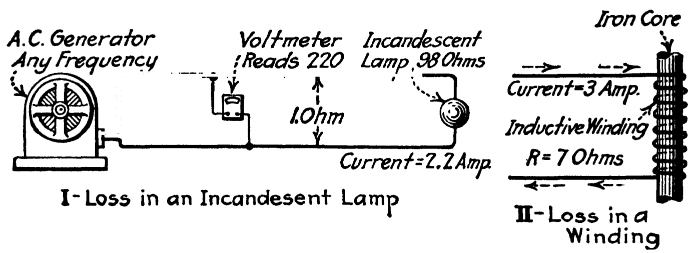


FIG. 96.—Illustrating watts power loss in conductors.

the possible purchasers of motors were more familiar with this unit than with the kilowatt. Motors are now, however, rated in kilowatts because this is the most convenient, logical and accurate unit. A *kilowatt* means precisely the same thing the world over while a *horse power* does not (see Table 163). Hence in the future the outputs of electric motors will be expressed in the kilowatts mechanical power available at the shaft. For practical purposes, the horse-power rating of a motor may be taken as four-thirds of its kilowatt rating.

169. Energy is capacity for doing work. Any body or medium which is of itself capable of doing work is said to possess energy. Energy can be expressed in foot-pounds or in units that can be reduced to foot-pounds.

EXAMPLES.—A clock spring that has been wound up or coiled possesses energy because in unwinding it will do work in driving the clock mechanism. A moving projectile possesses energy because it can overcome the resistance

offered by the air, by armor plate, etc., and thus do work. A charged storage battery possesses energy because it can produce electrical energy to operate a motor or to do many other kinds of work.

170. Energy of One Sort May Be Transformed into Energy of Another Sort.—Heat energy in coal may be transformed (but not without a certain loss) by a boiler, a steam engine and a generator, into electrical energy. The energy possessed by a stream of flowing water may be transformed, by a water wheel and a generator into electrical energy. There are definite numerical relations between the different sorts of energy.

Examples.—1 B.t.u. (*British thermal unit*, a unit of heat energy) = 778 ft.-lb. In electrical units, energy is expressed in *watt-hours* or in *kilowatt-hours*. Thus, 1 kw.-hr. = 2,655,000 ft.-lb. = 1.34 h.p.-hr.

171. A Kilowatt-hour represents the energy expended if work is done for 1 hr. at the rate of 1 kw. 1 kw.-hr. = 2,655,000 ft.-lb. = 1.34 h.p.-hr. A *watt-hour* is one-thousandth of a kilowatt-hour. 1 watt-hr. = 2,655 ft.-lb. = 0.001,341 h.p.-hr.

172. A Horse-power hour represents the energy expended if work is done for 1 hr. at the rate of 1 h.p. 1 h.p.-hr. = 1,980,000 ft.-lb. = 745.6 watt-hr. = 0.746 kw.-hr.

173. Torque is the measure of the tendency of a body to rotate. It is the measure of a turning or twisting effort and is usually expressed in *pounds-feet* or in pounds force at a given radius. Torque may exist even if there be no motion. Thus, in Fig. 97,*I*, the torque at the circumference of the drum is 50 lb. so long as the weight is supported, whether the drum is moving or standing still. It is assumed that the hoisting rope has no weight. Torque is sometimes expressed as the product of force introducing the tendency to rotate times the distance from the center of rotation to the point of application of the force.

Example.—In Fig. 97.*II*, the torque tending to turn the cylinder in the brick wall would be 100 lb. × 12 ft. = 1,200 *lb.-ft.* (In some text-books this would, inaccurately, be expressed as 1,200 ft.-lb.)

The cylinder can not turn and no work could be done, yet there is torque. Probably the best way of expressing torque is in terms of pressure (or force) and radius. Thus "100 *lb. force at 12 ft. radius.*" Ordinarily the expression is given for unit or 1 ft. radius. Because of the fact that many writers and engineers erroneously express units of both work and torque in foot-pounds a confusion sometimes exists regarding the distinction between

the two. *Work* (Art. 158) is properly expressed in foot-pounds (ft.-lb.) while torque should be expressed in pounds-feet (lb.-ft.), or preferably in pounds at a given radius.

EXAMPLE.—In Fig. 98, the tight side of the belt is pulling with a force of 50 lb. and the loose side with a force of 10 lb. The radius of the pulley is 2 ft.

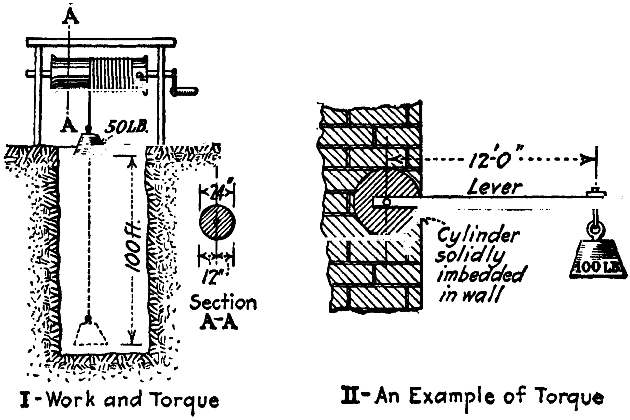


FIG. 97.—Examples of work and torque.

Hence the torque produced by the tight side tending to turn the shaft in the counter-clockwise direction is: 50 *lb.* × 2 *ft.* = 100 *lb.-ft.* The torque of the loose side tending to rotate the pulley in the clockwise direction is: 10 *lb.* × 2 *ft.* = 20 *lb.-ft.* The effective torque—in the counter-clockwise direction—is then: 100 *lb.-ft.* − 20 *lb.-ft.* = 80 *lb.-ft.* Or, solving the problem in another way: 50 *lb.* − 10 *lb.* = 40 *lb.* Then the effective torque is: 40 *lb.* × 2 *ft.* = 80 *lb.-ft.*

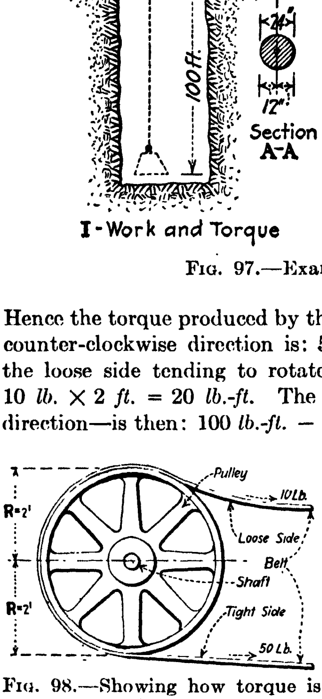


FIG. 98.—Showing how torque is exerted by a belt.

EXAMPLE.—The motor armature of Fig. 99 is developing 240 *lb.-ft.* torque. Then the pressure on the pinion and gear teeth at the pitch line which is 6 in. or half a foot away from the center of the motor shaft is: 240 *lb.-ft.* ÷ 0.5 *ft.* = 480 *lb.* The torque exerted on the shaft of the gear is: 480 *lb.* × 2 *ft.* = 960 *lb.-ft.* The power (Art. 159) developed by the gear is no greater—in fact it is a trifle smaller due to friction—than that developed by the pinion for the reason that the gear makes fewer revolutions per minute than does the pinion. The rate of doing work—the horse power —of the pinion and of the gear are practically the same. See Art. 677 on "Motor Horse Power."

EXAMPLE.—Fig. 100 indicates how an electric motor develops torque. This is treated more fully in Art. 627 and following articles. It is there shown

that when current flows in a conductor which is located in a magnetic field there is then a force tending to thrust the conductor from the field. Thus, in Fig. 100 the battery is forcing a current of electricity through the conducting loop which is located in a magnetic field due to two permanent magnets. The loop is free to turn on the shaft but is insulated therefrom. Due to the interaction of the current in the loop and the magnetic field a force develops tending to force A up out of the field and B down out of the field, thus producing a twisting moment or torque tending to rotate the loop.

Assume that the force tending to push A up was 10 lb., that the force tending to push B down was 10 lb. and that the distances R_1 and R_2 were each 6 in. or 0.5 ft. Then the A would exert: 10 *lb.* × 0.5 *ft.* = 5 *lb.-ft. torque.* Obviously B would exert the same and in the same direction. Hence, the total torque exerted by the loop would be: 2 × 5 *lb.-ft.* = 10 *lb.-ft. torque.*

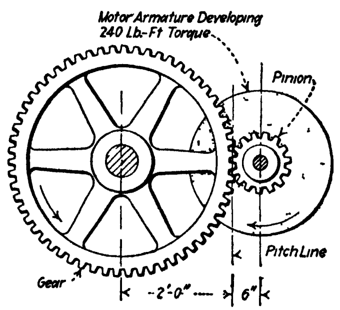


FIG. 99.—Transmission of torque by gears.

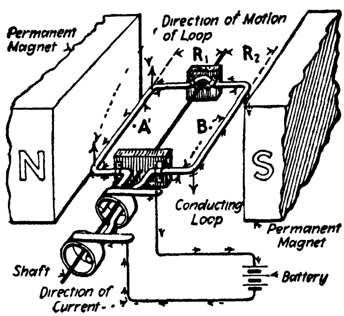


FIG. 100.—Illustrating the torque which produces the rotation of a motor.

174. The Prony Brake Formula Explained.*—First, consider the brake, B, clamped in place on the wheel, as shown in (Fig. 101), the wheel standing still. The man is lifting the end of the brake against friction with a force of, say, 200 lb. Assume that the distance, L, is 10 ft. In forcing the end of the brake through one complete revolution the man would do: 2 × 10 *ft.* × 3.1416 × 200 *lb.* = 12,566 *ft.-lb.* of work, regardless of the time consumed in doing it. But if he should move the brake around once every minute, his power could be measured in terms of horse power, for power (Art. 159) is the rate of doing work. One horse power is equivalent to 33,000 ft.-lb. of work per min. Therefore, if he did push the brake around once per minute he would develop: 12,566 ÷ 33,000 = 0.381 h.p. Thus, in order to do work at a rate equivalent to 1 h.p., the man would have to push the brake (against a resistance of 200 lb.) almost 3 r.p.m.

* N. G. Near, in the SOUTHERN ENGINEER, November, 1915.

NOTE.—The diameter of the circle through which his shoulder would have to move would be twice the radius L. Hence, the distance through which his shoulder would move would be equal to: *twice L multiplied by* 3.1416.

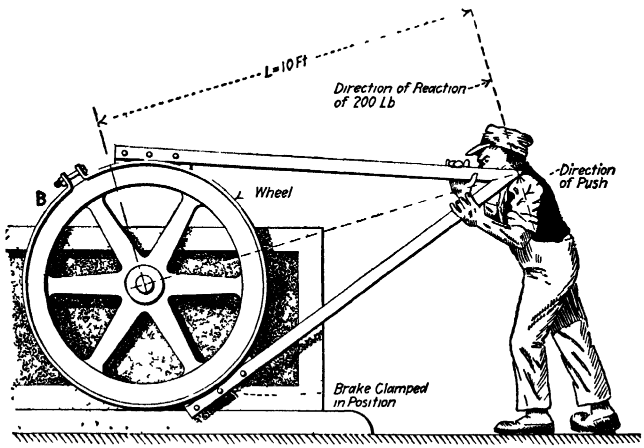


FIG. 101.—Illustrating the idea of torque.

If the *force* he exerts on the lever is represented by F, the work he would do during each revolution would be: $2 \times$ *distance* $L \times 3.1416 \times F$. Denoting the number of revolutions he makes

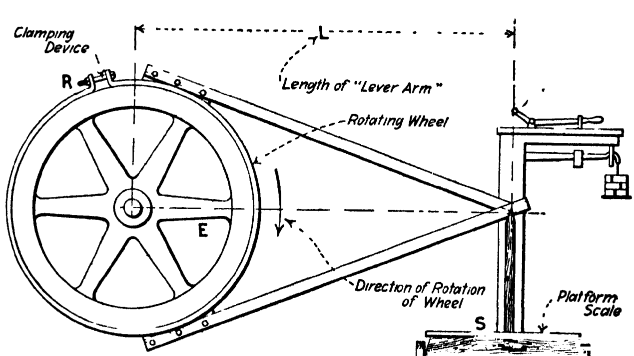


FIG. 102.—One common arrangement of a prony brake.

per minute by N, the number of foot-pounds of work he would do per minute would be: $2 \times L \times 3.1416 \times F \times N$. To reduce this horse power it is now necessary to divide by 33,000 thus:

(37) $$h.p. = \frac{2 \times L \times 3.1416 \times F \times N}{33{,}000} \quad \text{(horse power)}$$

This formula is used for finding the brake horse power of all types of engines and motors. When running a test, the brake lever is arranged (Fig. 102) to bear a scale, S, of some kind, and the pressure exerted upon the scale is regulated by means of the screw, R, on the brake where the ends of the bands are joined together. Each revolution of the engine flywheel, E, is equivalent to one complete revolution of the man (or of the scales) around the wheel. The same work would be done in either case.

175. Input.—Output is the useful energy delivered by a machine or device while input is the energy supplied to a machine. *Input* is what goes in and output is that portion which comes out and which is available for useful work.

176. Efficiency of an apparatus is the ratio of its net power output to its gross power input. No machine gives out as much power or energy as is delivered to it. There is always some loss due to friction or unuseful heating. It is for this reason that a perpetual-motion machine is an impossibility. There are always unavoidable losses even in the most perfectly constructed machines.

Refer to Fig. 93: If the water motor were frictionless its power output would equal its power input. Likewise, the output in horse power at the belt of the motor would be equal to the power input to the motor if there were no losses of power within the motor. Actually there are losses, sometimes large ones, in both water and electric motors and in all other machines. In making electrical estimates it is often convenient to consider that the mechanical power output of a machine is equal to its input. Often they *are* very nearly equal because the efficiencies of electrical machines are high. Some average efficiencies are given in Table 177. Efficiency is usually expressed as a percentage, thus, "the efficiency of a certain motor is 80 per cent." This means that only 80 per cent. of the electrical power received by the motor is delivered as useful power by the motor at the pulley. Stating this definition as a formula:

$$efficiency = \frac{output}{input} = \frac{output}{output\ and\ losses} \tag{38}$$

or

$$input = \frac{output}{efficiency} \tag{39}$$

or

$$output = input \times efficiency \tag{40}$$

NOTE that:—*input* = *output* + *losses*

EXAMPLE.—If 45 kw. is supplied to a motor and its output is found to be 54.2 h.p., what is its efficiency? SOLUTION.—Since 1 h.p. = 0.746 kw., 54.2 h.p. = 54.2 × 0.75 = 40.6 kw., then substituting in the formula (38). *efficiency* = *output* ÷ *input* = 40.6 ÷ 45 = 0.90 = 90 per cent. efficiency.

177. Average Efficiencies of Some Common Mechanical and Electrical Apparatus.—

Machine	Efficiency, per cent
MECHANICAL APPARATUS	
Steam engine	75 to 94
Gas engine	65 to 88
Water turbine	70 to 85
Reciprocating pump	60 to 90
Centrifugal pump	25 to 75
Fan	50
ELECTRICAL APPARATUS	
Generators, medium and large	80 to 96
Motors, medium and large	75 to 95
Transformers	93 to 98
Small motors, fan motors	35 to 60

SECTION 7

THE GENERATION OF ELECTRICAL ENERGY

178. Generation of Electrical Energy.—This subject is an important one. Hence, the student should peruse this division of the book very carefully. The reader should, before he proceeds, be certain that he has a good conception of the meaning of the word "energy" (defined in Art. 169), as it is used in engineering parlance.

179. The Real Meaning of the Term "Generation of Electrical Energy."—It should be understood that the term "generation of electrical energy" is, in a sense, misleading. Electrical energy can not be generated without the expenditure of some other kind of energy. What we really mean when we say that we are "generating electrical energy" is that we are transforming some other kind of energy into electrical energy. Thus, a generator or dynamo (Art. 509) is a machine whereby mechanical energy can be transformed into electrical energy. A cell or a battery (Art. 330) is a device whereby chemical energy is transformed into electrical energy.

EXAMPLE.—In Fig. 103,*I* the mechanical energy developed by the steam engine is transmitted to the generator by the belt. The generator transforms the mechanical energy into electrical energy which in turn is transmitted along the circuit wires to the incandescent lamps. The lamps are lighted because of the expenditure of electrical energy in them.

EXAMPLE.—The dry cell at *II* transforms chemical energy into electrical energy. When the button is pressed the bell rings, by virtue of the electrical energy transferred from the cell along the circuit wires. The chemicals and the metals comprising the cell contain chemical energy. As the cell is used this energy is consumed. After considerable use the chemical energy of the metals and chemicals of the cell will be "used up"—the cell will be "exhausted." Then new elements and chemicals must be supplied or a new cell must be installed if further energy is required.

When, then, a device or arrangement is said to "generate electrical energy," it should be remembered that although the device does, when considered in one way, generate electrical energy, it generates only by virtue of the expenditure of some other kind of energy.

180. Other Kinds of Energy Can Not be Transformed into Electrical Energy without Some Loss of Energy.—For example, all of the energy that the belt of Fig. 103,*I*, imparts to the generator will not be imparted by the generator to the circuit wires. If the generator has an efficiency (Art. 176) of 90 per cent. and 10,000 ft.-lb. of energy is imparted to it by the belt in an hour, only: $0.90 \times 10{,}000 = 9{,}000$ *ft.-lb.* of energy would be imparted to the circuit wires in the hour. See Art. 176 on "Efficiency." No machine or device can have an efficiency as great as 100 per cent. Most machines have an efficiency much less than 100 per cent.

Also, with the dry cell of Fig. 103,*II*, all of the chemical energy of its elements (Art. 373) can not be transformed into electrical energy which will be available at the circuit wires. There are certain losses of energy within the cell which, because of the

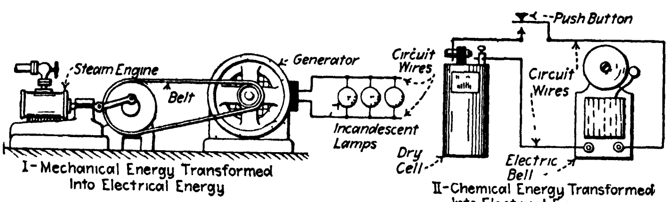


FIG. 103.—Transformation of other kinds of energy into electrical energy.

nature of things, can not be eliminated, though they may be minimized.

181. To Generate Electrical Energy, Electricity Must Be Forced to Move, that is, an electric current—a current of electricity—must be established. In Art. 90 it was noted that electricity may be thought of as a medium or agency for transmitting energy. When we generate energy we force some of this medium—electrons—to move. Electricity in motion constitutes an electric current (Art. 100). Hence to generate electrical energy a current must be forced through a circuit. But, as outlined below, a voltage is always necessary to force current through a conductor, that is, to establish a current.

182. The Establishment of a Voltage Is the First Requirement for the Generation of Electrical Energy.—It should be understood that electrical energy can not be generated directly. When it is desired to generate energy, a voltage, difference of potential, electric stress or pressure (these four terms are practically synony-

mous) must first be developed. The voltage will, if a suitable closed circuit (Art. 185) is provided, keep in motion along the conductors of this circuit a supply of electricity so long as the voltage is impressed. This supply of electrons may be considered as being already in existence in the conductors of the circuit and when a voltage is applied to the closed circuit, the supply is set in motion and it remains in motion so long as the voltage is applied and the circuit is closed. As outlined in Art. 181, electrical energy is developed when an electric current is maintained. Obviously, then, electrical energy is a result of an electric pressure or voltage forcing an electric current through a circuit for a given period of time.

When, therefore, it is desired to generate electrical energy, the first step is to develop a voltage by one of the methods described in Art. 184. Then, if a conductor is so arranged as to form a closed circuit, so that the voltage can force the electricity to circulate through it, electrical energy will be generated. Electrical energy can not be developed in an open circuit, that is, there can be no energy generated until current flows; this follows from the statements of Art. 181.

EXAMPLE.—If an electric generator, for example that of Fig. 103,*I*, be driven at its rated speed it will develop its rated voltage. However, it will develop no energy unless its external circuit be closed, that is, unless current flows. (It does develop a slight, negligible amount of energy, that necessary to excite itself, but it produces no energy for the external circuit unless the circuit is closed.) It follows then, that, since "generation of electrical energy" really means "transformation of energy," if there is no energy generated there is none transformed from mechanical into electrical energy. Therefore, the belt delivers no energy to the generator when the external circuit is open, except the negligible amount of energy required for the excitation of the generator.

It is evident, then, that a generator can develop a voltage without generating energy. This is true of any of the devices for generating electrical energy. All of them can, theoretically at least, develop voltage without generating energy.

183. Two Conditions Must Be Fulfilled if Electrical Energy Is to Be Generated, that is, if an electric current is to be made to flow (Art. 181). These conditions are, to repeat: (1) *There must be developed an electric pressure, difference of potential, e.m.f. or voltage; and* (2) *a suitable path* (*closed circuit*) *must be provided through which electric current can be circulated by this electric pressure.* These conditions are analogous to those necessary for the production of hydraulic energy in a hydraulic circuit. To

produce a flow of water there must be: (1) A hydraulic pressure or head; and (2) a path—pipe, canal, duct or flume—through which the water can be forced by the pressure.

184. Different Methods of Generating Electrical Energy.—As above noted, the initial step, if electrical energy is to be generated, is to produce an e.m.f. or voltage. It follows, then, that the different methods of developing e.m.fs., as outlined in Art. 119, are also the methods of generating electrical energy.

Thus:

METHODS OF DEVELOPING ELECTROMOTIVE FORCES
- (*a*) By *Contact of Dissimilar Substances* (Art. 310).
- (*b*) By *Chemical Action* (Art. 326).
- (*c*) By *Friction of Dissimilar Substances.*
- (*d*) By *Electromagnetic Induction* (Art. 416).

NOTE.—If the above four methods are arranged in the order of their commercial importance they are: (1) *Electromagnetic Induction,* (2) *Chemical Action,* (3) *Contact of Dissimilar Substances,* (4) *Friction of Dissimilar Substances.*

It is probable that methods (*a*), (*b*) and (*c*), above, are all merely manifestations of the development of e.m.f. due to contact of dissimilar substances which is specifically noted in method (*a*). It is an experimentally established fact that, whenever two dissimilar substances are placed in contact, an e.m.f. is established between them.

It should be understood that, fundamentally, the above four methods are merely methods of developing e.m.fs. or electric pressures. The e.m.f., if a suitable conductor be provided, will establish a current and energy will be generated when current flows. The amount of energy developed, assuming a constant pressure or e.m.f., will be proportional to the amount of electricity which flows. And, in turn, the current that flows will, by Ohm's law (Art. 134), be inversely proportional to the resistance of the circuit upon which the e.m.f. is impressed.

SECTION 8

ELECTRIC CIRCUITS

185. An Electric Circuit is the closed path, consisting of conductors, in which the electricity is moved in transferring energy (Arts. 90 and 169). Fig. 59 shows a simple, electric-bell circuit which contains the important elements found in nearly all electrical circuits: (1) Apparatus for generating electrical energy: *the battery;* (2) conductors for transmitting the electrical energy: *the wiring;* (3) a device for controlling the electrical energy: *the switch;* and (4) a device for utilizing or converting the electrical energy: *the bell.* Most electrical circuits are more complicated than this simple bell circuit.

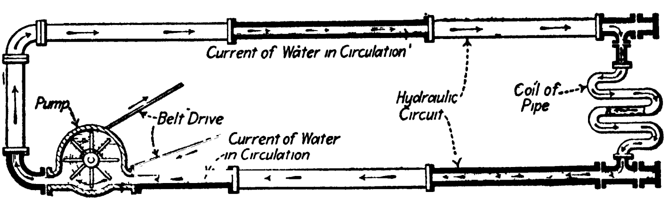


FIG. 104.—A smple hydraulic circuit.

186. Analogy between Hydraulic and Electric Circuits.—In Fig. 104 is shown a hydraulic or water circuit. In such a circuit some source of pressure or push must be provided to force the current of water to circulate around through the pipes against the opposition offered to it by the friction of the pipes. The rotary pump, driven by a belt, provides the push that circulates the current of water. Note that the pump does not create water; it only creates push or pressure. If the pump stops, the current of water will cease to circulate. Now note the similarity to the electricity circuit of Fig. 105. The electric "generator" forces a current of electricity around through the circuit against the opposition (*resistance*, Art. 124) of the conductors connected in the circuit. Note that the generator does not, strictly speaking, create electricity, although generators are sometimes incorrectly said to do so. The generator merely creates a pressure—a push

—(*voltage*, Art. 117) which causes a current of electrons to flow around in the circuit. Electrical pressure can be created in several other ways than by generators. See Art. 184. For instance, an electric battery of suitable size could be substituted for the generator of Fig. 105 and this battery would create an electrical push or pressure, just as the generator does.

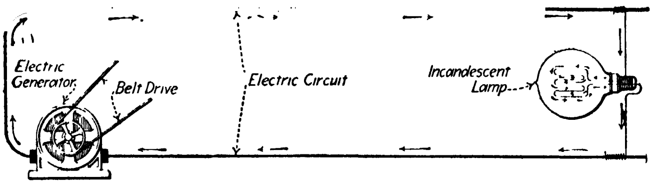


FIG. 105.—A simple electric circuit.

187. A Series Circuit is one in which the components are connected in tandem as in Figs. 106, 107, and 108. A series hydraulic circuit is shown in Fig. 106. The other illustrations show series electric circuits.

EXAMPLES.—Series circuits find their most important commercial application in series street lighting systems using series incandescent or arc lamps and are seldom if ever used in this country for the transmission of power. A constant-current generator or some other device is used for series lighting

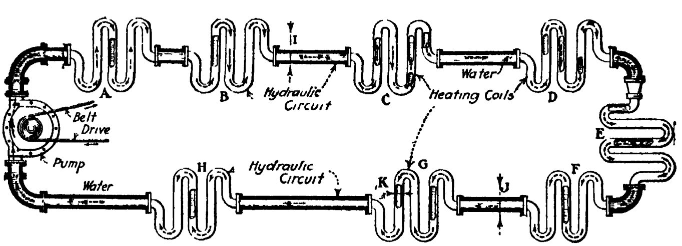


FIG. 106.—A series hydraulic circuit.

circuits. These devices will maintain the current flowing through the circuit at some certain value but will automatically vary the voltage impressed (Art. 102) on the circuit in proportion to the total resistance of the circuit so as to keep the current constant.

188. The Voltage of a Series Circuit equals the sum of the voltages across the components of the circuit. Hence, if it is desired to know what voltage must be impressed on a series circuit, or on a group of devices connected in series, to cause a certain current to flow through it: We must first ascertain the volts

required by each component by multiplying the resistance of the component by the current (Art. 134). All of the component voltages added together will give the total voltage required.

EXAMPLE.—Each of the eight incandescent lamps in the series circuit of Fig. 107 has a resistance of 4 ohms. The line wire has a total resistance of 1 ohm. What voltage must be impressed on the circuit to force a current of 5 amp. through it? SOLUTION.—The voltage required by each lamp will, by Ohm's law, be: $E = I \times R = 5 \times 4 = 20$ *volts* per lamp.

For the eight lamps, the voltage necessary will be $8 \times 20 = 160$ volts. The voltage required to force the current through the line wires will be: $E = I \times R = 5 \times 1 = 5$ *volts* for wire. Then adding to obtain total voltage required: $160 + 5 = 165$ volts. Hence, the generator must impress 165 volts on the circuit to cause 5 amp. to flow.

This problem could have been solved by adding together the component resistances thus: $8 \times 4 = 32$ ohms; 32 ohms + 1 ohm = 33 ohms, total. Now multiply this 33 ohms by the current 5 amp. $= 33 \times 5 = 165$ *volts*.

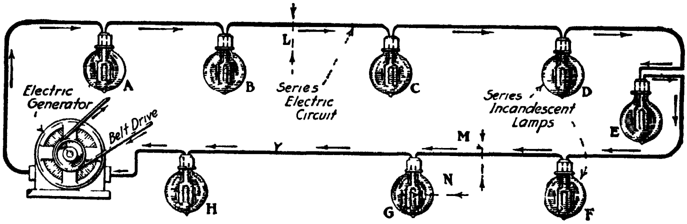


FIG. 107 —A series electric circuit.

189. The Current in a Series Circuit will be the same at all parts of the circuit (after the first fraction of an instant after the current commences to flow). The same current will flow through all components or devices that are connected in tandem or in series. The voltages and resistances of different series-connected components may be, and probably are, different but the current through each must be the same.

EXAMPLE.—It is evident in the hydraulic series circuit of Fig. 106 that the gallons per minute (current) flow must (assuming that the impressed hydraulic pressure is constant and that the pipes are kept full), be the same at all points of the circuit. The current past point I must be equal to that past any other point as J or K. Similarly in the electric circuit of Fig. 107, if 5 amp. is flowing through at L, 5 amp. must also be flowing past all other points in the circuit as for instance at M and N. When direct-current electricity flows through a conductor its action is just as if it spread out and permeated all portions of a conductor similarly to the way in which water may fill a pipe.

190. The Resistance of a Series Combination equals the sum of the resistances of the components. It is evident, from Fig. 108, for instance, that the two electric bells in series will have, assuming that both bells are just the same, twice the resistance of one bell. Connecting devices into a circuit in series increases the resistance of the circuit.

Example.—If in the series incandescent lamp circuit of Fig. 107 each of the eight lamps has a resistance of 4 ohms and the line wire has a total resistance of 1 ohm, the resistance of the complete circuit, exclusive of the resistance of the generator will be:—$4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 1 = 33$ *ohms.*

191. Parallel, Multiple or Shunt Circuits are those in which the components are so arranged that the current divides between them; see Figs. 110, 111 and 112. It follows that conductors so arranged in a circuit that there are as many paths for the current as there are conductors, may be said to be arranged in parallel or multiple. A parallel circuit is therefore a *divided circuit* (Art. 197). The principal distinctions between series and multiple, practical lighting and power circuits are: (1) In a series circuit, the current is automatically maintained constant and the voltage impressed on the circuit to force the current through varies as the load connected to the circuit varies; (2) in a parallel circuit, the current through the generator varies with the load and the voltage impressed on the circuit is automatically maintained practically constant. Nearly all power transmission circuits and interior lighting circuits are parallel circuits, hence a thorough understanding of the parallel circuit is essential.

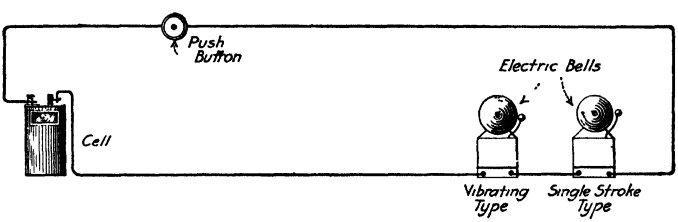


Fig. 108.—A series electric bell circuit.

192. A Hydraulic Analogy to a Parallel Circuit is shown in Fig. 109; the corresponding electric circuit is shown in Fig. 110. The belt-driven pump forces the current of water through the circuit. If pipes P_1 and P_2 are sufficiently large, there will be

practically the same pressure between a_1 *and* a_2, b_1 *and* b_2 and c_1 *and* c_2. It follows that practically the same current or amount of water will flow through each of the parallel-connected coils *A*, *B* and *C*, it being assumed that the coils are all alike. If, however, pipes P_1 and P_2 are small, a considerable portion of the pressure developed by the pump will be consumed in overcoming the opposition, friction, of the current of water against the interior of the pipe. There will be a loss or drop in pressure in P_1 and a similar loss in P_2. The pressure across a_1–a_2 will be less than that developed by the pump at P_1–P_2. And the pressure across c_1–c_2 will be less than that across a_1–a_2. The current of water that flows through coils *A*, *B* and *C* will be proportional to the pressure across them. It follows then that if P_1 and P_2 are small, a considerably smaller current will flow through *B* than through *A*. A considerably smaller current will flow

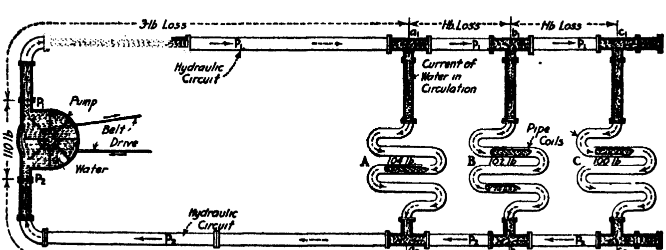


FIG. 109.—A parallel or multiple hydraulic circuit.

through *C* than through either *A* or *B*, because *C* is the furthest away from the pump.

EXAMPLE.—Assume that the pump (Fig. 109) develops 110 lb. per sq. in. pressure across P_1–P_2 and that the friction of the water in the pipes P_1 and P_2 causes the pressure to fall to 104 lb. per sq. in. across the nearest coil a_1–a_2. Correspondingly, the pressure might drop to 102 lb. per sq. in. across b_1–b_2 and to 100 lb. per sq. in. across c_1–c_2. The total drop or loss in pressure between P_1–P_2 and a_1–a_2 is $110 - 104 = 6$ lb. per sq. in. Half of this pressure is lost in P_1 and half in P_2 as shown on the illustration, it being assumed that both pipes are the same size. It follows that the pressure pushing water through *A* is 104 lb. per sq. in.; that pushing through *B* is 102 lb. per sq. in. and that through *C* is 100 lb. per sq. in.; whereas the pump develops 110 lb. per sq. in. Note that the piping system is so laid out that most of the loss of pressure (104, 102 and 100 lb.) occurs in the pipe coils *A*, *B* and *C* which are of very small diameter as compared with the main pipes P_1 and P_2. There is practically no loss of pressure (10 lb. total) in the main pipes.

The hydraulic circuit just described for distributing water through the pipe coils is, in general, analogous to the electric circuit shown in Fig. 110 for distributing electrical energy. In the hydraulic circuit the belt-driven pump creates a pressure that causes the current of water to flow. In the electric circuit the generator creates a pressure that causes electricity to flow. The hydraulic, pipe coils receive water at a practically constant pressure as do the electric lamps. The drop in pressure in the

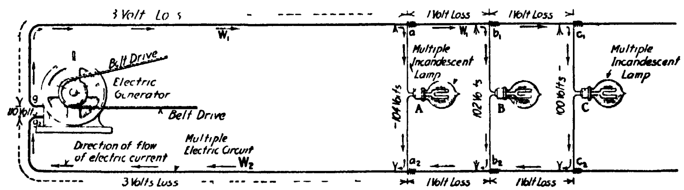


FIG. 110.—A parallel or multiple electric circuit.

pipe increases as the current of water increases. The drop in voltage in the mains increases as the current of electricity increases. If in either circuit, too many pipe coils or too many lamps are connected between the mains, the drop in pressure or voltage will be excessive and it will then be necessary to install larger supply pipes or wires to maintain the pressures across each of the coils or each of the lamps at practically the same

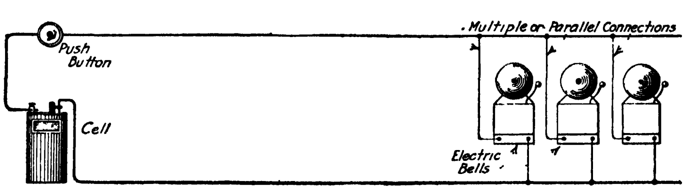


FIG. 111.—Electric bells connected in parallel.

value. It is assumed that the pump is of ample size to maintain the pressure across its outlets P_1 and P_2 at a constant pressure regardless of the magnitude of the load that is imposed on it. Likewise, it is assumed that the generator is big enough to maintain a constant terminal voltage for any load.

EXAMPLE.—In Fig. 110 the wires W_1 and W_2 correspond to the large pipes of Fig. 109. The incandescent lamps A, B and C correspond to the pipe coils. For illustration, the pressures in volts in this electric circuit have been made to correspond with the pressures in pounds per square inch in the hydraulic circuit. There is a certain drop in voltage, as shown, in the wires

that carry the current of electricity to the lamps. But the wires are big enough that this loss is relatively small and that the voltages impressed on all of the lamps by W_1 and W_2 are practically the same. As more lamps are connected between wires W_1 and W_2 the voltage drops in these wires will increase.

Lamp C receives only 100 volts; lamp B, 102 volts and lamp A, 104 volts. (The differences between these voltages are greater than are permissible in practice and are used only for illustration.) If c is much dimmer than A and B, it is an indication that W_1 and W_2 are too small. In practice several thousand lamps may all be connected in parallel to one generator; if the wires,

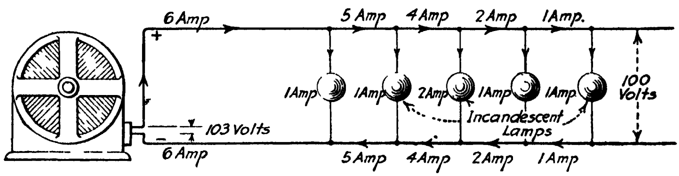


FIG. 112.—Distribution of current in a parallel circuit.

corresponding to W_1 and W_2, are large enough there will be no perceptible difference in the brilliancies of the lamps when they are all lighted. It follows that the voltage impressed on a circuit by a battery generator or other source must always be some greater than the voltage necessary at motors, lamps or other receivers. This is necessary to provide for a certain unavoidable *drop or "loss" of voltage* in the conductors between the source and the receivers. How to calculate wire sizes by the Ohm's law principle so as to keep this voltage drop within permissible limits is discussed in the author's AMERICAN ELECTRICIAN'S HANDBOOK.

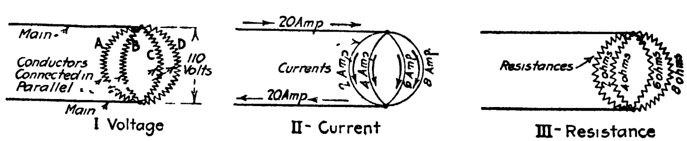


FIG. 113.—Voltage, current and resistance of parallel-connected receivers.

193. The Voltage Across a Group of Conductors Connected in Parallel is the same as the voltage across each member of the group. This is precisely true only where all of the conductors join the mains that serve them at exactly the same point, as in Fig. 113,*I*. Where the conductors that are in parallel do not connect to the mains at exactly the same point, as in Fig. 110, the voltages across the conductors will not be exactly the same but they may be very nearly the same.

EXAMPLE.—In Fig. 113,*I* the voltage across A, B, C and D is exactly the same.

194. The Total Current to a Parallel-connected Group of Conductors equals the sum of the currents in the conductors.

EXAMPLES.—The current values shown in Fig. 113,*II*, illustrate this principle. The current values shown on Fig. 112 also show the distribution of current in a parallel-connected circuit. Motors, heating devices or other receivers requiring electricity for their operation could be substituted for the incandescent lamps if the proper current values were substituted for those shown. Note that the current in the main conductors decreases toward the end of the run. The voltage at the end of the run is less than that impressed by the generator.

195. The Resistance of a Parallel-connected Group of Conductors is equal to the reciprocal of the sum of conductances of the conductors. A parallel-connected group of conductors constitutes a *divided circuit* (Art. 197).

EXAMPLE.—The resistances of the parallel-connected conductors of Fig. 113,*III* are respectively 2, 4, 6 and 8 ohms. The sum of their conductances

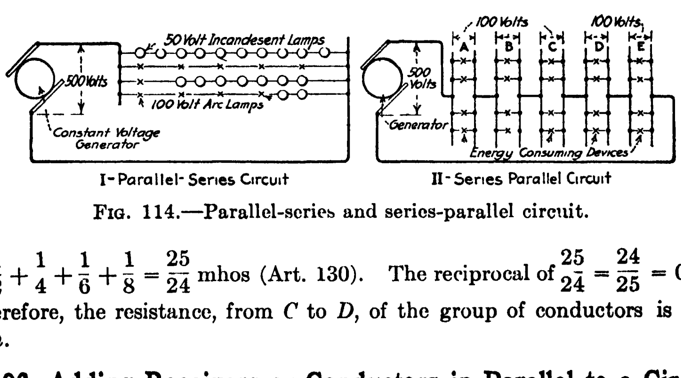


FIG. 114.—Parallel-series and series-parallel circuit.

$= \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} = \frac{25}{24}$ mhos (Art. 130). The reciprocal of $\frac{25}{24} = \frac{24}{25} = 0.96$. Therefore, the resistance, from C to D, of the group of conductors is 0.96 *ohm.*

196. Adding Receivers or Conductors in Parallel to a Circuit is really equivalent to increasing the cross-section of the imaginary conductor formed by all the receivers in parallel between the + and the − sides of the circuit. That is, it is equivalent to decreasing the resistance of the circuit.

197. A Divided Circuit (Figs. 113 and 114) is really one form of a multiple circuit (Art. 191). The distinction between the two sorts appears to be that, as ordinarily used, the term "divided" refers to an isolated group of a few conductors in parallel rather than to a group of a large number of widely distributed conductors in parallel.

198. To Compute the Resistance of a Divided Circuit or of a Number of Conductors in Parallel, the following formula, which

follows from the explanation of Art. 195, can be used. There should be as many terms in the denominator of the formula as there are conductors in parallel:

$$R = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4}, \text{ etc.}} \qquad \text{(ohms)} \tag{41}$$

Wherein R = the total resistance, in ohms, of the group. r_1, r_2, r_3, etc., are the respective resistances, in ohms, of the parallel-connected conductors forming the group.

EXAMPLES (see Art. 195 for an example).—What is the joint resistance of the conductors in the divided circuit shown in Fig. 115. In other words what is the resistance from A to B. SOLUTION.—Substitute in the formula (41):

$$R = 1 \div \left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right) = 1 \div \left(\frac{1}{5} + \frac{1}{10} + \frac{1}{15}\right) = 1 \div \left(\frac{6}{30} + \frac{3}{30} + \frac{2}{30}\right)$$
$$= 1 \div \frac{11}{30} = 1 \times \frac{30}{11} = 2.73 \textit{ ohms.}$$

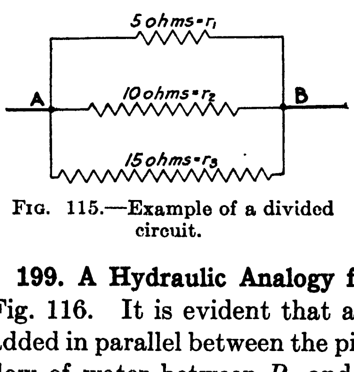


FIG. 115.—Example of a divided circuit.

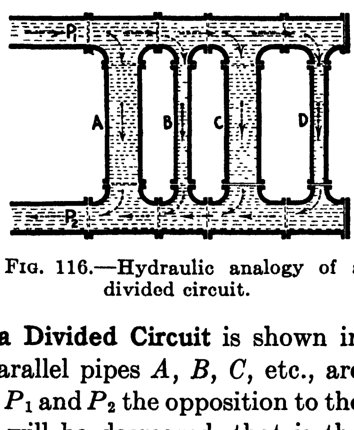


FIG. 116.—Hydraulic analogy of a divided circuit.

199. A Hydraulic Analogy for a Divided Circuit is shown in Fig. 116. It is evident that as parallel pipes A, B, C, etc., are added in parallel between the pipes P_1 and P_2 the opposition to the flow of water between P_1 and P_2 will be decreased, that is the resistance between P_1 and P_2 is decreased. A similar decrease in resistance occurs when electric conductors are added in parallel between two wires so as to form a divided or multiple circuit.

200. *A Parallel-series or a Multiple-series Circuit (Fig. 114,*I*) consists of a number of minor circuits in series with each other and which are then connected in parallel. Or, a parallel-series circuit consists of a number of series circuits connected in parallel. Arc and incandescent lamps for exterior illumination are sometimes arranged in this way. For example, five arc lamps each requiring a pressure of 100 volts, or ten incandescent lamps

* See footnote, p. 138.

requiring 50 volts each, and then these series groups are connected across a 500-volt railway circuit.

201. *A Series-parallel or a Series-multiple Circuit (Fig. 114,*II*) consists of a number of minor circuits connected in parallel and several of these parallel circuits connected in series. Or a series-parallel circuit comprises a series connection of a number of multiple circuits.

202. Kirchoff's Laws, so-called in honor of the man who developed them, are derived from Ohm's law and are very important. They are: (1) *At any point in a circuit the sum of the currents directed toward the point is equal to the sum of the currents directed away from the point.* This law is illustrated by Fig. 117,*I*,

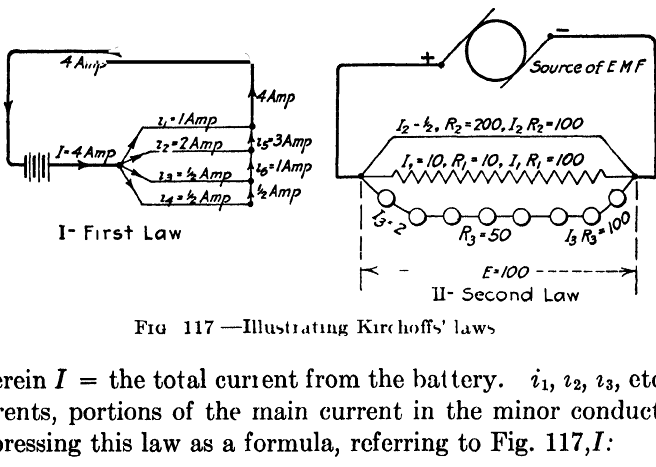


FIG 117 —Illustrating Kirchoffs' laws

wherein I = the total current from the battery. i_1, i_2, i_3, etc. = currents, portions of the main current in the minor conductors. Expressing this law as a formula, referring to Fig. 117,*I*:

$$I = i_1 + i_2 + i_3 + i_4 \qquad \text{(amp.)} \tag{42}$$

or

$$I = i_1 + i_5 \qquad \text{(amp.)} \tag{43}$$

or

$$i_6 = i_3 + i_4 \qquad \text{(amp.)} \tag{44}$$

Similar formulas can be applied to any circuit. The second law is: (2) *In any closed circuit, the sum of the IR (current × resistance) drops around any one path is equal to the e.m.fs. impressed*

* There has been some confusion regarding the exact meanings of the terms "parallel-series" and "series-parallel" The definitions as above given represent the best opinions that it has been possible to secure. Mr Steinmetz, consulting engineer of the General Electric Company, and Mr Lamme, chief engineer of the Westinghouse Company agree on the above definitions It is understood that a majority of the American Institute of Electrical Engineers Standardizing Committee favor the above given definitions.

on that path. See Fig. 117,*II* for an illustration. This law expressed as a formula (a similar formula can be written for any circuit), applying to the lettering of Fig. 117,*II*, becomes:

(45) $$E = I_1 \times R_1 = I_2 \times R_2 = I_3 \times R_3$$

203. Polarity of Direct-current Circuits Can Be Determined by the Evolution of Hydrogen at the Negative Conductor.*—As shown in Fig. 118, if the two ends of conductors connecting respectively to the sides of a circuit, having impressed on it a direct-

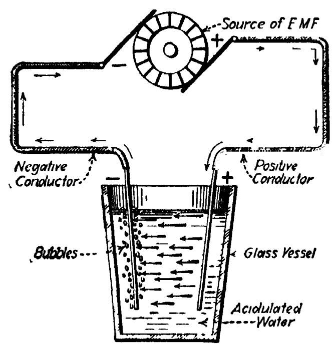


FIG. 118.—Determination of polarity by presence of hydrogen bubbles.

current voltage, are dipped in a vessel of water, hydrogen bubbles will form but only at the end of the conductor that connects to the negative (−) side of the circuit. The current flows toward the wire end on which the bubbles form. This also constitutes a method of detecting a direct e.m.f. Where the voltage is very low—such as that produced by a primary cell—it may be necessary to dissolve some common salt in the water to render it sufficiently conducting that hydrogen will be formed. Care must be taken that the wire ends do not touch and make a "short-circuit."

* See also articles "Practical Tests for Proving Polarity" in SOUTHERN ENGINEERS, February and March issues, 1917.

SECTION 9

ELECTROMAGNETISM

204. Current Electricity Always Produces a Magnetic Field—or, in other words, electrons in motion always produce a magnetic field. Wherever there is a current of electricity, there must also be a stream of magnetic lines of force—or a magnetic current. The magnetic field produced by a current of electricity always lies at right angles to the current that produces it. The truth of these statements can be demonstrated by many experiments, for instance:

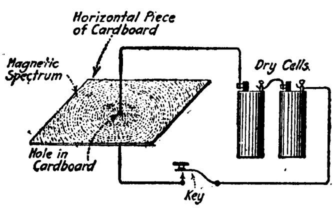


FIG. 119.—Magnetic spectrum about a conductor carrying a current.

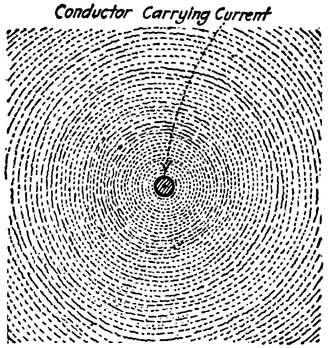


FIG. 120.—Showing magnetic spectrum of field about a conductor carrying current.

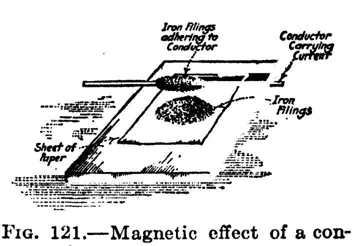


FIG. 121.—Magnetic effect of a conductor carrying current.

EXAMPLE.—If a conductor be arranged as shown in Fig. 119 and, the key being closed, iron filings are sifted on the horizontal cardboard, a magnetic spectrum (Fig. 120) will form. The directions of the lines of force about the conductor will be in concentric circles. Two or three dry cells will supply sufficient current. If a conductor carrying the current (Fig. 121) of two or three cells be dipped in iron filings some of the filings will adhere to it indicating that it now has magnetic properties. If the current ceases to flow through the conductor, the filings will drop off.

205. The Magnetic Field About a Straight Wire carrying current will be somewhat as indicated in Fig. 122. If the direction of

the current through the wire were reversed, the direction of the lines of force would be reversed. Each line of force is a complete closed circle around the conductor. If an isolated north pole (this is an imaginary conception because such an "*isolated*" pole can not exist) were placed in this circular field enclosing the conductor, it would whirl around and around in a circular path in

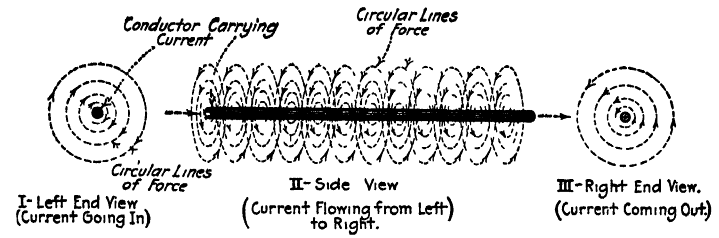


FIG. 122.—Magnetic field or lines of force about a straight conductor carrying current.

the direction of the lines of force. Note that, in *I* and *III*, the circular lines of force become further and further apart as the distance from each to the conductor increases. This is a graphic way of showing that the strength of magnetic field around the conductor decreases as the distance from the conductor increases.

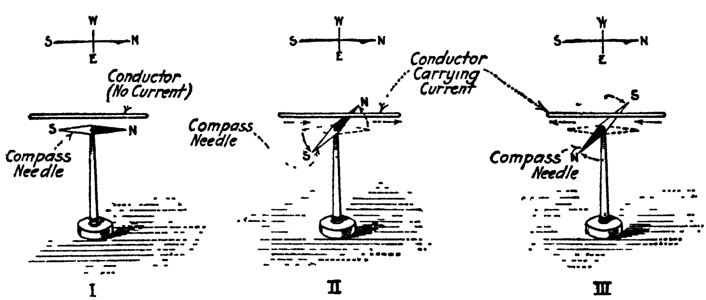


FIG. 123.—Deflection of a compass needle when near a conductor carrying current.

206. Experimental Proof of the Existence of a Magnetic Field About a Conductor Carrying Electricity.—If a pocket compass (Fig. 123) be brought close to a wire through which no current is passing, the compass needle will remain in its normal position as at *I*. That is, it will rest with its north-seeking end pointing toward the north pole of the earth. Now if a relatively strong electric current be forced through the conductor, the compass

needle will swing to a position at right angles to the conductor as at *II*. If the direction of flow through the conductor be reversed, the compass needle will reverse as at *III*. This shows that there is a magnetic field, a flux of lines of or a "magnetic current"—associated with the conductor.

207. The Correct Conception of a Field of Magnetic Flux Around a Conductor Carrying Current.—It should not be assumed that the circular lines of force enshrouding the conductor permeate only the space immediately adjacent to the conductor as diagrammed in Fig. 122. Actually these lines permeate all space, extending out from the conductor to an infinite distance. But the field becomes weaker and weaker—as the square of the distance—as the distance from the current-carrying conductor increases. That is, the circular lines become

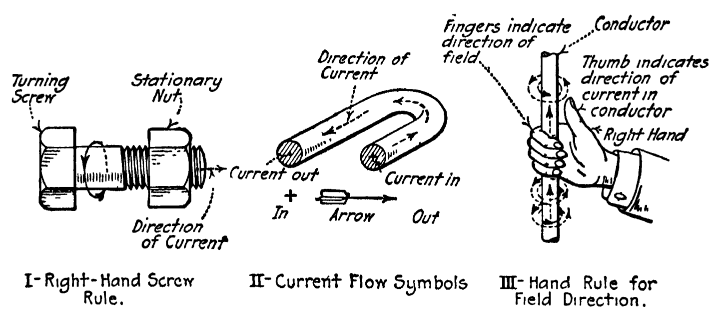


FIG. 124.—Methods of determining direction of magnetic field, current direction being known, and vice versa.

further and further apart as the distance from the conductor increases. Fig. 122 is only an illustrative diagram because the *actual* conditions could not be represented in the space available for the illustration.

In practice, the field usually becomes so weak a short distance away from the conductor—a few inches to a few feet—that it can be assumed for all practical purposes that there is no field except within the volume of space quite close to the conductor. Also, the field is distributed uniformly along all portions of a conductor in which there is the same current in amperes. See Arts. 226 and 56 for definitions of "a line of force."

208. Rules for the Direction of a Magnetic Field About a Straight Wire.—The field always lies at right angles to the conductor as shown in Fig. 122. The direction of the field bears

the same relation to that of the current that the direction of rotation of a right-hand screw (Fig. 124,*I*) bears to its forward or backward motion. If a wire through which electricity is flowing is so grasped (Fig. 124,*III*) with the right hand that the thumb points in the direction of the current flow, the fingers will point in the direction of the magnetic field and *vice versa.* If one looks along a current-carrying conductor in the direction of the current, the direction of the magnetic field will be clockwise, that is in the same direction as the hands of a watch.

209. A Compass Can be Used to Determine the Direction of the Current Flow in a Wire.—If the compass is placed under a conductor (Fig. 123) in which electricity is flowing from south to north, the north-seeking end of the needle will be deflected to the west. If the compass is placed over the conductor, the north-

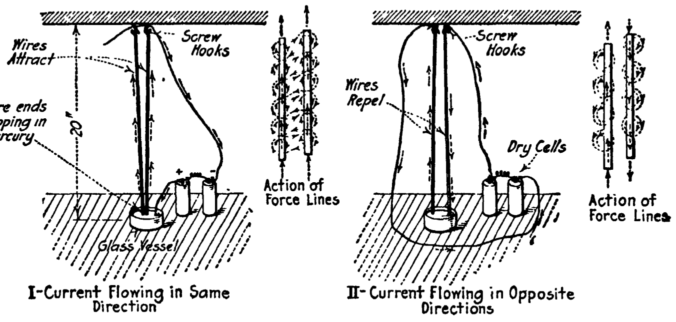


FIG. 125.—Illustrating action of parallel currents.

seeking end of the needle will be deflected to the east. If the direction of current flow in the conductor is reversed, the direction of deflection of the needle will be reversed correspondingly. A magnetized needle always tends to place itself at right angles to a conductor carrying current. The north, that is the north-seeking, end of the needle will point in the direction of the magnetic field. This direction can be determined by any of the rules of Art. 208.

210. Laws of Action Between Currents.—Conductors carrying currents attract or repel each other—or tend to—because of the magnetic fields (lines of force) generated around the conductors by the action of the currents. These three very important laws are:

1. *Parallel currents flowing in the same direction attract.* This is demon-

strated by the experiment of Fig. 125,*I*. Where the currents are flowing in the same direction in both conductors, the lines of force tend to encircle both conductors and contract like stretched rubber bands drawing the conductors together.

2. *Parallel currents flowing in opposite directions repel* as shown in Fig. 125,*II*. The oppositely flowing currents generate circular lines of force about them that are swirling in the same direction, hence they repel.

3. *Currents making an angle with each other tend to become parallel and to flow in the same direction.* This law is demonstrated by the experiment of Fig. 126,*I* and *II*.

All of the three above laws may be incorporated into one statement, which is one of the so-called Maxwell's laws, thus: *Any two circuits carrying current tend to so dispose themselves that they will include the largest possible number of lines of force common to the two.*

211. Another of "Maxwell's" Laws: *Every electromagnetic system tends to change its configuration so that the exciting circuit*

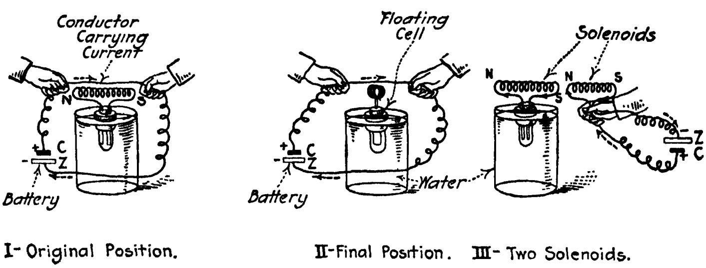


FIG. 126.—Illustrating the magnetic properties of solenoids.

will embrace the maximum number of lines of force in a positive direction. This means that any coil or loop which carries current will always tend to turn shift or distort into such a position or shape that the flux enclosed by it will be the greatest possible and that the flux lines will all be in the same direction. This law is very important because it explains effectively the operation of electric motors (Art. 625) measuring instruments and other essential phenomena. The law applies to every arrangement of closed circuits and magnetic field, irrespective of whether the fields are produced wholly or partially by the circuit or by other means.

EXAMPLES.—In Fig. 127, the helix and the magnetized knitting needle tend to turn into such positions that the exciting circuit—the helix—will enclose the maximum flux and so that the flux due to the helix will be in the same direction as that due to the knitting needle. In this case the flux is

due partly to the exciting circuit and partly to the permanent magnet, the needle which is held in the paper stirrup.

Now consider the apparatus of Fig. 128 where the flux is due wholly to the exciting circuit. The copper-wire bridge floats on the mercury and makes electrical connection with it. With no current flowing, the bridge wire is placed

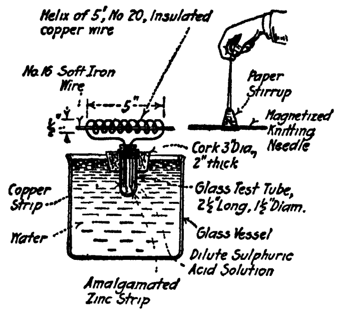


FIG. 127.—Construction of a floating battery.

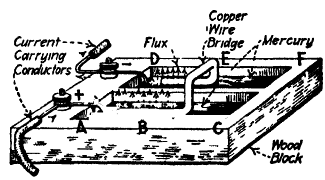


FIG. 128.—Apparatus for illustrating Maxwell's law.

in the position shown in the illustration. Now a current is forced through the mercury and bridge in the direction indicated. A flux is then produced by current as pictured and the bridge is immediately thrust—due to the sideways crowding tendency of the flux lines (Art. 56)—to the position *BCFE*. Note that the flux enclosed by the exciting circuit was originally only that in

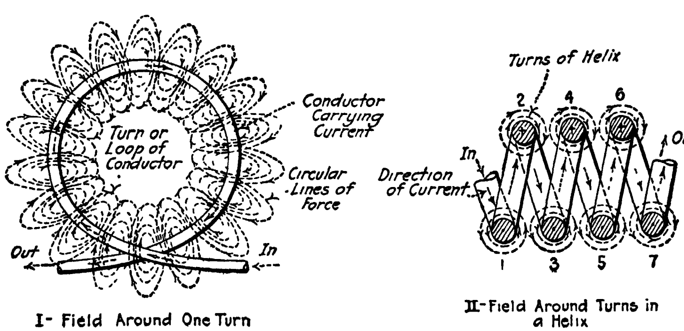


FIG. 129.—Magnetic fields around looped conductors.

area *ABEJD*. However, obeying its tendency to embrace the maximum flux, the area enclosed by the circuit enlarges until it is that designated by *ACFD*.

212. The Magnetic Field About a Conducting Loop or Turn carrying electricity is roughly represented in Fig. 129,*I*. The same circular lines of force or magnetic field surrounds the conductor as when it was straight, shown in Fig. 122. However,

when the conductor is bent into a loop (because the circular lines of force all whirl around the conductor in the same direction) all of the lines *enter* at one face of the loop and *leave* at the other face of the loop. This creates a north pole at one face of the loop and a south pole at the other face (Art. 215) in accordance with the provisions of the right-hand rule of Art. 216. The

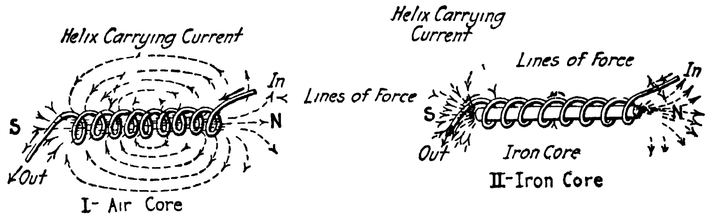


FIG. 130.—Illustrating effect of an iron core in a helix.

conductor of Fig. 129, *I* might be considered a helix (Art. 213) of 1 turn.

213. A Helix is any coil of wire having circular turns and carrying a current; Figs. 130, 131 and 132 show examples. Sometimes this term "helix" is applied only to a spiral having a length greater than its diameter and comprising but one layer of wire,

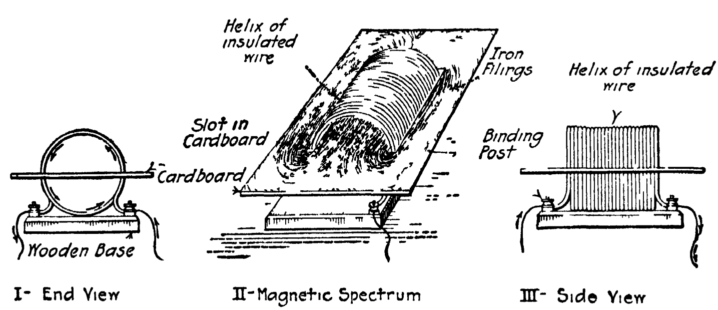


FIG. 131 —The magnetic spectrum of a helix.

but in this book the word "helix" will mean any coil as above defined. A *solenoid* is, as the term is generally understood, a helix that has very considerable length, as compared with its diameter, and a large number of turns, usually wound close together. It may have more than one layer of wire. A *toroid* is a helix bent into circular form as in Fig. 135,*I*.

214. Magnetic Field of a Helix.—If several loops or turns of wire are so wound as to form a helix (Fig. 133,*I*) a considerable

number of the flux lines produced by each of the turns will encircle the entire helix instead of encircling only the turn that generates them. Now if the turns are wound as close together as is possible (Fig 133,*II*), a great many more of the lines will

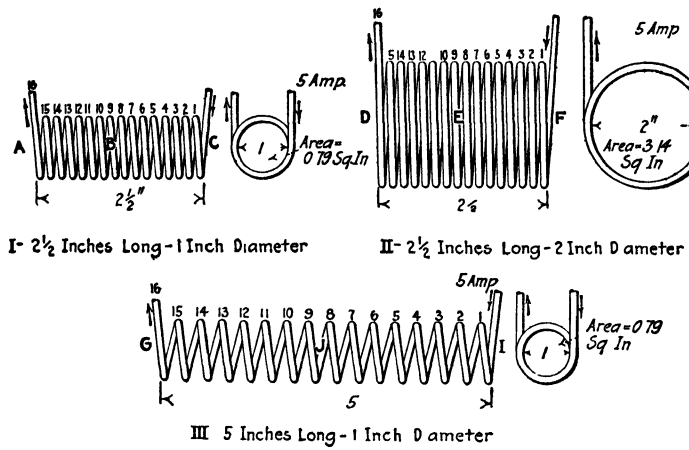


FIG 132—Helices carrying current (each helix has 16 turns)

encircle the entire helix, they will pass through the cylindrical space inside of the turns and return in the space outside of the turns Obviously, the field at N or at S is stronger (in Fig

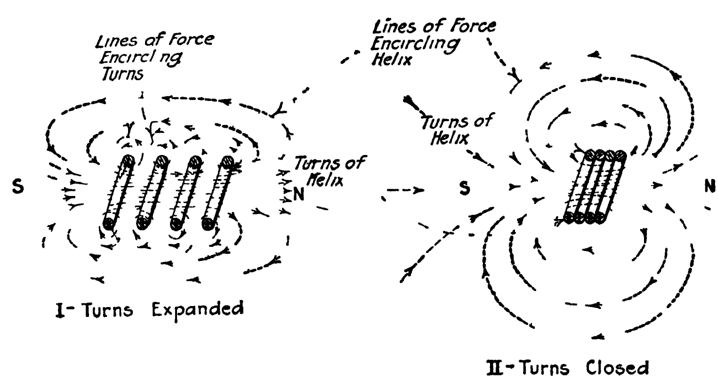


FIG 133—Illustrating increase in flux due to closing together the turns of a solenoid

133) at *II* than at *I*. Fig. 129,*II* shows why arranging the turns close together causes many of the lines to encircle the entire helix. The current circulates in the turns shown in the directions indicated by the arrows and produces the circular lines of

force illustrated. Consider that turns 3 and 5 are pushed close together. Since the circular lines about 3 have a direction such that they oppose those around 5, they will tend to neutralize each other if 3 and 5 are pushed close together. It is evident, then, that there is a tendency toward the neutralization of the field between the adjacent turns of a helix that are close together. Many of the lines of force must, therefore, flow in spaces which are not between adjacent turns, hence a considerable number of the lines generated by a helix encircle the entire helix. The total magnetic field developed by the current in a helix is, obviously, due to the combined action of all of the lines of force generated by each of its individual turns.

215. Properties of a Helix or Solenoid.—A helix or a solenoid (Art. 213) through which current electricity is passing has all of the properties of a permanent magnet. This can be shown with the floating battery apparatus of Figs. 126 and 127. A north pole is developed at one end of the helix and a south pole at the other, in accordance with the hand rule of Art. 208.

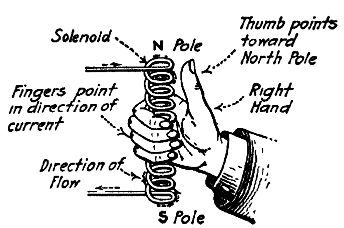


FIG. 134.—Hand rule for determining the polarity of a solenoid.

EXAMPLES.—The zinc and copper in the test-tube of Fig. 127 constitute a cell (Art. 330) that forces a current of electricity through the helix. This helix will be repelled or attracted by a permanent magnet or will repel and attract a permanent magnet just as if it were one itself. It behaves exactly like a magnet. If, as in Fig. 126,*I*, a conductor carrying current is placed above and parallel with the helix, the helix will immediately swing around and assume a position at right angles to the conductor as shown in *II*. When the helix and conductor are in the position of *I*, the currents in them are at right angles to one another and their lines of force oppose (see hand rules of Arts. 208 and 216) which causes the helix to swing around.

Assume now that a helix carrying current is brought near the floating helix as at *III*. If the currents in both helices are in the same direction, as shown, they will attract. If the current in one is in the opposite direction to that in the other they will repel—just as do permanent magnets when their unlike poles are brought together.

126. The Hand Rule for Determining the Polarity of a Helix, Solenoid or Electromagnet Is: If (Fig. 134) the solenoid or electromagnet be so grasped with the *right* hand that the fingers point in the direction of electricity flow, the thumb will point toward the north (or north-seeking) pole of the magnet.

217. If Iron is Placed Within a Helix, as in Fig. 130,*II*, the magnetic properties of the helix are then very much more pronounced. This is because iron is the best conductor of magnetism or lines of force that is known; it is a better conductor than air by several hundred times. The presence of iron in a magnetic circuit (Art. 219) decreases the opposition to the flow of lines of force and the number of lines is thereby very greatly increased; see "Permeability" (Art. 241). Where there is no iron core within a helix, some of the lines leak out of its sides between the turns and do not extend through it from end to end. Not only does the iron decrease this *magnetic leakage* but it also increases the number of lines in the magnetic circuit as above noted because iron is a better "conductor" of magnetism than is air. (Fig. 130,*II* indicates diagrammatically the increase in lines due to the presence of iron; it does not show the increase quantitatively because with an iron core the number of lines is increased several hundred times above the number, Fig. 130,*I*, with the air core.)

218. A Helix Surrounding an Iron Core Constitutes an Electromagnet.—A bar of hard steel can be permanently magnetized by being placed in a solenoid carrying current.

218A. The Electron Theory of Magnetism may, insofar as underlying principles are concerned, be explained thus: Electrons in motion constitute an electric current, Art. 101. Also, any electric current generates a magnetic field, Art. 204. Therefore, the electrons revolving about the nucleus of an atom (Art. 13) create a magnetic field through or in the atom—in about the same way as an electric current, circulating around through the turns of a coil of wire, creates a magnetic field through the coil.

EXPLANATION.—Hence each atom—or molecule—thus is, of itself, a minute magnet (note that this is supplementary to the explanation of Art. 68). However, in substances which exhibit no external evidence of magnetization, the atoms lie "every which way," so that the minute magnetic field due to any one of them is neutralized by the field of some other one, which is in such a position that their fields oppose and annul one another. In magnetic substances (Art. 45), such as iron or steel, the atoms are capable of being "lined-up," by any process which produces magnetization, so that their fields will all be in the same direction and thus act in unison. With non-magnetic substances, such "lining-up" can not be effected. Magnetizable (Art. 45) metals, such as steel, are those in which, after once being lined up, the atoms will remain so. In non-magnetizable substances, although the atoms may be lined up, they will *not* remain so unless the application of the magnetizing agent is continued.

SECTION 10

THE MAGNETIC CIRCUIT

219. The Laws of the Magnetic Circuit are similar to (but not the same as) those of the electric circuit. Thus, it will become evident as the reader proceeds, that the same general underlying principles which govern phenomena of electric circuits also govern those of magnetic circuits. (The term *magnetic circuit* was defined in Art. 57.) The reader will find that the *flux*—lines of force (—Arts. 70 and 225) in a magnetic circuit is analogous to the current in an electric circuit and that the *magnetomotive force*—the force to which the flux in a magnetic circuit is due—is analogous to the electromotive force, or voltage, of an electric circuit. Furthermore, he will learn that just as electric circuits have a property which is termed resistance, so every magnetic circuit has an analogous property which has been termed *reluctance.*

Hence, instead of having to become familiar with a new and distinct set of principles and ideas in order to understand magnetic-circuit phenomena, the reader will merely have to apply to a new group of quantities and units the same old general principles which he has already learned in connection with the study of the electric circuit. Understand that the phenomena, units and quantities for magnetic circuits *are not* the same as those for electric circuits, but they are analogous and the same essential underlying general ideas apply for both.

NOTE.—Acknowledgment is due Prof. V. Karapetoff for proposing a logical and rational development of the theory of the magnetic circuit and for placing magnetic-circuit theory on a firm rational engineering basis. The treatment given in this book is based on that outlined by Karapetoff in his most excellent mathematical treatise, "THE MAGNETIC CIRCUIT."

EXAMPLE.—Assume that suitable instruments, which can not be described here, are arranged somewhat as shown in Fig. 135. By reading the instruments and making certain calculations one can readily determine the flux generated by the toroid or circular helix when it has an air core as at *I*, and also when it has an iron core, as at *II*. The same current is assumed to flow in the helix in both instances. Also the toroid of *I* is of the same diameter and has the same number of turns as that of *II*. It is obvious, then, that

each of the toroids should have the same magnetizing effect. Now, it can be shown that the flux developed in the iron core of *II* is very much greater than that in the air core of *I*. In fact, under certain conditions, and with the same current flowing in *I* and *II*, assuming the cross-sectional area of each of the cores to be 1 sq. in., a flux of 23 lines will be established within the air core of *I*, but when an iron core is inserted in the toroid, as at *II* and with the same current flowing, the flux then developed within the toroid will be 40,000 lines. That is, by the insertion of the iron core, the flux has been

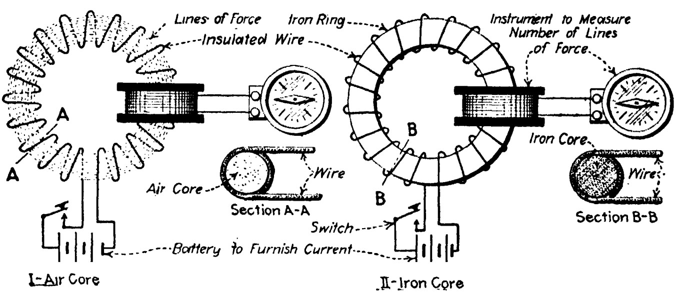


FIG. 135.—Arrangement for determining permeability of iron.

increased 1,739 times. The natural inference is that air offers a much greater opposition to the development of magnetic flux in it than does iron. This inference will be verified in a following article. That is, the "magnetic resistance" of iron, its reluctance, is much less than that of air.

220. Magnetomotive Force (abbreviated m.m.f.) is that force, or agent, due to which flux, lines of force, or magnetism, are set up in a magnetic circuit. In an electric circuit there must be an e.m.f. (Art. 182) impressed on the circuit before electrons (a current) can be forced to flow in it. Similarly, in a magnetic circuit there must be a m.m.f. before there can be flux. M.m.f. is the cause, flux is the effect. Thus, it is evident that m.m.f. in a magnetic circuit is analogous to electric pressure, voltage, or e.m.f., in an electric circuit.

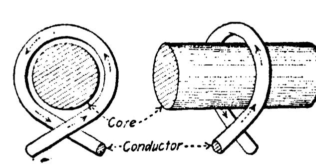


FIG. 136.—Showing one electromagnetic turn.

NOTE.—To produce an effect with any medium—water, air, electricity, or magnetism—a force, push, or pressure, is always necessary. With the electric circuit this pressure is called voltage (Art. 117) and with a magnetic circuit it is called *magnetomotive force.*

221. A Turn in electromagnetic parlance implies one wrap of a conductor around a core, which may, in the case of a solenoid, be an air core, or, in the case of an electromagnet, an iron core.

EXAMPLE.—Fig. 136 shows 1 turn around a core and Fig. 137,*I* and *II*, shows, respectively, 2 turns and 5 turns around a core.

222. The Practical and Rational Unit of M.M.F. Is the Ampere-turn.—That is, the ability to produce a flux of lines of force in an electromagnetic circuit is determined by the number of ampere-turns magnetizing that circuit. If the number of lines of force produced by a given helix be ascertained mathematically or experimentally, as in Fig. 135, it will be found that the number of these lines is, in the last analysis, proportional to just two factors: (*a*) *the current, in amperes, flowing in the helix*, and (*b*) *the number of turns in the helix.* The voltage impressed on the helix, and the size of wire used in—that is, the resistance of—the helix have nothing to do with the situation, except indirectly. The flux developed within a coil like that of Fig. 135,*I*, will not change if the current (ampere) and the number of turns in the "exciting" winding so vary that their product remains the same.

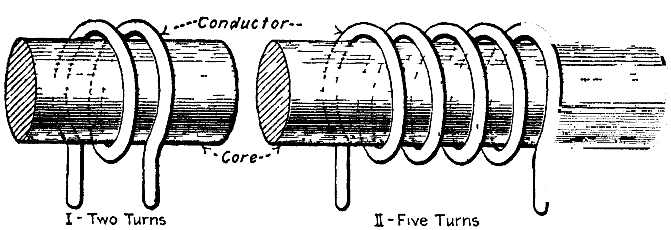


FIG 137 —Illustrating the meaning of the term "turn"

EXAMPLE.—If a current of 20 amp. circulates around a coil of 5 turns, the m.m f. of that coil in ampere-turns is 20 × 5 = 100 *amp.-turns.* Also, if 1 amp. circulates in a coil of 100 turns, the m m.f. of that coil is 1 × 100 = 100 *amp.-turns.* If 10 amp. circulate in a coil of 10 turns, again, the m.m.f. developed by this coil is 10 × 10 = 100 *amp.-turns.* If 50 amp. circulate in a coil of 2 turns, the *m.m.f.* of the coil is 50 × 2 = 100 *amp.-turns.*

EXAMPLE.—In Fig. 132, each of the three helices shown has 16 turns and a current of 5 amp. is flowing through each helix. Hence, the ampere-turns of each of the three is: 5 *amp* × 16 *turns* = 80 *amp.-turns.*

NOTE.—A given number of ampere-turns will produce a flux comprising more lines of force in a short magnetic circuit, or in one of large cross-sectional area, than in a magnetic circuit which is long or of small cross-sectional area. But, for a given magnetic circuit, the greater the number of ampere-turns, the greater will be the flux produced.

NOTE.—Where very large m.m.fs. are involved, as for example, in the magnetic circuits of electrical machines, it is customary to specify these m.m.fs. in *kiloampere-turns.* One kiloampere-turn is equal to 1,000 amp.-turns, hence, 20,000 amp.-turns = 20 kiloamp.-turns.

223. Why Magnetizing Effect Is Proportional to the Product of Amperes and Turns may be evident from a consideration of Figs. 138, 139 and 140. Consider the helix of square wire containing 20 turns wound around a core as shown in Fig. 138. The m.m.f. of this helix is 20 amp.-turns. The turns of the helix are insulated from one another and from the core. With a current of 1 amp. flowing in this helix, it is obvious that it will produce a certain

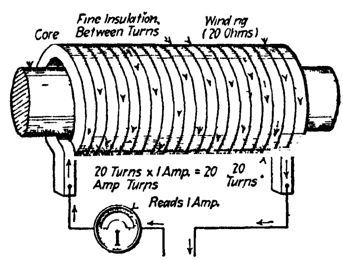


FIG. 138—20 turns and 1 amp.

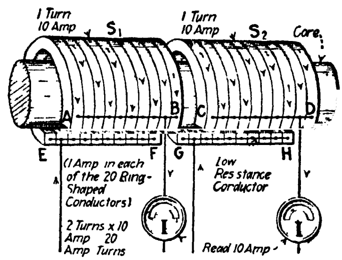


FIG. 139.—2 turns and 10 amp.

magnetizing effect in the core. Now, assume that the helix is divided into two sections, S_1 and S_2, as shown in Fig. 139. Furthermore, assume that each turn is cut through with a fine saw. (The width of the cut is exaggerated in Fig. 5 so as to bring out the details clearly.) Now, assume that the turns are connected in parallel, as shown, by the very low resistance conductors, *AB*, *CD*, etc. When connected in this way, each of the two sections, S_1 and S_2, really constitutes a single turn around the conductor. Now, it is evident that if a current of 10 amp. be forced through each of the sections, the ampere-turns of the arrangement will be: 2 *turns* $\times$ 10 *amp.* = 20 *amp.-turns,* the same m.m.f. as with Fig. 138. Furthermore, it is also apparent that in each individual ring conductor around the bar, a current of 1 amp. will flow, so that the magnetizing effect of the arrangement of Fig. 139 must be the same as that of Fig. 138.

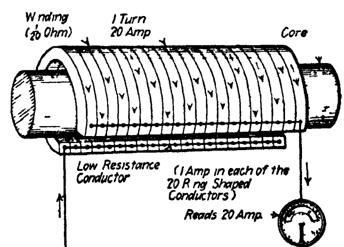


FIG. 140.—1 turn and 20 amp.

Also, if the 20 turns are all connected in multiple, as shown in Fig. 140, and each one opened with a saw cut, the arrangement is equivalent to 1 turn around the conductor. With 20 amp.

forced through this 1 composite turn, its m.m.f. would be 20 *amp.-turns.* However, just 1 amp. would flow in each elemental turn, and the magnetizing effect would, obviously, be the same as that with coils of Figs. 138 and 139.

224. The Formulas for the Ampere-turn follow from what has preceded, thus:

$$M = I \times N \qquad \text{(amp.-turns)} \tag{46}$$

$$I = \frac{M}{N} \qquad \text{(amp.)} \tag{47}$$

$$N = \frac{M}{I} \qquad \text{(turns)} \tag{48}$$

Wherein M = m.m.f., in ampere-turns. I = current, in amperes, flowing in the winding under consideration. N = number of turns in the winding under consideration.

EXAMPLE.—If a certain solenoid is developing a m.m.f. of 1,200 amp.-turns, and there are 60 turns in the solenoid, the current in that solenoid would necessarily be from equation (47): $I = M \div N = 1{,}200 \div 60 = 20$ *amp.*

EXAMPLE.—Assume that it is necessary to produce a m.m.f. of 2,600 amp.-turns, and that only a series circuit which always carries a constant current of 6.6 amp. is available. How many turns would a solenoid of this circuit have to contain in order to develop the required m.m.f.? SOLUTION.—Substitute in equation (48): $N = M \div I = 2{,}600 \div 6.6 = 394$ *turns.*

225. Flux has been referred to in the preceding Art. 70. Flux is the total "magnetism" or total number of lines of force in the magnetic circuit passing through a cross-section taken at right angles to the direction of the lines of force. Just as in a series electric circuit the current flowing in every part of the circuit is the same, likewise in a series magnetic circuit, the total flux flowing in every part of the circuit is the same, at any part of the complete circuit. It should be noted (see Art. 267, "Magnetic Leakage"), however, that since there is no "insulator" for magnetism, it is impossible to confine flux in a definite path in the same way that an electric current may be confined in a conductor. For this reason, it may be difficult in many cases to ascertain the flux at a certain cross-section of a magnetic circuit because of the difficulty of accurately defining the extent of the cross-section. The cross-section may include both magnetic and non-magnetic materials.

226. A Line of Force, or a maxwell, has been defined in a general way in a preceding section (Art. 60). It will now be defined

quantitatively because the *line of force is a unit of magnetic flux*, just as the ampere is the unit of electric current. The definition which will be given is based on the observed experimental fact (which will be discussed in detail in a following section, Art. 418) that when a conductor is moved across a flux so as to cut through the lines of force, an e.m.f. or voltage will be induced in the conductor. A magnetic flux of one hundred million (100,000,000) lines of force—or a hundred million maxwells—has been arbitrarily defined as that uniform flux which, if a conductor is moved through it, so as to cut across the lines at a uniform speed of just 1 sec., the e.m.f. induced in the conductor will remain constant during the second and be equal to just 1 volt. *Hence, a flux of* 1 *line is that flux which would induce an e.m.f. of* $\frac{1}{100,000,000}$ *volt in a conductor moved through the flux in* 1 *sec.*

NOTE.—The following definition from Karapetoff's "THE MAGNETIC CIRCUIT" which really has the same meaning as that above given, is worth noting: "*A flux through a turn of wire changes at a uniform rate of* 100,000,000 *lines per sec. when the e.m.f. induced in the turn remains constant and equal to* 1 *volt.*"

NOTE.—A kiloline is equal to 1,000 lines of force. A megaline is equal to 1,000,000 lines of force. Examples illustrating quantitatively how e.m.fs. are induced in conductors which cut through flux are given in a following section of this book which relates to the induction of e.m.f. by cutting flux.

227. Reluctance (how to compute reluctance will be explained later) is the name that has been given to that property of materials which opposes the creation of magnetic flux in them. The symbol for reluctance is $\mathcal{R}$. Reluctance, then, indicates the "difficulty" encountered in creating magnetic flux in a material. With electric circuits, the property of substances which opposes or limits the flow of current in them is called resistance. The analogous property of substances in magnetic circuits is reluctance. In electric circuits nearly all substances have different resistance properties, some offering little, and others great opposition to the establishment of electric currents. In magnetic circuits, nearly all substances except the magnetic metals (Art. 45) have practically the same reluctances. Iron has a relatively low reluctance, while air and all other non-magnetic materials have the same and a relatively high reluctance. Numerically, reluctance is the reciprocal of permeance (Art. 63), that is (see Art. 238): $\mathcal{R} = 1 \div \mathcal{P}$.

228. The Practical Unit of Reluctance is the Rel.—A magnetic circuit has a reluctance of 1 rel when a m.m.f. of 1 amp.-turn

generates in it a flux of 1 line. The rel is analogous to the ohm. Just as the ohm is the resistance of a column of mercury 41.85 in. long and 0.00049 in. in diameter (Art. 126), the rel is the reluctance of a prism of air, or any other non-magnetic material, Fig. 141, 1 in. square and 3.19 in. long. The reluctances of different materials may be determined by tests similar in general to the methods used in determining the resistances of substances.

EXAMPLES.—The reluctance from A to B, Fig. 141, of a prism of air, wood, glass, or other non-magnetic material, 1 in. square and 3.19 in. long, is 1 rel. A bar of mild steel or wrought iron, 1 in. square and 460 ft. long has, under the most favorable conditions, a reluctance of 1 rel. A bar of cast iron 1 in. square and 50.7 ft. long has, under the most favorable conditions, a reluctance of about 1 rel.

229. Reluctance Is Not Always Constant in magnetic materials, but it is in all non-magnetic materials. In magnetic materials the reluctance varies with the flux density (Art. 246). Variation of resistance with temperature is a somewhat analogous condition. The greater the flux density in a magnetic material the greater (within certain limits) will be the reluctance of the material.

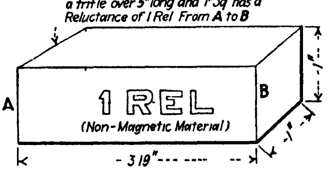


FIG. 141.—A graphic definition of the rel.

230. The Distinction Between the Electric Circuit and the Magnetic Circuit Should Not Be Disregarded.—This article is here inserted to caution the reader that while analogous and computed by the same general processes, the magnetic and electric phenomena are entirely distinct and separate conditions. Hence, it must not be inferred from the preceding that an electric current is the same thing as a flux of lines of force. In fact, they are entirely different things, except that the flow of an electric current is governed by laws similar to those which govern the development of magnetic flux. It is true that electric currents and magnetic fluxes are closely related. Electricity moving in the conductor of an electric circuit, can, as has been shown, produce a magnetic flux. Conversely, if the flux of a magnetic circuit cuts a conductor of a closed circuit, a current of electricity will be produced in that electric circuit (Art. 226).

231. The "Ohm's Law" of the Magnetic Circuit is the term which is sometimes applied to the fundamental law of the magnetic circuit, which will now be recited. The same underlying

natural principles which govern the phenomena in electric circuits also govern the phenomena in magnetic circuits, and for that matter, in all circuits (note under Art. 134) whatsoever. For all circuits—electric, magnetic, hydraulic, pneumatic, heat, and what not—the same general law holds. That is, it is always true that *the result produced is directly proportional to the effort and inversely proportional to the opposition.* It has already been shown (Art. 134) that for an electric circuit the so-called Ohm's law applies. That is:

$$(49) \qquad current = \frac{electromotive\ force}{resistance} \qquad \text{(amp.)}$$

It can be demonstrated experimentally and, in fact, it follows from the nature of things, that for a magnetic circuit a similar law holds; that is:

$$(50) \qquad flux = \frac{magnetomotive\ force}{reluctance} \qquad \text{(lines)}$$

The similarity between the electric-circuit equation (49) and the magnetic-circuit equation (50) is obvious. The essential concepts of the magnetic circuit can, as will be shown, readily be developed in much the same way as the electric circuit ideas were developed from the simple fundamental law stated in the note under Art. 134. Using symbols instead of words, the formula (50) becomes:

$$(51) \qquad \phi = \frac{M}{\mathcal{R}} \qquad \text{(lines)}$$

$$(52) \qquad M = \mathcal{R} \times \phi \qquad \text{(amp.-turns)}$$

$$(53) \qquad \mathcal{R} = \frac{M}{\phi} \qquad \text{(rels)}$$

Wherein ϕ = the flux in the magnetic circuit, or any portion of the magnetic circuit, under consideration, in lines (lines of force) or maxwells. M = m.m.f., in ampere-turns, to which ϕ is due. $\mathcal{R}$ = reluctance, in rels, of the magnetic circuit or the portion of the magnetic circuit under consideration.

EXAMPLE.—If, Fig. 142,*I*, the m.m.f. in a certain magnetic circuit is 500 amp.-turns and the reluctance of the circuit is 0 02 rel, what flux would be developed in this circuit? SOLUTION.—Substitute in equation (51): $\phi = M \div \mathcal{R} = 500 \div 0.02 = 25{,}000$ *lines.*

EXAMPLE.—If, Fig. 142,*II*, a flux of 1,000,000 lines is required in a magnetic circuit which has a reluctance of 0.004 rel, what m.m.f. would be necessary to develop this flux? SOLUTION.—Substitute in equation (52): $M = \mathcal{R} \times \phi = 1{,}000{,}000 \times 0.004 = 4{,}000$ *amp.-turns.*

EXAMPLE —In a certain air-core magnetic circuit, a m m f of 45,000 amp - turns develops a flux of 2,500 lines. What is the reluctance of this circuit? SOLUTION.—Substitute in equation (53): $\mathcal{R} = M \div \phi = 45,000 \div 2,500 =$ 18 *rels.*

232. Reluctivity (symbol is ν, pronounced nu) is specific reluctance. This property is similar to resistivity (Art. 126A) which is specific resistance. Reluctivity in practical work is measured in rels per inch cube.

233. The Reluctivities of Different Substances must, obviously, vary since the reluctances of different substances vary. However, the reluctivity of all non-magnetic substances is the same, namely 0.313 rel per in. cube. The reluctivities of magnetic substances vary with flux density, the greater the flux density

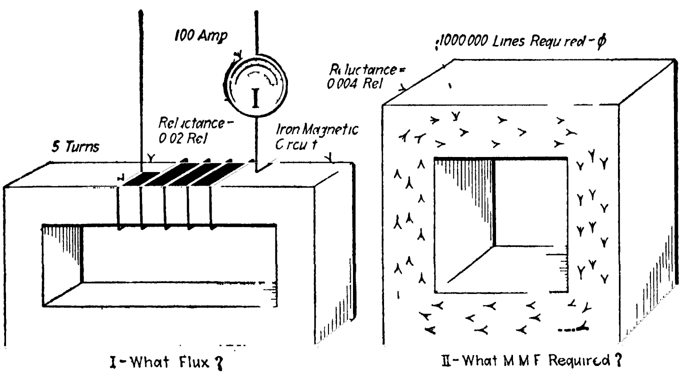


FIG 142 —Magnetic circuit problems

the greater the reluctivity within certain limits. The reluctivity of mild steel or wrought iron is, under the most unfavorable conditions, about 0.00018 rel per in. cube. The reluctivity for cast iron, under the most unfavorable conditions, is about 0.00164 rel per in. cube. Just how reluctivity varies with flux density can be appreciated from a consideration of the permeability (permeability is the reciprocal of reluctivity) values given in Table 249 and in Fig. 143.

234. The Computation of Reluctance, Using the Quantity "Reluctivity" involves the same general process as that described in Art. 143A, wherein it was shown how the resistance of any conductor may be figured on the basis of the resistivity of the material of that conductor. The following formulas are based on these truths: *The reluctance of any magnetic path is: (A) in-*

creased as the length of the magnetic path is increased, (B) increased as the cross-sectional area of the magnetic path is decreased, and (C) increased as the reluctivity of the material comprising the path increases. That is, "the reluctance is directly proportional to the average length of the lines of force, is inversely proportional to the cross-sectional area of the path, and varies with the material of the path."

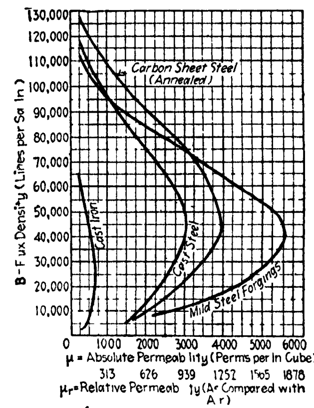


FIG. 143.—Graphs showing relation of permeability to flux density in certain irons and steels.

EXAMPLE.—Consider the two simple magnetic circuits of Fig. 144, the rings of both of which have the same cross-sectional area and are composed of the same kind of iron. The m.m.f. in each case is 100 amp.-turns. Now, if this 100-amp.-turn m.m.f. produced a total flux of 5,300 lines in the iron ring of *I*, which is a magnetic circuit of a mean length of 12 in., the flux in the ring of *III*, which is twice as long (24 in.) would be only about one-half of 5,300 lines, or 2,650 lines. It would be exactly one-half of 5,300 lines were there not certain corrections (Art. 267, "Leakage") which must be made,

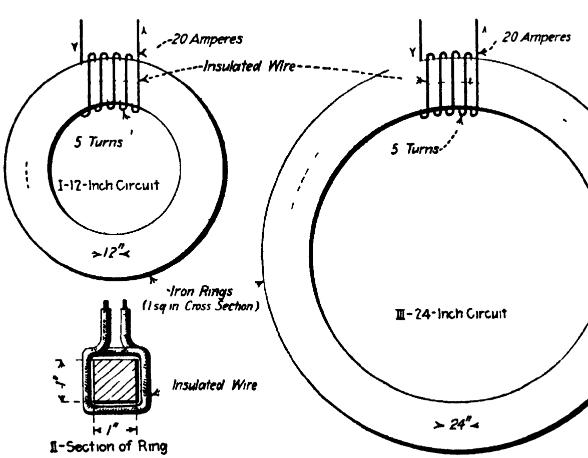


FIG. 144.—Examples of magnetic circuits.

and were it not for the fact that (Art. 242) reluctivity may vary with the flux density. It is apparent that if the length of the circuit—that is, the opposition to magnetization—is doubled, the flux would be one-half. Similarly, if an iron core of larger area were substituted for that of *I*, the

ampere-turns being maintained the same, the opposition or reluctance offered by the core would be correspondingly decreased, hence, the total flux would be correspondingly increased. Furthermore, if the m.m.f. of either *I* or *III* were doubled, that is, increased to 200 from 100, the flux would be doubled in each case. Corrections for leakage (Art. 267) must be made in order to ascertain the actual flux with great accuracy, but in many cases such corrections can be disregarded, and in any event, they do not materially affect the general truth of the above statement.

235. The Formulas for Figuring Reluctance of a Path on the Basis of Reluctivity, Length and Area of the Path are these (obviously, they follow from the facts outlined in the preceding Art. 234 and example):

(54) $$\mathcal{R} = \frac{\nu \times l}{A} \qquad \text{(rels)}$$

(55) $$\nu = \frac{A \times \mathcal{R}}{l} \qquad \text{(rels per in. cube)}$$

(56) $$l = \frac{A \times \mathcal{R}}{\nu} \qquad \text{(in.)}$$

(57) $$A = \frac{\nu \times l}{\mathcal{R}} \qquad \text{(sq. in.)}$$

Since (Art. 241) $\nu = 1 \div \mu$, substituting this in (54), the result is

(58) $$\mathcal{R} = \frac{l}{\mu \times A} \qquad \text{(rels)}$$

Wherein $\mathcal{R}$ = reluctance, in rels, of the magnetic path or portion thereof under consideration. ν = reluctivity of the material of the path under the existing conditions, in rels per inch cube. l = length of the magnetic path or portion thereof, under consideration, in inches. A = area of the magnetic path under consideration, in square inches. μ = permeability, in perms, per inch cube.

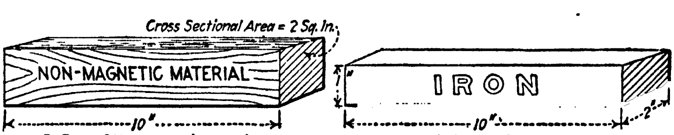


FIG. 145.—Example in computing reluctance.

EXAMPLE.—What is the reluctance (Fig. 145) of an air (or any other non-magnetic material) core 2 sq. in. in cross-sectional area and 10 in. long? SOLUTION.—The reluctivity of air or of any other non-magnetic material

(Art. 233) is always 0.313 rel per in. cube. Therefore, substituting in equation (54): $\mathcal{R} = (\nu \times l) \div A = (0.313 \times 10) \div 2 = 3.13 \div 2 = 1.57$ *rels.*

EXAMPLE.—If the reluctivity of a certain specimen of iron is 0.001 under given conditions, what is the reluctance of a piece of this material, Fig. 145, *II*, 1 in. $\times$ 2 in. in cross-section and 10 in. long? SOLUTION.—The cross-sectional area of the piece is 1 in. $\times$ 2 in. = 2 sq. in. Substitute these values in equation (54): $\mathcal{R} = (\nu \times l) \div A = (0.001 \times 10) \div 2 = 0.01 \div 2 = 0.005$ *rel.*

236. The Joint Reluctance of a Number of Magnetic Paths in Parallel may be calculated by a method identical to that of Art. 198 for figuring the joint resistance of a number of conductors in parallel. The joint reluctance of a number of such parallel paths is equal to the reciprocal of the sum of the reciprocals of the reluctances of the separate paths. That is, the joint reluctance is equal to the reciprocal of the sum of permeances (Art. 238) of the separate paths.

237. The Joint Reluctance of a Number of Magnetic Paths in Series is equal to the sum of the reluctances of the individual paths. Reluctances in series are thus added to obtain the total reluctance, just as resistances of a number of conductors in series are added to obtain the joint resistance of the series combination, as described in Art. 190.

238. Permeance (symbol is $\mathcal{P}$) is that property of materials which is the "opposite" of reluctance. Reluctance implies the difficulty encountered in developing magnetic flux, whereas, permeance implies the ease or readiness with which the flux may be developed. Permeance in magnetic circuits is analogous to conductance in an electric circuit. Permeance is numerically equal to the reciprocal of reluctance. That is: $\mathcal{P} = 1 \div \mathcal{R}$. In other words, permeance might be called the "magnetic conductance" of a material. The perm—which is analogous to the mho—has been suggested as the unit of permeance. Conductance values are useful in electric circuit computations when it is desired to determine the joint resistance of a number of conductors in parallel. In the same way, permeance values are convenient when it is desired to determine the joint permeance (or joint reluctance, which is the reciprocal of the joint permeance) of a number of magnetic paths which are in parallel.

239. Iron and Steel Are Materials of High Permeance.—It is exceedingly fortunate that it has so happened that one of the commonest of metals, iron, is the one with the greatest permeance. If it were not for the fact that this relatively cheap metal

were also very permeable, it is probable that the present electrical development would be impossible. If it had so happened that some expensive metal—gold, for instance—were required to provide paths of high permeance for magnetic circuits of our electrical machinery and devices, instead of iron which is now used, electrical machinery would be very costly. Hence, extensive electrical development would, because of the economics of the situation, be impossible.

240. The Magnetic Circuit Equations Involving Permeance Instead of Reluctance follow from those given in Art. 231. The derivation of the three following formulas will be apparent if one remembers, Art. 238 above, that permeance is the reciprocal of reluctance, that is, that $\mathcal{P} = 1 \div \mathcal{R}$.

$$\phi = \mathcal{P} \times M \qquad \text{(lines)} \tag{58a}$$

$$\mathcal{P} = \frac{M}{\phi} \qquad \text{(perms)} \tag{59}$$

$$M = \frac{\phi}{\mathcal{P}} \qquad \text{(amp.-turns)} \tag{60}$$

Wherein the symbols have the same meanings given hereinbefore except that $\mathcal{P}$ = permeance, in perms, per inch cube.

241. Permeability (symbol is μ, pronounced mu) is specific permeance. Absolute permeability is conveniently measured in *perms per inch cube.* (Do not confuse absolute permeability with relative permeability which is treated below.) Permeability is analogous to conductivity. In other words, absolute permeability is the permeance of a 1-in. cube. From what precedes, it follows that numerically, permeability is the reciprocal of reluctivity, that is: $\mu = 1 \div \nu$ or $\nu = 1 \div \mu$.

242. The Absolute Permeability of Different Substances varies in the case of magnetic materials with the flux density or saturation as is obvious from a consideration of the fact that reluctivity of magnetic substances varies with the flux density. See Fig. 143. The absolute permeability of air and of all other non-magnetic materials is 3.19 perms per in. cube, Fig. 146. Table 249 shows how the absolute and relative permeabilities of magnetic materials vary with flux density.

243. The Distinction Between Absolute Permeability and Relative Permeability should be noted. In many text-books on electromagnetism the permeability of air is given as 1. This

value of 1 is, however, the *relative* permeability and not the *absolute* permeability. Absolute permeability, like conductivity, depends upon the unit selected and with the *ampere-turn*, the flux *line* and the *inches* taken as units, the absolute permeability of air comes out as equal to 3.19 perms per in. cube. However, it is also perfectly legitimate to express the permeability of iron in terms of that of air, in which case the relative permeability of air becomes unity, or 1. An analogous situation is this: The conductivity of aluminum may be expressed either in ohms per inch cube or the relative conductivity may be referred to that of copper, which is then taken as unity. An absolute permeability value is always larger than the corresponding relative permeability value. Thus, where an absolute permeability value in perms per inch cube is given, this value will be 3.19 times greater than the corresponding relative (as compared with air) permeability value.

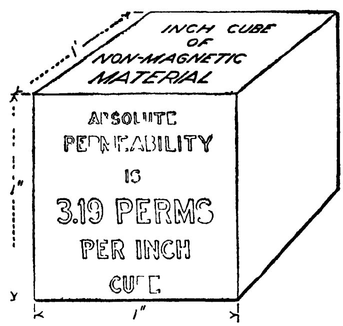


FIG. 146.—Illustrating permeability of non-magnetic materials.

NOTE.—To reduce an absolute permeability value, expressed in perms per inch cube, to a relative (as compared with air) permeability value, divide the relative permeability value by 3.19 or multiply it by 0.313. To reduce relative permeability to absolute permeability in perms per inch cube, divide by 0.313, or multiply by 3.19.

244. To Compute Permeance by Utilizing Permeability Values, the following formulas may be used:

(61) $$\mathcal{P} = \frac{\mu \times A}{l} \qquad \text{(perms)}$$

(62) $$\mu = \frac{l \times \mathcal{P}}{A} \qquad \text{(perms per in. cube)}$$

(63) $$l = \frac{\mu \times A}{\mathcal{P}} \qquad \text{(in.)}$$

(64) $$A = \frac{l \times \mathcal{P}}{\mu} \qquad \text{(sq. in.)}$$

Wherein all of the symbols have the same meanings as given before.

245. Gradient or Magnetic Intensity (symbol H) is a quantity analogous to voltage gradient or electric intensity in an electric circuit, which is described in Art. 157*a*, which should be reviewed. In a magnetic circuit, the m.m.f. is "consumed" gradually along each inch length of the circuit in a way similar to that in which the voltage impressed on an electric circuit may be considered as being consumed in each unit length of the circuit conductor. The m.m.f. gradient is, then, a quantity indicating the m.m.f. expended per unit length of the magnetic path. The real significance of m.m.f. gradient is explained in Art. 261. Thus (compare the following equations with those for the electric circuit, given in Art. 157*b*):

$$H = \frac{M}{l} \qquad \text{(amp.-turns per in. length)} \tag{65}$$

$$M = H \times l \qquad \text{(amp.-turns)} \tag{66}$$

$$l = \frac{M}{H} \qquad \text{(in.)} \tag{67}$$

EXAMPLE.—If a magnetic circuit is 40 in. long and is excited by a m.m.f. of 2,000 amp.-turns, the average m.m.f. gradient in this circuit would be equation (65): $H = M \div 1 = 2{,}000 \div 40 = 50$ *amp.-turns per in. length.*

246. Flux Density (symbol B) has already been briefly considered in Art. 73 where it was shown that with a uniform distribution of flux through the magnetic path:

$$B = \frac{\phi}{A} \qquad \text{(lines per sq. in.)} \tag{68}$$

$$\phi = B \times A \qquad \text{(lines)} \tag{69}$$

$$A = \frac{\phi}{B} \qquad \text{(sq. in.)} \tag{70}$$

Wherein all of the symbols have the same meanings as before given, except that B = flux density, in lines, per square inch.

247. The Magnetic Saturation of Iron, that is, its ability to carry lines of force, may be likened to the ability of a sponge to absorb water. When there is very little water in the sponge it will readily soak up more but when the sponge is almost saturated, it will absorb additional water only with difficulty. Likewise, with iron, Fig. 147, when the flux density is low, the slight increase in m.m.f. gradient (or m.m.f.) will cause a material increase in the flux density or in the number of lines of force per square inch. However, when the flux density is high, it requires a great increase in m.m.f. to produce a material increase in flux density.

248. The Flux Density Beyond Which It Is Impracticable to Magnetize a Magnetic Material Is Called the Magnetic Saturation Point of that material. It is possible to magnetize magnetic materials beyond their saturation points, but the m.m.f. required to effect such magnetization is then out of all proportion to the

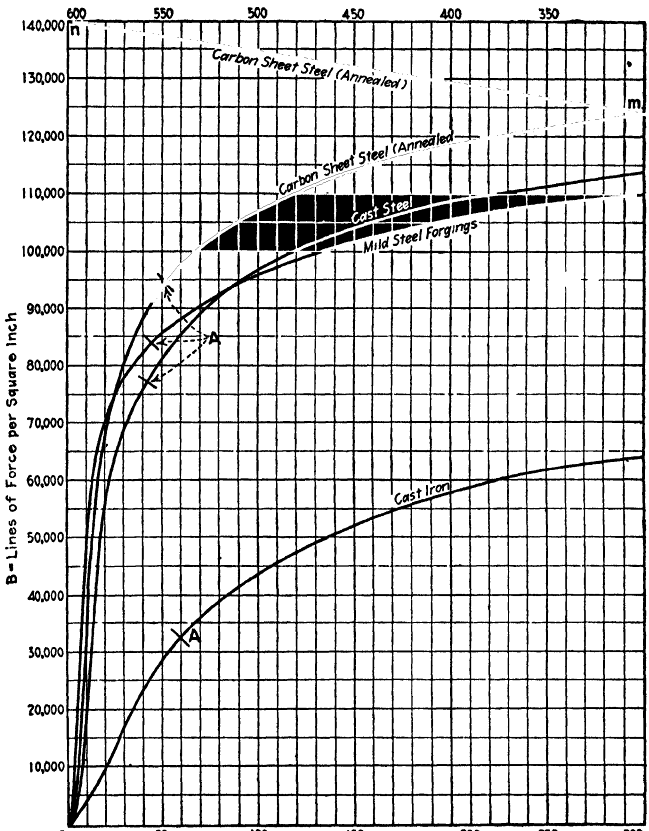


FIG. 147.—Magnetization graphs of various irons and steels.

magnetization obtained. The approximate location of the saturation points on the graphs of Fig. 147 are indicated by the letters *A*.

EXAMPLES.—Referring to the values for mild steel forgings of Table 249, which values are those from which the graphs of Fig. 147 were plotted, a m.m.f. gradient of 7.20 − 5.64 = 1.56 *amp.-turns per in.* length is required to effect a net increase of 10,000 lines per sq. in. (from 30,000 to 40,000) in

flux density because a flux density of 40,000 lines per sq. in. is below the saturation point. However, a m.m.f. gradient of 134.7 − 62.6 = 72.1 *amp.-turns per in. length* is required to increase the flux density from 90,000 to 100,000 lines per sq. in. because a flux density of 100,000 lines per sq. in. is considerably above the saturation point. It requires about 46 times as great a magnetic gradient in one case as in the other.

249. Magnetic Properties of Iron and Steel.

(See Figs. 143 and 147 for graphs illustrating these properties)

Carbon sheet steel (annealed)				Cast steel			
B Flux density (lines per sq. in.)	H M.m.f. gradient magnetic intensity (amp.-turns per in. length)	μ Absolute permeability (perms per in. cube)	μ_r Relative permeability (as compared with that of air)	B Flux density (lines per sq. in.)	H M.m.f. gradient magnetic intensity (amp.-turns per in. length)	μ Absolute permeability (perms per in. cube)	μ_r Relative permeability (as compared with that of air)
10,000	5.01	1,996.0	625	10,000	5.64	1,774.0	556.0
20,000	7.20	2,775.0	870	20,000	8.77	2,278.0	714.0
30,000	8.77	3,416.0	1,071	30,000	10.90	2,791.0	875.0
40,000	10.30	3,866.0	1,212	40,000	13.40	2,967.0	930.0
50,000	13.20	3,796.0	1,190	50,000	16.90	2,954.0	926.0
60,000	16.60	3,611.0	1,132	60,000	22 50	2,657.0	833.0
65,000	19.00	3,420.0	.....	65,000	26 00	2,500.0	797.0
70,000	21.30	3,283.0	1,029	70,000	31.00	2,255.0	707.0
80,000	29.40	2,715.0	851	80,000	45.70	1,745.0	547.0
90,000	43.20	2,080.0	652	90,000	70.50	1,276.0	400.0
100,000	67.00	1,490.0	467	100,000	117.40	851.7	267.0
110,000	117.00	937.9	294	110,000	228.60	481.7	151.0
120,000	227.00	526.4	165	115,000	317.90	360.5	113.0
125,000	365.00	370.0	116				
Mild steel forgings (wrought iron)				**Cast iron**			
10,000	3.76	2,567.0	833	10,000	20.00	497.6	156.0
20,000	4.70	4,252.0	1,333	20,000	32.90	609.3	191.0
30,000	5.64	5,091.0	1,596	30,000	51.40	583.4	183.0
40,000	7.20	5,547.0	1,739	40,000	82.10	488.1	153.0
50,000	9.40	5,318.0	1,667	50,000	134.00	370.0	116.0
60,000	13.90	4,351.0	1,364	60,000	224.00	266.7	83.6
65,000	16.00	4,070.0	1,275	65,000	322.00	201.3	63.1
70,000	20.40	3,436.0	1,077				
80,000	32.60	2,453.0	769				
90,000	62.60	1,436.0	450				
100,000	134.70	743.2	233				
105,000	197.20	532.7	167				
110,000	324.20	338.1	106				

250. Explanation of the Application of the Above Table of Magnetic Properties of Iron and Steel.—The columns headed "H" show the number of *ampere-turns required for each* 1 *in. length of magnetic circuit* to produce through each square inch of sectional area of the magnetic circuit the corresponding flux density, B.

EXAMPLE.—See the first line and section of the table. 5.01 amp.-turns will produce through each square inch sectional area of a carbon-sheet-steel magnetic circuit 1 in. long, 10,000 lines.

251. M.m.f. Gradients Required to Produce Different Flux Densities in Air.—The values under B were computed by substituting different values for flux density in formula (78) $H = B \div \mu$. That is, for air, $H = B \div 3.19$, because permeability of air is always 3.19 perms per in. cube. Thus with B taken as equal to 10,000: $H = 10{,}000 \div 3.19 = 3{,}135$.

B Flux density (lines per sq in)	H M.m.f. gradient (amp.-turns per in length)	B Flux density (lines per sq. in.)	H M m.f. gradient (amp -turns per in. length)
5,000	1,567	60,000	18,810
10,000	3,135	65,000	20,377
15,000	4,702	70,000	21,944
20,000	6,270	75,000	23,512
25,000	7,837	80,000	25,080
30,000	9,404	85,000	26,647
35,000	10,972	90,000	28,214
40,000	12,540	95,000	29,782
45,000	14,107	100,000	31,350
50,000	15,675	105,000	32,917
55,000	17,242	110,000	34,485

252. Permeability of Iron Is Not Constant, but varies with the flux density and with the different kinds of iron. A consideration of the permeability values in Table 249 and of the graphs of Fig. 143 will verify this assertion. When the number of lines of force per square inch is small, that is, when the flux density is quite low, the permeability of the iron is low. However, this permeability increases as the flux density increases, up to a certain saturation (Art. 247) or degree of flux density. At this flux density, the permeability of iron is greatest. With higher flux densities the permeability decreases.

EXAMPLE.—Consider the values of permeability in Table 249 for mild steel forgings. The absolute permeability at the low flux density of 10,000 lines per sq. in. is 2,657 perms per in. cube. At the flux density of about 40,000 lines, the permeability is then 5,547 perms per in. cube, which is about the maximum permeability obtainable with this sample of mild steel. Now, as the flux densities increase still further, the permeability decreases until at a flux density of 110,000 lines the permeability is then 338.1 perms per inch cube.

253. A Table Showing Magnetic Qualities of Iron and Steel (Table 249 and Figs. 143 and 147) indicates values for the grades of these materials that are ordinarily obtainable around the average shop. Different grades of a certain material, mild steel, for example, will have different magnetic qualities. Irons may be obtained that show materially better values than those of Table 249. On the other hand, some irons show values that are much worse. Irons that have exceedingly good magnetic qualities may be so expensive that their commercial application is not feasible.

254. The Working Formulas for the Magnetic Circuit, the derivation of which is given below, are these:

$$(71)\qquad I \times N = \frac{\phi \times l}{\mu \times A} \qquad \text{(amp.-turns)}$$

$$(72)\qquad I = \frac{\phi \times l}{\mu \times A \times N} \qquad \text{(amp.)}$$

$$(73)\qquad N = \frac{\phi \times l}{\mu \times A \times I} \qquad \text{(turns)}$$

$$(74)\qquad \phi = \frac{\mu \times A \times I \times N}{l} \qquad \text{(lines)}$$

$$(75)\qquad l = \frac{\mu \times A \times I \times N}{\phi} \qquad \text{(in.)}$$

$$(76)\qquad \mu = \frac{\phi \times l}{I \times N \times A} \qquad \text{(perms per in. cube)}$$

$$(77)\qquad A = \frac{\phi \times l}{I \times N \times \mu} \qquad \text{(sq. in.)}$$

Wherein I = exciting current, in amperes. N = number of turns in exciting winding. ϕ = the total flux threading the magnetic circuit, in lines. l = the length of the magnetic circuit, in inches. ϕ = the permeability of the material of the circuit, in perms per inch cube. A = the cross-sectional area at right an-

gles to the direction of the flux of the magnetic circuit, in square inches.

THE DERIVATION OF THE ABOVE EQUATION IS THIS.—As based on the fundamental equation (52) $M = \mathcal{R} \times \phi$. Now from (46) $M = I \times N$ and also from (58) $\mathcal{R} = l \div (\mu \times A)$. Then substituting these values in the equation given below:

$$I \times N = \frac{l}{\mu \times A} \times \phi = \frac{l \times \phi}{\mu \times A}$$

255. In Computing the Flux in a Magnetic Circuit formula (51) may be used as outlined in the examples under Art. 231. However, for reasons given in Art. 257 it is not usually feasible to compute magnetic circuits directly from the formula (51): $\phi = M \div \mathcal{R}$. The modification, (74) above, of the fundamental formula can, however, be readily applied.

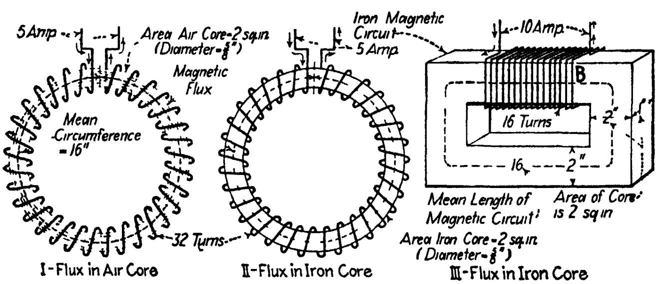


FIG. 148.—Examples of magnetic circuits.

EXAMPLE.—What flux will be produced in the air-core toroid Art. (210) of Fig. 148,*I* which has 32 turns, assuming that a current of 5 amp. flows in these turns? The length of the magnetic circuit, that is the mean circumference of the toroid, is 16 in. The area of the air core is 2 sq. in. SOLUTION.—The permeability of air is always (Art. 242) 3.19 perms per in. cube. Hence, substituting in formula (74): $\phi = (\mu \times A \times I \times N) \div l = (3.19 \times 2 \times 5 \times 32) \div 16 = 63.8$ *lines*.

EXAMPLE.—What flux will be produced in the core of the toroid of Fig. 148,*II* which is exactly the same as that of the preceding example, except that it has an iron core. Assume the permeability to be 3190.0 perms per inch cube. SOLUTION.—Since the only difference in the two examples is that the permeability is 3190.0 in this case instead of 3.19 in the preceding, the flux with the iron core would be 1,000 times as great, or: $1{,}000 \times 63.8 = 63{,}800$ *lines*. This example shows how the flux is increased by using iron instead of air for the path in a magnetic circuit.

EXAMPLE.—What flux will form in the magnetic circuit of Fig. 148,*III*? Turns = 16. Current = 10 amp. Area = 2 sq. in. Mean length of magnetic circuit is 16 in. Assume the permeability of the iron to be 3190.0 perms per inch cube. SOLUTION.—Substituting in equation (24): $\phi = (\mu \times A \times I \times N) \div l = (3190.0 \times 2 \times 10 \times 16) \div 16 = 63{,}800$ *lines*.

This (63,800 lines) is the same result as that obtained in the previous example. Inspection would have predicted this, because all of the values except "amperes" and "turns" are the same in both examples, but the amp.-turns are the same for both problems because $5 \times 32 = 160$ and also $10 \times 16 = 160$. Actually, however, the flux would be uniform in the ring of *II* because the winding is distributed. With *III* there would be some leakage, that is, while 63,800 lines would be developed at *B*, somewhat less than this number, because of leakage, would exist in the lower limb of the circuit.

256. In the Practical Design of Magnetic Circuits the total flux, ϕ, is usually known or assumed. The problem is then to lay out a circuit which will carry this flux effectively and compute the ampere-turns necessary for its development in the magnetic circuit. The general proportions of the circuit are tentatively assumed, the dimensions being based on previous experience and trial calculations. An examination of similar magnetic circuits already in successful operation will be of assistance. The cross-sectional area of the magnetic path of the iron in the circuit must be so selected that the iron will not be oversaturated (Art. 247). There is no direct method of designing a magnetic circuit. The first tentative plan is developed to a conclusion and, if it does not work out as desired, it must then be altered and recalculated accordingly. Because permeability varies with the saturation it is almost impossible to effectively design magnetic circuits without consulting data similar to that in Table 249.

257. In the Practical Calculation of Magnetic Circuits it is, as above suggested, often necessary to determine the ampere-turns, $I \times N$, that would drive a certain flux, ϕ, through the circuit. In solving such problems formula (71) $I \times N = (\phi \times l) \div (\mu \times A)$ might be used in the form just given. However, it is usually more convenient to compute on the basis of the flux density, that is, on the basis of the number of lines per square inch cross-section in the magnetic circuit, because tabular data like that in Table 249 can be most conveniently compiled on this basis. Furthermore, it is often inconvenient to make the computation for a complete magnetic circuit in one calculation. It is usually more practical to consider the component parts of the circuit, one part at a time, that is, to compute the ampere-turns necessary to develop the required flux in each part. After the ampere-turns necessary to force the flux through each of the parts has been figured, these component ampere-turn values are totaled. The resulting total will then be the number of ampere-

turns required to force the flux through the entire circuit. The following examples illustrate the process.

EXAMPLE.—How many ampere-turns will be required to develop a flux of 400,000 lines in the magnetic circuit of Fig. 149,*I*? SOLUTION.—Consider the component parts of the circuit one at a time. First consider:

The wrought iron yoke which has a sectional area of 2 × 2 = 4 *sq. in.* Therefore, if the total flux is to be 400,000 lines, the flux density or lines per square inch will be: 400,000 ÷ 4 = 100,000 *lines per sq. in.* Now, Table 249 shows that, in wrought iron, with a flux density of 100,000 lines (100 kilolines) per sq. in., the m.m.f. gradient is 134.7. That is, there would be required 134.7 amp.-turns per in. length of the magnetic circuit to produce a flux density of 100,000 lines. Hence, the amp.-turns required to magnetize the 20-in. yoke of the specified flux density would be: 20 × 134.7 = 2,694 *amp.-turns.*

The air gaps are each ⅛ in. long and, it will be assumed, are 2¼ in. square. The lines of force spread out at an air gap and will there occupy a cross-sec-

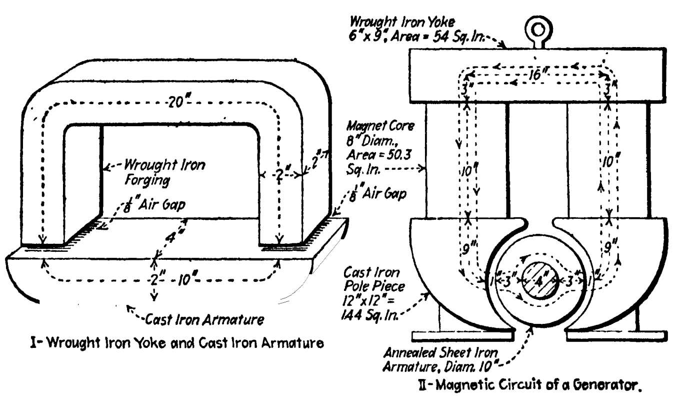


FIG. 149.—Examples of magnetic circuits.

tional area greater than that of the iron yoke. Hence, the area of the magnetic circuit at the air gap is: 2.25 × 2.25 = 5+ *sq. in.* The flux density in the air gap is: 400,000 ÷ 5 = 80,000 *lines per sq. in.* From Table 251, to produce a flux density of 80,000, the ampere-turns per inch length would be 25,080. For the ⅛-in. air gap there would be required: 25,080 ÷ 8 = 3,134 *amp.-turns.* Then for the two ⅛-in. air gaps: 2 × 3,134 = 6,268 *amp.-turns* are necessary.

The cast-iron armature has a sectional area of 4 × 2 = 8 *sq. in.* Hence, the flux density in it is: 400,000 ÷ 8 = 50,000 *lines per sq. in.* From Table 249, to produce a flux density of 50,000 lines in cast iron requires 134 amp.-turns per in. length of the magnetic circuit, hence, to magnetize this 20-in. long yoke there will be required: 134 × 20 = 2,680 *amp.-turns.*

Totaling the ampere-turns required for the different components:

Wrought-iron yoke	2,694 amp.-turns
Two air gaps	6,268 amp.-turns
Cast-iron armature	2,680 amp.-turns
Total	11,642 amp.-turns

Therefore, 11,642 amp.-turns are necessary to drive a flux of 400,000 lines through the magnetic circuit of Fig. 149,*I*.

EXAMPLE.—How many ampere-turns will be required to develop a flux of 4 megalines (4,000,000 lines) in the magnetic circuit of the generator of Fig. 149,*II*. SOLUTION.—Only the general principles will be illustrated. It is not practicable to here consider the details of electrical machine design, which is a very complicated subject. Consider the machine a part at a time:

The wrought-iron yoke has an area of 54 sq. in. Hence, the flux density in it is: 4,000,000 ÷ 54 = 74,000 *lines per sq. in.*, say 70,000 lines per sq. in. To produce this flux density in wrought iron (Table 249) 20.4 amp.-turns are necessary for each inch length of the magnetic circuit. Hence, for the 22-in. magnetic circuit of this example: 20.4 × 22 = 449 *amp.-turns* are necessary.

Magnet Cores.—Each has an area of 50.3 sq. in. Flux density = 4,000,000 ÷ 50.3 = 79,000, say 80,000 *lines per sq. in.* Ampere-turns per inch length for this density in wrought iron is 32.6. Length of core = 10 in. Then, the ampere-turns required for each core must be: 32.6 × 10 = 326. Hence for both cores there would be required: 2 × 326 = 652 *amp.-turns.*

Pole-pieces.—Cast iron. Area = 144 sq. in. Flux density = 4,000,000 ÷ 144 = 27,800, say 30,000 lines per sq. in. For cast iron at this density 51.4 amp.-turns per in. length are required. Length of magnetic path = 9 in. Then, ampere-turns required for each pole-piece: 9 × 51.4 = 462.6. Then to magnetize both pole-pieces there would be required: 2 × 462.6 = 925.2 amp.-turns.

Air Gap.—Assume the magnetic circuit comprises one-third of the circumference at each side of the armature. Armature has a diameter of 11 in. Circumference of an 11-in. circle = 34.5 in. Then: $\frac{1}{3}$ × 34.5 = 11.5 in. Hence the length of each pole face is 12 in. each air gap has an area of: 12 *in.* × 11.5 *in.* = 138 *sq. in.* To compute the ampere-turns required to excite this air gap we may either use the values of Table 249 or Formula (71). Thus: $I \times N = (\phi \times l) \div (\mu \times A) = (4{,}000{,}000 \times 1) \div (3.19 \times 138) =$ 9,091 *amp.-turns.* For the air gaps there would be required: 2 × 9,091 = 18,182 *amp.-turns.*

Armature Iron.—Carbon sheet steel, sometimes called annealed sheet iron. Area = 6 × 12 = 72 *sq. in.* The shaft area is subtracted because the steel in the shaft is considered ineffective. Flux density is: 4,000,000 ÷ 72 = 55,000, say 60,000 *lines per sq. in.* It requires for sheet steel 16.6 amp.-turns per in. length of magnetic circuit to produce a density of 60,000 lines per sq. in. The average path of the magnetic circuit through this armature is 9 in. long. Therefore, there are required for magnetizing the armature: 9 × 16.6 = 149.4 *amp.-turns.*

Totaling the ampere-turns required for the different parts:

Wrought-iron yoke	449 amp.-turns
Two magnet cores	652 amp.-turns
Two pole-pieces	925 amp.-turns
Two air gaps	18,182 amp.-turns
Armature iron	149 amp.-turns
Total	20,357 amp.-turns

Therefore, if there were no leakage, 20,357 amp.-turns would be necessary to develop a flux of 4,000,000 lines in the magnetic circuit of Fig. 149,*II*. In practice the correction for leakage should be made as suggested in Art. 269. The winding on one of the magnet cores should furnish one-half of the required ampere-turns and the one on the other core should furnish the other half. The design of a magnet winding to furnish a specified number of ampere-turns is treated briefly in Art. 277.

258. The Difficulties Encountered in Calculating Magnetic Circuits should be considered. Magnetic circuits can not be computed with the same exactness as can electric circuits because of two conditions.

First.—Magnetic leakage must be considered (Art. 267). It is impossible to confine a magnetic flux to certain paths as we can confine an electric current. It is possible to compute, with accuracy, the m.m.f. that a given helix will develop. But it is not, because of leakage, possible to compute exactly, the effective flux that this force will push through a magnetic circuit.

Second.—The reluctance (Art. 227) of iron and its permeability (Art. 241) varies with its saturation. The variation of permeability with flux density or saturation is discussed in Art. 248.

259. Permissible Flux Densities in Magnetic Circuits should not be exceeded. As a general proposition, the cross-sectional area of any portion of a magnetic circuit should be so proportioned that the flux density in it will be such that the iron in it is worked somewhat below its saturation point (Art. 247). For average work with grades of iron ordinarily obtainable, Table 249 and Fig. 147, the flux density should not exceed about 110,000 per sq. in. for annealed sheet iron; 90,000 per sq. in. for unannealed cast steel and wrought-iron forgings and 50,000 lines per sq. in. for gray cast iron.

260. The Relations Between Flux Density, *B*, and Magnetic Gradient, *H*, and Permeability, μ, may be expressed by the following formulas, the derivation of which is given below:

(78) $H = \frac{B}{\mu}$ (amp.-turns per in. length)

(79) $B = \mu \times H$ (lines per sq. in.)

(80) $\mu = \frac{B}{H}$ (perms per in. cube)

or substituting ν which equals $1 \div \gamma$ for μ in the above formulas:

(81) $H = B \times \gamma$ (amp.-turns per in. length)

(82) $B = \frac{H}{\nu}$ (lines per sq. in.)

(83) $\nu = \frac{H}{B}$ (rels per in. cube)

As Karapetoff suggests (in his THE MAGNETIC CIRCUIT) *formulas* (78) *and* (81) *state Ohm's law for a unit magnetic path, for instance a path* 1 *in. long and* 1 *sq. in. in cross-sectional area. H is the m.m.f. between the opposite faces of the cube. μ is the permeance (permeance and permeability are of the same value for a* 1*-in. cube) of the cube and B is the flux passing through the cube."* Note that formulas (78) and (81) above are analogous with similar formulas (23*e*) and (23*d*) for the electric circuit. Refer to following Art. 261 for further information relative to this situation.

THE DERIVATION OF THE ABOVE EQUATIONS is this: Again we start with the fundamental equation (62), thus:

(*a*) $M = \mathcal{R} \times \phi$

Now from (58) $\mathcal{R} = l(A \times \mu)$. Also, from (69) $\phi = A \times B$. Then substituting these values for $\mathcal{R}$ and ϕ in the above equation (*a*):

(*b*) $M = \frac{l}{A \times \mu} \times (A \times B) = \frac{l \times A \times B}{A \times \mu} = \frac{l \times B}{\mu}$

Therefore, dividing both terms of the above equation by l:

(*c*) $\frac{M}{l} = \frac{B}{\mu}$

But, from (65), $M \div l = H$, then substituting this new value for $M \div l$ in the above:

(*d*) $H = \frac{B}{\mu}$

261. The Real Significance of M.m.f. Gradient, H, should be thoroughly understood. In all the text preceding (except in the article just ahead of this) only the phenomena relating to entire magnetic circuits or considerable portions thereof have been

considered. Now, it is often desirable, particularly for the purposes of comparison and computation, to consider unit portions, that is 1-in. cubes, of uniform magnetic circuits and to examine the magnetic performance of these unit cubes. In fact, reference tables and graphs (like those of 249 and Figs. 143 and 147) which show the magnetic properties of magnetic-circuit materials are always so compiled that they, as will be shown, give data applying to these 1-in. cubes. Obviously, magnetic-properties data and tables can be useful for general work only if compiled on unit basis.

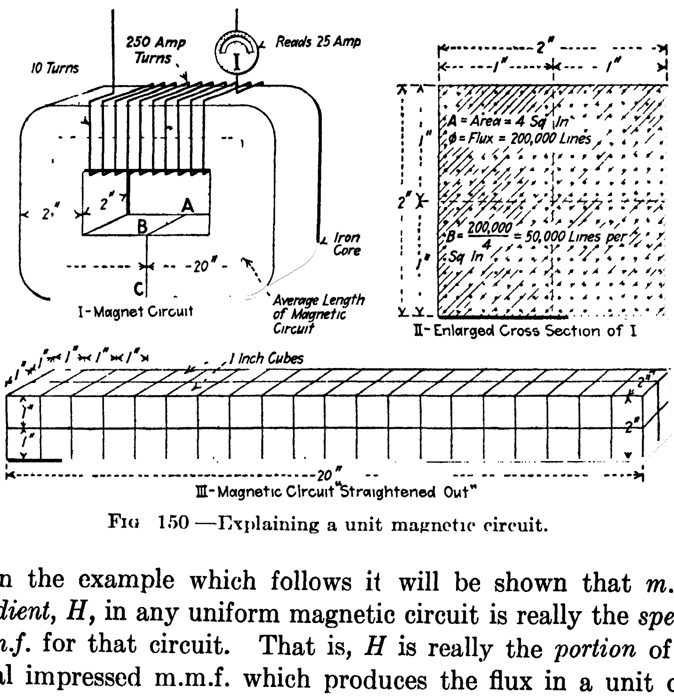


FIG 150 —Explaining a unit magnetic circuit.

In the example which follows it will be shown that *m.m.f. gradient, H*, in any uniform magnetic circuit is really the *specific m.m.f.* for that circuit. That is, *H* is really the *portion* of the total impressed m.m.f. which produces the flux in a unit cube (1-in. cube) of the magnetic circuit material. It is the m.m.f. impressed on a 1-in. cube and to which the flux in that unit cube is due.

EXAMPLE.—Consider the magnetic circuit of Fig. 150,*I*. Assume that it is desired to produce a flux, ϕ, of 200,000 lines in this circuit. It will be assumed that there is no magnetic leakage and that the permeability of the circuit material is 4,000 perms per in. cube. The average length of the circuit is 20 in. and its cross-sectional area is 4 sq. in. Then to develop the flux of 200,000 lines, the m.m.f., in ampere-turns, would, from (71), be:

(a) $$I \times N = \frac{\phi \times l}{\mu \times A} = \frac{200{,}000 \times 20}{4{,}000 \times 4} = 250 \textit{ amp.-turns.}$$

Hence, in this circuit, the m.m.f. gradient would, formula (20), be:

(b) $$H = \frac{M}{l} = \frac{I \times N}{l} = \frac{250}{20} = 12.5 \textit{ amp.-turns per in. length.}$$

We will now proceed to show that this 12.5 *amp.-turns*, or H, is actually the m.m.f. impressed on each unit or 1-in. cube of this magnetic circuit.

If the magnetic circuit of I were cut through at the plane ABC and "straightened out," its equivalent would then be, as shown at Fig. 150,*III*, a prism 20 in. long and 2 in. square. This prism can be considered as being composed of 80 unit cubes as shown. Let us compute the m.m.f. which would be required to produce the same flux, that is carried in I, in any one of these unit cubes. Since the total flux in I is 200,000 lines, and the cross-sectional area is 4 sq. in., the flux through any one cube would be: $200{,}000 \div 4 = 50{,}000$ *lines*. Now compute the m.m.f., in ampere-turns, necessary to develop 50,000 lines through one of the cubes. Each cube has a length of 1 in. and an area of 1 sq. in., thus:

(c) $$I \times N = \frac{\phi \times l}{\mu \times A} = \frac{50{,}000 \times 1}{4{,}000 \times 1} = 12.5 \textit{ amp.-turns.}$$

Note that 12.5 *amp.-turns per in. length* was the result obtained at (b) for H and that the result in (c), the m.m.f. required for a unit cube, is 12.5 *amp.-turns*, which is, of course, for a 1-in. length. It will, then, be evident, from a consideration of the facts brought out in this example, that H is actually, in a uniform magnetic circuit, the specific m.m.f., that is, the m.m.f. required to develop the flux in one unit cube of that circuit.

NOTE.—It follows, therefore, that, in a uniform unit-cube magnetic circuit, B, or flux density (or flux through a unit cube) may be considered as the effect, the cause of which is H or m.m.f. gradient (m.m.f. impressed on a unit cube). A similar situation exists in every magnetic circuit in which ϕ or total flux is the effect of which $I \times N$ or total m.m.f. is the cause. Also note in Art. 157 the analogous condition where voltage gradient is the cause and current density the effect in a 1-in.-cube electric circuit.

262. The Flux Produced by a Helix in air will now be considered. If current flows around in a helix like in one of those of Fig. 132, it will create a magnetic field or flux (Art. 214). This fact can be readily demonstrated by making a magnetic spectrum as suggested in Fig. 131. A considerable portion of the flux lines generated by such a helix will circulate the entire length of the space inside of the helix and complete their circuit entirely around it. However, some lines will leak out (magnetic leakage, Art. 267) and follow shorter paths between turns, as suggested in Fig. 130,*I*.

It follows, then, that the number of lines of force threading the air core of a helix will be different through different cross-sections

along its length. More lines will pass through the air core at the center of the helix, for example, at B, Fig. 132,I, than thread the core at any other point along its length. It can be shown experimentally and mathematically that the number of lines in the air core at the center of the helix is approximately twice the number in the core at the ends, as at A and C, Fig. 132,I.

Moreover, with the same current flowing in both cases, if helix I, Fig. 132, be pulled out until it is twice as long, as in III, the number of flux lines in the air core at the center and the number of flux lines at the ends G and I will be only half as great as at the corresponding locations in the shorter helix of I. The reason for this is that I and III, each has the same number of ampere-turns, hence develops the same total m.m.f. But the length of core—or the length of magnetic circuit—in which the flux must be developed is twice as great in III as in I, hence, the total flux developed by I will be twice as great as that developed by III.

Furthermore, the solenoid of II, Fig. 132, will develop more flux than will that of I, the ampere-turns being the same in each case, because the area of II is greater than that of I, hence, the reluctance of II is less than that of I. The flux density—number of lines per square inch—in II will be the same as that in I, but there are more square inches in II than I, therefore there will be more lines developed in II.

263. To Calculate the Flux Developed by an Air-core Helix, the same fundamental principles which have been discussed are applied. As stated in Art. 262, flux density varies at different locations within the air core of the helix, being twice as great at the center as at the ends. It logically follows that the magnetic gradient at these locations will be in proportion. At the center of the air-core helix:

$$H = \frac{M}{l} \quad \text{(amp.-turns per in. length)} \tag{84}$$

Therefore, since from equation (78) $H = B/\mu$, at the center of the air-core helix $B = (M \times \mu) \div l$. Also, μ for air is 3.19 perms per in. cube. Therefore, the flux density *at the center of ah air-core helix is:*

$$B = \frac{3.19 \times M}{l} = \frac{3.19 \times I \times N}{l} \quad \text{(lines per sq. in.)} \tag{85}$$

Since the flux density at the ends is one-half as great as at the

center, the flux density *at the ends of an air-core helix may be computed from this formula:*

$$B = \frac{1.6 \times M}{l} = \frac{1.6 \times I \times N}{l} \quad \text{(lines per sq. in.)} \tag{86}$$

Wherein B = flux density, in lines of force per square inch, in the air core of a helix. M = m.m.f., in ampere-turns of the helix. I = the current, in amperes, flowing in the helix. N = the number of turns in the helix. l = the length of the helix, in inches.

NOTE.—The above formulas give very accurate results for air-core solenoids which are 100 times, or more, longer than their diameters provided the turns are wound close together. However, they give approximate results for short solenoids or helices, even if the turns are spread apart. All magnetic-circuit calculations give approximate results because of leakage (Art. 267). The application of the higher mathematics is necessary for the computation of the flux density produced by an air-core solenoid at locations other than the center and the ends of the core.

264. To Obtain the Total Flux in an Air-core Helix at the center or at the ends, multiply the flux density at the given location, which may be computed with the above formulas, by the area in square inches of the air core of the helix. That is, $\phi = A \times B$.

EXAMPLE.—In Fig. 132, three different air-core helices are shown. Each has 16 turns and the current in each is 5 amp. What is the flux density in the core, at the center and at the two ends, of each helix? What is the total flux, ϕ, in the core, at the center and at the ends, of each helix?

SOLUTION.—*For Helix I.*—Substitute in formula (85), the flux density at the center is: $B = (3.19 \times I \times N) \div l = (3.19 \times 5 \times 16) \div 2.5 = 255.2 \div 2.5 = 102.1$ *lines per sq. in.* To compute the total flux at the center, B, of this helix, which has an area of 0.79 sq. in.: $\phi = A \times B = 0.79 \times 102.1 = 80.6$ *lines.* The flux density at ends A and C of the helix will be: $B = (1.6 \times 5 \times 16) \div 2.5 = 51.1$ *lines per sq. in.* Note that the flux density at the ends is just one-half of that at the center. The total flux at the ends A and C is $\phi = A \times B = 0.79 \times 51.1 = 40.3$ *lines.* This is, it will be noted, just one-half of the total flux at the center of the helix.

For Helix II.—This has the same number of ampere-turns but is of larger diameter; the flux density at the center is: $B = (3.19 \times 5 \times 16) \div 2.5 = 102.1$ *lines per sq. in.*, which is the same flux density as computed above for the helix *I*. But, now compute the total flux in the air core (which has an area of 3.14 sq. in.) at the center E: $\phi = A \times B = 3.14 \times 102.1 = 320$ *lines* = total flux at center.

NOTE.—The larger the diameter or the area inside of the coil, the greater will be the total flux, even if the flux density is the same. In each of the ends, D and F, of *II*, the flux density will obviously be the same as that at

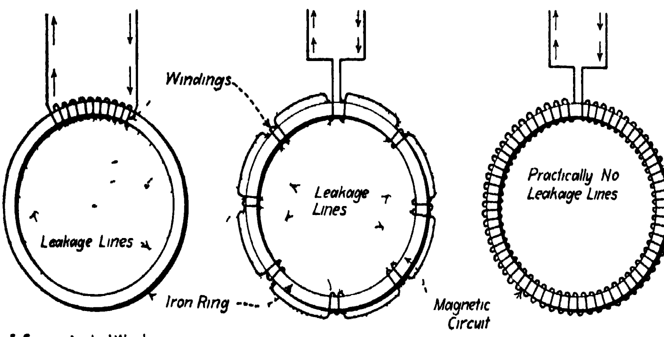
the ends of I, namely 51.1 lines per sq. in., but the total flux at the ends will be: $\phi = A \times B = 3.14 \times 51.1 = 160.5$ *lines* = total flux at ends.

For helix III, which has the same number of ampere-turns as the other two, I and II, but is twice as long as I, and is one-half the diameter of II, the flux density at its center J will be: $B = (3.19 \times 5 \times 16) \div 5 = 51.1$ lines per sq. in., which is just one-half the density at the center, B, of I, because the length of II is twice that of I. Now, the total flux at the center J of this helix, which has an area of 0.79 sq. in., will be: $\phi = A \times B = 0.79 \times 51.1 = 40.3$ *lines* = the total flux at center. The flux density at the ends G and I will be: $B = (1.6 \times 5 \times 16) \div 5 = 25.5$ *lines per sq. in.* This is just half the flux density at the center J. It follows that the total flux at the ends G and I will be: $\phi = A \times B = 0.79 \times 25.5 = 20.2$ *lines* = total flux at ends.

SUMMARY.—From a consideration of the above examples it is evident that (*a*) the total flux, and likewise the flux density, is different at different locations along its length in an air-core solenoid: (*b*) the greater the diameter or core of a solenoid, the greater is the total flux that is developed—although the flux density developed by a given number of ampere-turns is always the same, regardless of its diameter, at a given position in the solenoid of this example.

265. The Magnetic Circuit of an Air-core Helix is Not Accurately Defined because of the paths taken by the leakage lines, as shown in Fig. 133,*I*. However, the total flux produced by the helix at its center and its ends can be readily computed as suggested, by applying the above equation. It can also be shown that the total reluctance, in rels, of the magnetic circuit of a long, straight, air-core helix is equal to *the length of the helix in inches ÷ the area of the core of the helix, in square inches.* In other words, if the total m.m.f. of such a helix is divided by its reluctance, in rels, obtained as just described, the total flux produced, that threading the helix at its center, will be the result. All of this flux flows through the air core of the straight helix at its center (Fig. 133,*I*) but it spreads out so that at other locations than the center within the air core, there is less flux than the total amount developed. As has been shown, the flux at the ends is but one-half the total developed. If, however, an air-core helix is bent into a circular form or *toroid*, Fig. 135,*I*, practically all of the lines of force, developed by the ampere-turns, remain inside of the helix (Art. 269) and then the path and the area of the air magnetic circuit are very definitely defined. Hence, the reluctance of a toroid magnetic circuit can be very accurately computed and the flux in the air core at any location around the core, that a certain number of ampere-turns will develop, can be easily calculated.

266. When the Turns of a Helix Are Wound Around Iron and Are Evenly Distributed Along the Magnetic Circuit, as for example, in Fig. 151,*III*, or even where there are but a few turns close together, or the turns are concentrated as in *I*, nearly all of the flux will follow the iron path because the surrounding air has much greater reluctance than the iron. In making magnetic



I-Concentrated Winding   II Semi Distributed Winding   III Distributed Winding

FIG 151.—Leakage from a magnetic circuit surrounded by air.

circuit calculations, it is usually first assumed that all of the flux stays in the iron. Then, corrections such as experience and experimental evidence have shown to be necessary are, if required, made for leakage as described in Art. 271. There will be considerably more leakage with the arrangement of *I* than with that of *III*, but for any ordinary practical problems the leakage in *I* might be disregarded.

SECTION 11

MAGNETIC LEAKAGE

267. Magnetic Leakage.—There is no known insulator for magnetism (lines of force) as is outlined in Art. 225. In dealing with electric currents we can direct them definitely by using conductors where we wish the current to flow and by interposing extremely poor conductors or insulators to prevent the electricity from flowing where we do not desire it. Fortunately, air is a splendid insulator of electricity. Unfortunately, in this respect, there is no very poor conductor or insulator for magnetism. Air is a fairly good conductor of magnetism, although it is not nearly as good as iron; see "Permeability" (Art. 241).

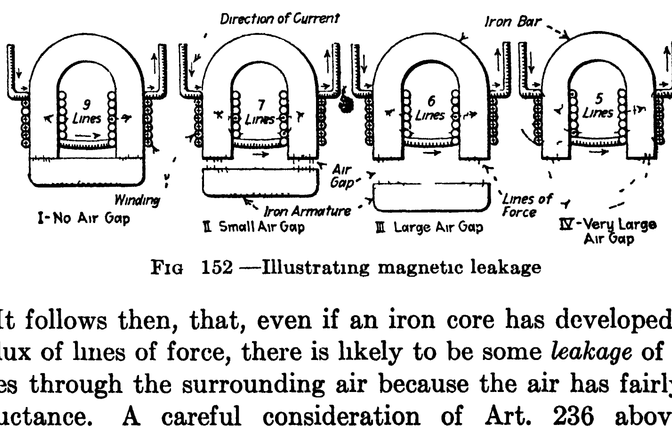


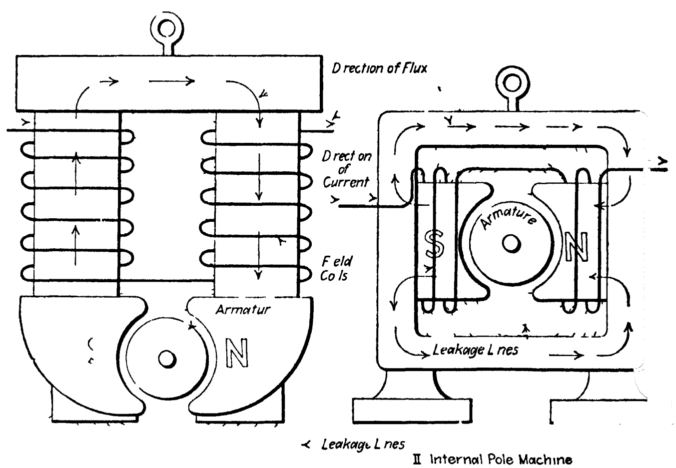
FIG 152 —Illustrating magnetic leakage

It follows then, that, even if an iron core has developed in it a flux of lines of force, there is likely to be some *leakage* of these lines through the surrounding air because the air has fairly low reluctance. A careful consideration of Art. 236 above on divided magnetic circuits will make it clear why this is so. An iron magnetic circuit surrounded by air is analogous to an uninsulated-copper electric circuit immersed in impure water. There would be some leakage of the electric current of such a circuit through the water just as there is some leakage of magnetic flux through the air. Because of this leakage all of the lines developed by the m.m.f. of a helix cannot always be confined to the iron of a magnetic circuit.

268. An Example of Magnetic Leakage is shown in Fig. 152. A certain current is flowing around the helices on the magnet

cores and produces a flux around the magnetic circuit. This picture indicates the principle of magnetic leakage but the values given are only illustrative and must not be taken as accurate or absolute:

Example.—At *I* the armature is held tightly against the poles, there is practically no leakage and a total flux of 9 lines circulates around the entire magnetic circuit. If the armature is pulled a short distance away from the poles as at *II* the reluctance of the circuit is increased which decreases the flux to 7 lines and there is some leakage. As the armature is pulled further away as at *III* the reluctance is still further increased reducing the flux to 6



Note: The Useful Lines Those Which Pass Through the Armature are not Shown
I-Obsolete Edison Bipolar Machine

Fig. 153.—Leakage lines of generators and motors.

lines and there is considerable leakage. If the armature is now entirely removed the reluctance of the magnetic circuit is very greatly increased because of the very large air gap, the total flux is thereby reduced to 5 lines and there is a great amount of leakage.

Examples of leakage lines in generators and motors are shown in Fig. 153.

269. Magnetic Leakage Can Be Practically Eliminated by Distributing the Winding which develops the m.m.f. (Art. 220) uniformly around the magnetic circuit (Fig. 151,*III*)—this applies only where the reluctance (Art. 227) is uniform along the entire length of the magnetic circuit. If the reluctance of certain portions of a circuit is different from that of other portions, the

winding should be distributed along the portions of the circuit, each portion being provided with a part of the winding in proportion to its reluctance. That is, the portions of the circuit that have the greatest reluctances should have the greatest proportion of the ampere-turns. The portions of the circuit that have little reluctance should be provided with correspondingly few ampere-turns. In practice it is seldom possible to distribute a winding over the parts of a magnetic circuit in proportion to their reluctances but it should be done in so far as practicable.

EXAMPLE.—Why it is that distributing the winding eliminates leakage will be evident from a study of Figs. 151 and 154 which show analogous electric and magnetic circuits. The electric circuits are immersed in a tank of water with a little acid in it, which renders the water conducting to electric currents. This is analogous to the usual condition of magnetic circuits which are always surrounded by air or by some other medium that is a fairly good

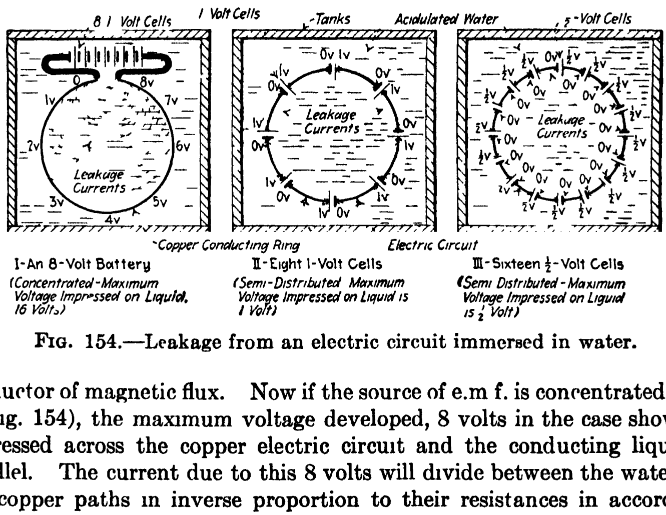


FIG. 154.—Leakage from an electric circuit immersed in water.

conductor of magnetic flux. Now if the source of e.m f. is concentrated as at *I* (Fig. 154), the maximum voltage developed, 8 volts in the case shown, is impressed across the copper electric circuit and the conducting liquid in parallel. The current due to this 8 volts will divide between the water and the copper paths in inverse proportion to their resistances in accordance with the law of divided circuits (Art. 197). Much more current will flow through the copper than through the water because the copper has, probably, much the lower resistance. However, some *leakage current* will flow through water as shown.

Current will flow through the water from locations in the copper ring of *I* which are at a high potential (Art. 96) to those of a lower potential, as shown. The current that flows between any two locations will be proportional to the difference of potential-voltage—between the two locations. Evidently, then, the most current will flow through the water from one end of the copper ring to the other, at the points where the battery joins the ring and where the difference of potential is 8 volts. Obviously, if it were possible to so arrange this submerged electric circuit that all locations in it would

be at the same potential there would be no leakage currents through the water.

If the eight cells are arranged around the circuit as at Fig. 154,*II*, the greatest possible difference of potential is 1 volt and the leakage currents are correspondingly reduced. However, by Ohm's law (Art. 134), the same current is flowing through the copper conductor in *I* as in *II* because the total voltage impressed on the circuit is the same in both cases and the resistance is the same. Now, if, as in *III*, 16, ½-volt cells are uniformly arranged around the submerged electric circuit, the maximum difference of potential is reduced to ½ volt and the leakage currents are therefore practically eliminated—nearly all the current will flow through the copper conductor. It is conceivable that a still greater number of cells, of correspondingly lower voltage, could be cut into the circuit and the maximum potential difference between any two locations in the circuit might thereby be reduced to practically 0 (zero), in which case of course, there could be no leakage currents through the water. In other words, if the cells that produce the e.m.f. were uniformly distributed around the electric circuit there could be no leakage currents.

A similar set of conditions obtains with the analogous magnetic circuit of Fig. 151. With the ampere-turns concentrated as at *I* a large difference of magnetic potential or m.m.f. (Art. 220) is concentrated across the ends of the winding and there would be a correspondingly great leakage. With the winding semi-distributed as at *II* the maximum possible difference of magnetic potential is reduced and the magnetic leakage is decreased accordingly, just as the maximum difference of electric potential was reduced in Fig. 154, *II*. Now with a winding uniformly distributed as in *III*, all parts of the magnetic circuit are at practically the same magnetic potential and there can be no magnetic leakage.

Therefore, where possible, the winding for developing a m.m.f. in a magnetic circuit should be so distributed that the m.m.f. will be consumed in that portion of the circuit where it is applied. Then there can be no leakage. Even with an air core, when a helix winding is uniformly distributed as in Fig. 135,*I*, there is no appreciable leakage.

270. Computation of Magnetic Leakage is complicated and almost impossible in many cases because of the difficulty of determining the area of the path, of the magnetic circuit in air, that the leakage lines traverse. The lines spread out in passing through the air and how great or how small is the cross-sectional area that they actually occupy cannot be readily estimated or calculated. In practice, leakage is usually determined experimentally. A magnetic circuit having been built, its leakage or *leakage factor* (Art. 271) can be ascertained by electrical and magnetic measurements.

271. The Leakage Factor of a Magnetic Circuit is the value by which the number of *useful* lines of force must be multiplied to ascertain the number of lines of force that must be developed by the ampere-turns of the developing helix. That is, the leakage factor of a magnetic circuit is equal to the number of lines developed in the circuit divided by the number of useful lines of the circuit.

EXAMPLE.—If, in the magnetic circuit of Fig. 153,*I*, it is necessary to design the field magnet coils to produce 140 lines of force for every 100 lines

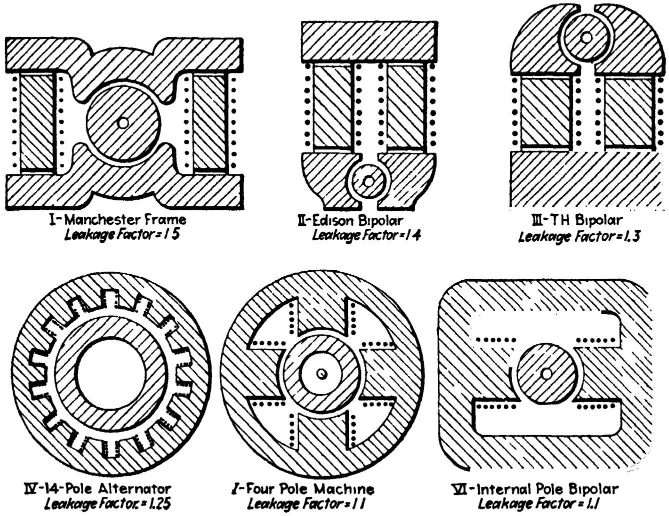


FIG. 155.—Leakage factors of magnetic circuits of electrical machines.

that are useful, that is, that pass through the armature, the leakage factor of that magnetic circuit is 1.4.

272. Leakage Factor Values vary with the arrangement and characteristics of a magnetic circuit. As a general proposition, the greater the pole-piece surface exposed to the air, the greater will be the leakage. See the illustration of Fig. 155. In commercial generators and motors the leakage factor of the magnetic circuit may vary from 1.1 to 1.5. With modern machines, it probably varies between 1.1 and 1.3. Fig. 155 indicates approximate leakage factors for some typical magnetic circuits.

SECTION 12

CALCULATION OF MAGNET WINDINGS

273. The Design of Windings to Excite Electromagnets is a rather complicated subject and can not be treated in detail in this book. However, directions are given in following articles whereby windings to satisfy ordinary conditions can be calculated. As in all other branches of engineering, experience is a great asset and is a necessary equipment of one who designs windings most effectively and economically.

274. The Requirements in Designing a Winding.—Usually the number of ampere-turns necessary to produce a certain flux is known, this number having been estimated as outlined in preceding articles. The problem is, then, to determine the size of wire to use for the winding and the number of turns for the winding so that, with a given voltage applied to the winding, a current will flow that will develop the required ampere-turns. The winding must also be such that it will not heat excessively and that it will fit into the space prepared for its reception.

275. With a Given Size Wire, a Given Length of Mean Turn and a Given Applied Voltage, Changing the Number of Turns Does Not Change the Ampere-turns.—To change the ampere-turns of a winding, with the conditions as above, it is necessary to either use a different size wire or to change the applied voltage. If a wire twice as large is used the ampere-turns will thereby be doubled. If the applied voltage is doubled, the ampere-turns will be doubled. The following example explains the situation.

EXAMPLE.—Consider the winding spaces of Fig. 156. The diameters of all of the spaces are as at *I*, hence the average diameter of coil or mean length of turn is, in each case, 9.42 in. Suppose that the 1 in. by 1 in. space of *II* is so wound with insulated wire that it will contain just 100 turns (No. 11 single, cotton-covered wire, Table 283, will about meet this requirement). See Note below for detail calculations. The resistance of the entire coil, 100 turns, in *II* would be about 0.1 ohm. Assume that the impressed voltage is 1 volt. Then by Ohm's law, the current that would be pushed through the coil by this voltage would be: $I = E \div R = 1\ volt \div 0.1\ ohm = 10\ amp.$ Therefore, the ampere-turns of the winding would be: $100\ turns \times 10\ amp. = 1{,}000\ amp.\text{-}turns.$

Now if the coil were made twice as long, as at *III*, it would contain twice as many or 200 turns. But its resistance would also be twice as great. Hence, although the turns have been doubled, the current has been halved, hence the ampere-turns, assuming the same applied voltage (1 volt) would remain 100. With the winding of *IV*, the turns are increased eight times but the resistance is also thereby increased eight times. Hence, one-eighth the current will flow and the ampere-turns will remain 200.

NOTE.—The resistance of the above-mentioned coil was calculated as follows: The length of mean turn for *I*, Fig. 156, is 3 in. × 3.1416 = 9.42 in.

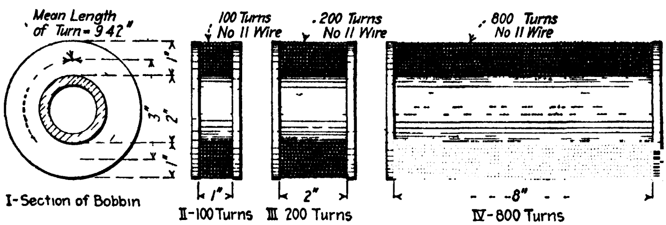


FIG. 156.—Illustrating effect of changing number of turns, the mean length of turn remaining constant.

If the 1 in. by 1 in. space of *II* is wound with No. 11 single, cotton-covered magnet wire, Table 283, it will contain just about 100 turns. The average length of each turn is: 9.42 in. ÷ 12 = 0.79 ft. Then in the entire coil there are 0.79 ft. × 100 turns = 79 ft. The resistance of 1,000 ft. of No. 11 wire (Table 157) 1.26 ohms or the resistance of 1 ft. is 0.00126 ohm. Then the resistance of the entire coil is: 0.00126 ohm × 79 ft. = 0.1 ohm—the value used above.

276. Increasing the Number of Turns in a Coil, Having a Given Length of Mean Turn and with a Given Applied Voltage, Decreases the Heat Developed by the Coil.—Decreasing the number of turns increases the heat. Therefore, the amount of wire on a coil, the conditions being as above, determines whether it will operate hot or cool. The size of wire (Art. 275) merely determines the ampere-turns. The reason of this is that the heat developed by a current of electricity in any conductor varies as the square of the current (Art. 167). That is, watts heat developed always equals: $I^2 \times R$. Doubling the turns halves the heating. Other increases or decreases in the number of turns change the heating produced proportionately.

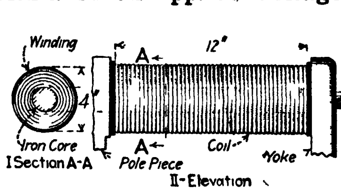


FIG 157.—Example in determining radiating ability of a coil.

EXAMPLE.—(1) Assume a coil having 200 turns and, say, 20 ohms resist-

ance. If this coil were connected across 100 volts, 5 amp. would flow through it. The heat developed in the coil would then be: $I^2 \times R = 5 \times 5 \times 20 = 500$ watts. (2) Now consider another coil of the same size wire, and having the same mean length of turn, but wound with twice as many or 400 turns. Its resistance will be twice as great or 40 ohms. If connected across 100 volts, 2.5 amp. will flow through it. The heat developed by this coil will, therefore, be: 2.5 amp. $\times$ 2.5 amp. $\times$ 40 ohms $=$ 250 watts. This is just half of the heat developed by the coil having 100 turns.

277. Calculation of Size Wire for a Magnet Coil That Will Provide a Required Number of Amperes-turns When Connected Across a Given Voltage.—The working formula is the last one in this paragraph. Its derivation will be given: It follows from the formula of Art. 144 for computing the resistance of any conductor that:

$$(87) \qquad R = \frac{K \times N}{cir.\ mils} \times \frac{l}{12} = \frac{K \times N \times l}{12 \times cir.\ mils} \qquad \text{(ohms)}$$

Wherein R = resistance, in ohms, of all of the turns of any magnet winding. K = a constant, numerically equal to the resistance in ohms of a circular mil-foot of the conductor of the winding. N = number of turns in the winding. l = length in *inches* of an average turn of the winding—or the length of a mean turn. *Cir. mils* = cross-sectional area of the conductor in circular mils.

If the magnet coil is to operate on some certain fixed voltage, as magnet coils usually do, the current through the coil will be, by Ohm's law, $I = E \div R$. Now substituting the expression for R obtained above in this Ohm's law formula, we have:

$$(88) \qquad I = \frac{E}{R} = \frac{E}{\dfrac{K \times N \times l}{12 \times cir.\ mils}} = \frac{E \times cir.\ mils \times 12}{K \times N \times l} \qquad \text{(amp.)}$$

Wherein I = current, in amperes, through the coil. E = e.m.f. impressed on the coil, in volts. Now multiplying both sides of this last equation by N, we have:

$$(88a) \qquad IN = \frac{E \times cir.\ mils \times 12 \times N}{K \times l \times N} \qquad \text{(amp.-turns)}$$

The two N's in the right-hand member cancel out giving:

$$(88b) \qquad IN = \frac{E \times cir.\ mils \times 12}{K \times l}, \qquad \text{(amp.-turns)}$$

and it follows that:

$$(88c) \qquad cir.\ mils = \frac{I \times N \times K \times l}{12 \times E} \qquad \text{(cir. mils)}$$

But, for soft-drawn copper wire operating at about 130 deg. F., K becomes (see Fig. 90) 12 ohms. Therefore, where a winding will operate at about 130 deg. F., which is a fair average operating temperature: $cir.\ mils = I \times N \times l \times 12 \div E \times 12$. The two 12's cancel out giving as the working formula:

$$cir.\ mils = \frac{I \times N \times l}{E} \qquad \text{(circular mils)} \tag{88d}$$

278. Effects of Heat on a Magnet Winding.—As outlined in Art. 277, the formulas there given for determining the size wire necessary to produce a certain number of ampere-turns are based on the assumption that the winding will operate at a temperature of about 130 deg. F. If the winding actually operates at a lower temperature, the wire size obtained by using the Art. 277 formula will be larger than necessary. This is because the resistance of copper decreases as its temperature decreases as outlined in Art. 147; the graph of Fig. 90 shows how the resistance of a circular mil-foot of copper varies with the temperature. By substituting the proper value for K, from Fig. 90, in the formula of Art. 277, the ampere-turns necessary with any other operating temperature than 130 deg. F. can be easily determined. If the wire for a winding determined in accordance with the formula of Art. 277 actually operates at a temperature greater than 130 deg. F., the wire will then be too small rather than too large.

279. The Amount of Heat That Can be Dissipated By a Magnet Coil without an excessive temperature rise is determined largely by the amount of surface that the coil exposes to the air. It is frequently the practice to assume that there should be the equivalent of 1 sq. in. coil surface exposed to the air for every 0.8 watt of I^2R loss or heat developed by the coil. Experience shows this rule to be safe under ordinary conditions and where it is followed a coil will not acquire a temperature great enough to injure its insulation. A safer practice is to allow 1 sq. in. equivalent surface for every 0.5 watt of I^2R loss in the coil. It is usually assumed that the heat dissipated through surfaces of the magnet structure, in addition to that dissipated directly from the surface of the coil, amounts to from 50 to 75 per cent. of that radiated directly from the coil surface. As a general proposition, no coil that is to carry relatively heavy currents should be thicker than 2 in.

EXAMPLE.—How many watts heat loss can the coil of Fig. 57 dissipate

without becoming excessively hot? SOLUTION.—Coil is 4 in. in diameter, hence is: 4 × 3.14 = 12.6 in. in circumference. It is 12 in. long. Therefore, the surface of the coil exposed to the air is: 12.6 × 12 = 151 sq. in. It will be assumed that the heat conducted into and radiated from the yoke and pole-piece will be 75 per cent. of that radiated direct. Then, the total equivalent radiating surface = 1.75 × 151 sq. in. = 270 sq. in. Assuming each square inch surface will radiate 0.8 watt, the total watts that can be dissipated by the coil will be 0.8 × 270 = 216 *watts.*

280. The Maximum Permissible Thickness of Magnet Coils, of cotton-covered wire, that are wound solid—without ventilating ducts—and that may carry continuously relatively heavy currents, is 2 in. Where coi's are thicker than this, the heat developed in the inner turns travels slowly to the surface from which it may be radiated. The consequence is, that the inner turns of such coils may become excessively hot. Where the wire comprising the winding is insulated with a non-combustible or heat-resisting material windings may be thicker than 2 in.

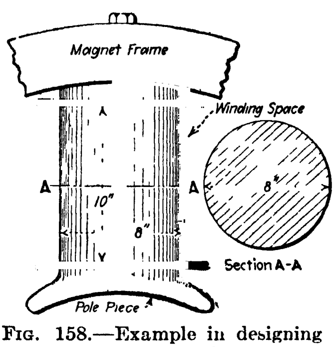


FIG. 158.—Example in designing a magnet winding.

281. Example Illustrating the Design of a Constant-voltage Magnet Coil to produce a certain number of ampere-turns is given in the following articles:

EXAMPLE.—Design a winding to produce 11,000 amp.-turns for the winding space of Fig. 158. Assume the voltage available to be 110. SOLUTION.—(1) DETERMINING WIRE SIZE.—Assume that the coil will be 2 in. thick, the maximum permissible thickness (Art. 280). The magnet core is 8 in. in diameter. Hence, the diameter of mean turn is 10 in. The circumference of mean turn equals: 10 × 31.4 = 31.4 in. Substituting these values in the formula of 277:

$$(a) \qquad \text{cir. mils} = \frac{IN \times l}{E} = \frac{11{,}000 \times 31.4}{110} = 3{,}140 \ \textit{cir. mils}$$

Hence to produce 11,000 amp.-turns under the conditions outlined a 3,140-cir. mil winding should be used. Referring to Table 157, a 3,140-cir. mil conductor lies between Nos. 15 and 16, American Wire Gage. We will use the larger wire, No. 15. Remember, Art. 275, that a 3,140-cir. mil conductor will then produce 11,000 amp.-turns, regardless of how few or how many turns of this conductor are wound into the coil.

2. ASCERTAIN JUST HOW MANY AMPERE-TURNS THE WIRE SIZE AS ABOVE WILL PRODUCE.—Since with a given voltage, wire size and mean length of turn, the amount of this No. 15 wire wound on the coil will not

affect the number of ampere-turns developed, we will find the number of ampere-turns developed by 1 lb. of wire. Then the ampere-turns developed by a greater or lesser amount of the wire will be the same (Art. 275) number.

The length of a mean turn is 31.4 in., is: 31.4 ÷ 12 = 2.62 ft. From Table 157, 1 lb. of No. 15 wire contains 101.63 ft. Then 1 lb. of No. 15 would provide: 101.63 ÷ 2.62 = 38.8 turns. A coil containing 1 lb. of No. 15 would have a resistance (Table 157) of 0.32 ohm. Then a 1-lb. coil would, on 110 volts, pass 110 ÷ 0.32 = 344 amp. Therefore, the ampere-turns of No. 15 wire for the conditions of this example are: 344 amp. × 38.8 turns = 13,350 amp.-turns. Whether 1, 10, 100 or 1,000 turns of No. 15 wire were wound on the core of Fig. 158 (with a mean diameter of turn of 10 in. and an applied voltage of 110) the ampere-turns would remain 13,350.

3. DETERMINE HEAT RADIATING SURFACE OF COIL.—The outside diameter of the coil will be 12 in. Therefore, its circumference will be: 12 in. × 3.14 = 37.7 in. The exposed area of the coil will be: 37.7 in. × 10 in. = 377 sq. in. Assume that the pole-piece and frame provide a radiating surface 75 per cent. as great as that of the winding (Art. 279). Then the total, equivalent radiating surface is: 377 × 1.75 = 660 sq. in.

4. DETERMINE WATTS POWER LOSS PERMISSIBLE IN THE COIL.—Assume (Art. 279) that each square inch of equivalent coil surface will radiate the heat produced by 0.5 watt. Then the coil can effectively radiate the heat due to: 0.5 × 660 = 330 watts.

5. DETERMINE PERMISSIBLE CURRENT IN COIL.—With a pressure of 110 volts, the current that will develop 330 watts is (Art. 164): $I = P \div E =$ 330 watts ÷ 110 volts = 3 amp. Therefore, the permissible current in the coil is 3 amp.

6. DETERMINE AMOUNT OF WIRE REQUIRED.—Through a coil of 1 lb. of No. 15 wire (as calculated in (2)), 344 amp. will flow. As determined in (5), the permissible current through the coil of this example is 3 amp. To pass a current of 3 amp., a coil of No. 15 wire weighing: 344 ÷ 3 = 115 lb. would be required. We will then use 115 lb. of No. 15 if it will fit in the winding space available. Since (Table 157) there are 101.6 ft. in 1 lb. of No. 15 bare copper wire, the length of wire in the 115-lb. coil required in this problem would be: 115 × 101.6 = 11,700 ft.

7. CHECK WIRE SIZE TO ASCERTAIN IF IT CAN BE WOUND IN SPACE AVAILABLE.—We must find room for 11,700 ft. of No. 15 wire which must be insulated. The thickness of the coil is 2 in. and its length is 10 in. The cross-section of the coil is, then: 2 × 10 = 20 sq. in. Assume that single cotton-covered magnet wire will be used. From Table 283, 1 sq. in. will contain 249 such wires or turns. The 20 sq. in. will contain: 20 × 249 = 4,980 turns. The mean turn has, as determined in (2), a length of 2.62 ft. Then, total length of wire that can be wound in coil is: 2.62 × 4,980 = 12,050 ft. It is, then, evident that there is ample room for the 11,700 ft. that is necessary, as calculated in (6).

282. Magnet Coils Operating on Constant Current, such as coils of constant-current or series generators and series street lighting system magnets, always have practically the same current flowing through them. Therefore, with such coils the

wire size merely determines the $I^2 \times R$ loss or heating in the coil. Where such a coil is to be designed, divide the ampere-turns required by the amperes flowing in the constant-current circuit; the result will be the number of turns required. Use a size wire that will carry this current without excessive heating.

283. Cotton-covered Annealed-copper Magnet Wire.

American or B & S gage	Bare		Single cotton-covered			Double cotton-covered			Triple cotton-covered		
	Dia mils d	Area cir mils d^2	Dia over ins mils dx	Wires per in $\frac{1,000}{dx}$	Wires per sq in $\left(\frac{1,000}{dx}\right)^2$	Dia over ins mils dx	Wires per in $\frac{1,000}{dx}$	Wires per sq in $\left(\frac{1,000}{dx}\right)^2$	Dia over ins mils dx	Wires per in $\frac{1,000}{dx}$	Wires per sq in $\left(\frac{1,000}{dx}\right)^2$
0000	460.0	212,000.0							478.0	2.09	4.36
000	410.0	168,000.0							428.0	2.33	5.42
00	365.0	133,000.0							383.0	2.61	6.81
0	325.0	106,000.0				339.0	2.94	8.64	343.0	2.91	8.46
1	289.0	83,700.0				303.0	3.30	10.8	307.0	3.25	10.5
2	258.0	66,400.0				272.0	3.67	13.4	276.0	3.62	13.1
3	229.0	52,600.0				242.0	4.13	17.0	247.0	4.04	16.3
4	204.0	41,700.0	211.0	4.73	22.3	216.0	4.62	21.3	220.0	4.54	20.6
5	182.0	33,100.0	189.0	5.29	27.9	194.0	5.15	26.5	198.0	5.05	25.5
6	162.0	26,300.0	169.0	5.91	34.9	174.0	5.74	32.9	178.0	5.61	31.4
7	144.0	20,800.0	151.0	6.62	43.8	156.0	6.41	41.0	160.0	6.25	39.0
8	128.0	16,500.0	136.0	7.35	54.0	141.0	7.09	50.2	145.0	6.89	47.4
9	114.0	13,100.0	121.0	8.26	68.2	126.0	7.93	62.8	130.0	7.69	59.1
10	102.0	10,400.0	108.0	9.25	85.5	112.0	8.92	77.5	116.0	8.02	64.3
11	90.7	8,230.0	97.0	10.3	106.0	101.0	9.90	98.0	105.0	9.52	90.6
12	80.8	6,530.0	87.0	11.4	129.0	91.0	10.9	118.0	95.0	10.5	110.0
13	71.9	5,180.0	78.0	12.8	163.0	82.0	12.1	146.0	86.0	11.6	134.0
14	64.1	4,110.0	70.0	14.2	201.0	74.0	13.5	182.0	78.0	12.8	163.0
15	57.1	3,260.0	63.0	15.8	249.0	67.0	14.9	222.0	71.0	14.0	196.0
16	50.8	2,580.0	56.0	17.8	316.0	59.0	16.9	285.0	63.0	15.8	249.0
17	45.3	2,050.0	50.0	20.0	400.0	53.0	18.8	353.0	57.0	17.5	306.0
18	40.3	1,620.0	45.0	22.2	492.0	48.0	20.8	432.0	52.0	19.2	368.0
19	35.9	1,290.0	39.0	25.6	655.0	43.0	23.2	538.0	47.0	21.2	449.0
20	32.0	1,020.0	36.0	27.7	767.0	40.0	25.0	625.0	44.0	22.7	515.0
21	28.5	810.0	32.5	30.7	942.0	36.5	27.3	745.0	40.5	24.6	605.0
22	25.3	642.0	29.0	34.4	1,180.0	33.0	30.3	918.0	37.0	27.0	729.0
23	22.6	510.0	26.6	37.5	1,400.0	30.6	32.6	1,060.0	34.6	28.9	835.0
24	20.1	404.0	24.1	41.4	1,710.0	28.1	35.5	1,260.0	32.1	31.1	967.0
25	17.9	320.0	21.9	45.6	2,070.0	25.9	38.6	1,480.0			
26	15.9	254.0	19.9	50.2	2,520.0	23.9	41.8	1,740.0			
27	14.2	202.0	18.2	54.9	3,010.0	22.2	45.0	2,020.0			
28	12.6	160.0	16.6	60.2	3,620.0	20.6	48.5	2,350.0			
29	11.3	127.0	15.3	65.3	4,260.0	19.3	51.8	2,680.0	Wires as small as these are seldom, if ever, covered with three layers of cotton.		
30	10.0	101.0	14.0	71.4	5,090.0	18.0	55.5	3,080.0			
31	8.93	79.7	12.9	77.5	6,000.0	16.9	59.1	3,490.0			
32	7.95	63.2	11.9	84.0	7,050.0	15.9	62.8	3,940.0			
33	7.08	50.1	11.1	90.0	8,100.0	15.1	66.2	4,380.0			
34	6.31	39.8	10.3	97.0	9,400.0	14.3	69.9	4,880.0			
35	5.62	31.5	9.6	104.0	10,800.0	13.6	73.5	5,400.0			
36	5.00	25.0	8.5	117.0	13,600.0	12.0	83.3	6,930.0			
37	4.45	19.8									
38	3.97	15.7									
39	3.53	12.5									
40	3.15	9.89									

SECTION 13

APPLICATIONS OF ELECTROMAGNETS

284. The Most Important Application of Electromagnets is for the field coils of electricity generators and motors. See index. The most economical method of developing electrical energy where relatively large quantities are required is to revolve suitably arranged conductors in intense magnetic fields, Art. 447.

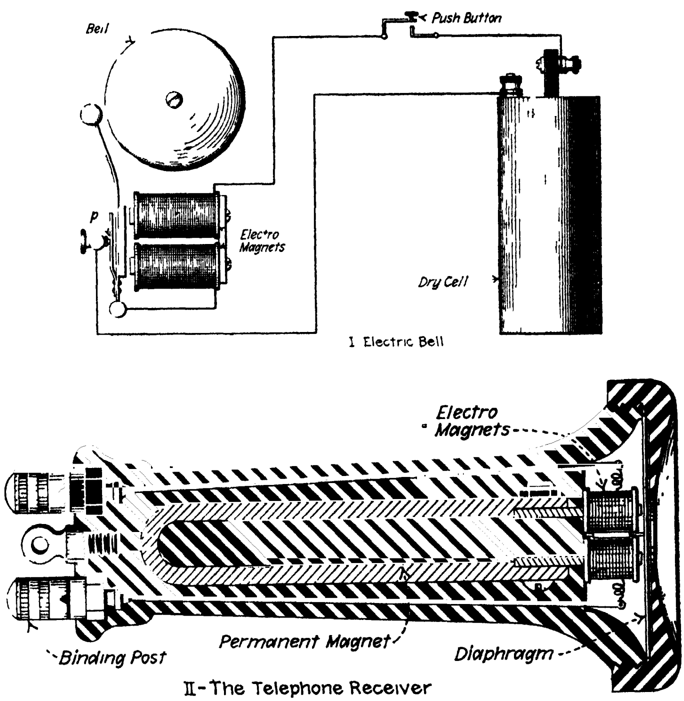


FIG. 159.—Practical applications of electromagnets.

Electromagnets are used to produce these fields; Art. 548 shows the field coils and a portion of the magnetic circuit of a direct-current generator or motor. Strong magnetic fields, produced by electromagnets are also necessary in electric motors.

285. Action of the Electromagnets in an Electric Vibrating Bell.—When the button B is pressed (Fig. 159,I) a circuit is

completed through the cell (sometimes though incorrectly called battery), button and bell and electricity flows. It magnetizes the iron cores of the magnets *A* by flowing around the magnet coils. Then the armature *S* is attracted and the circuit is broken at the contact point *P*. The electricity ceases to flow through the coils, the magnet cores lose their magnetization and the leaf spring on which the armature is mounted forces it back again against the contact point *P*. The circuit is thereby again completed and the process of attraction and "springing back" is thus continued so long as the button is pressed down completing the circuit. In commercial electric bells, one of the binding posts is usually electrically connected to the iron frame of the bell and the iron frame constitutes a portion of the circuit.

286. The Application of the Electromagnet in the Electric Telegraph is illustrated in Figs. 160 and 161. The *key* or transmitter (Fig. 160,*I*) is simply a current interrupter similar to a

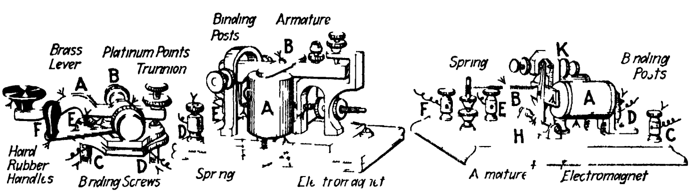


FIG. 160.—Telegraph instruments.

push button. It comprises a brass lever *A* which turns on a trunnion *B*. It is connected in the line by the binding screws *C* and *D*. On pressing down the lever, a platinum point, extending from the lower face of the lever, makes contact with another platinum point *E*. The circuit is thereby closed. When the key is not in use the path through the key is closed by shifting the lever *F*.

THE SOUNDER (Fig 160,*II*) or receiver comprises an electromagnet *A* with a pivoted armature *B*. When the circuit is closed through the terminals *D* and *E*, the armature is attracted toward the magnet This produces a clicking sound as the armature bar strikes between the two adjustable screws that limit its action When the circuit is broken, the spring *c* pulls the armature away from the electromagnet

THE RELAY (Fig. 160,*III*) is used to "relay" the line current in a long telegraph line. When the resistance of the line is great, the current may not be sufficiently strong to operate directly a sounder, which requires a relatively large current. Furthermore, there is always a leakage of current, from a

telegraph line, due to imperfect insulation. This leakage may be so pronounced that only a fraction of the current that leaves the sending station reaches the receiving station. To overcome these difficulties a relay is used. It consists of an electromagnet *A* having coils of many turns of small-diameter, copper wire, which is connected into the circuit (Fig. 161) by means of the binding posts *C* and *D*. As its armature *H* oscillates between the points *K*, it opens and closes a "local" circuit, connected through *E* and *F*, in which a battery and sounder are inserted. Thus, through the agency of a relay, a strong, local current, that flows or ceases to flow in step with the line current, is provided. This strong current operates the sounder.

THE BATTERY for a telegraph line ordinarily consists of a number of gravity cells (Art. 369) connected in series. If the line is long a considerable number of cells is used. Usually the battery is divided into two sections, one at each terminal station, each in series with the line.

A TYPICAL DIAGRAM OF A TELEGRAPH SYSTEM is shown in Fig. 161 which shows how the above described components are interconnected. The earth is ordinarily used for a return path for the current and an iron or copper

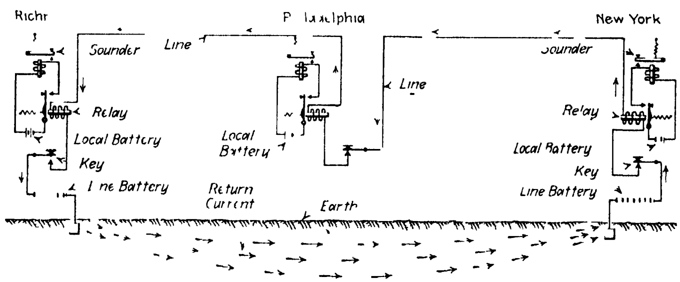


FIG. 161.—Diagram of an electric telegraph system.

wire supported on glass insulators is used for the other side of the circuit. An earth-return path has practically no resistance.

287. The Electromagnet in a Telephone Receiver is shown in Fig. 159,*II*. In this device the magnetic field produced by an electromagnet is superimposed on the field due to a permanent magnet of hard steel. The hard rubber case is in three parts: the base, the shell and the cap. Two permanent bar magnets are usually employed and they are so fastened together at one end as to constitute one horseshoe magnet. To each of the free ends of this horseshoe magnet are clamped the soft-iron pole-pieces. Each pole-piece is wound with a coil of small-diameter, insulated copper wire. The soft-sheet-iron diaphragm is supported in front of the pole pieces. The diaphragm forms a part of the magnetic circuit. Where the lines of force enter the one face of the diaphragm a south pole is induced. Where they

leave the other face of the diaphragm a north pole is induced. The diaphragm is really an armature and the attraction of the permanent magnet "dishes" it toward the pole-pieces.

NOTE.—The coils on the pole-pieces are wound in opposite directions. When a current flows around them in one direction it weakens the field of the permanent magnet. When the current flow is in the other direction the field of the permanent magnet is strengthened. The "talking" current in a telephone line is alternating. Hence, when such a current flows through the receiver magnet coils the field produced by the joint effect of the permanent and the electromagnets is alternately strengthened and weakened. This causes the diaphragm to vibrate and to reproduce the sound of the human voice.

When the field is strengthened, the diaphragm is attracted toward the pole-pieces. When the current ceases, the diaphragm assumes its normal position. When the field is weakened, that is, when the field produced by the electromagnets opposes the field of the permanent magnets, the diaphragm springs further away from the pole-pieces.

288. The Permanent Magnets in a Telephone Receiver increase the sensitiveness of a telephone receiver. It can be shown that* the sensitiveness is increased many times by the incorporation of the permanent magnets. Furthermore, if a receiver is not equipped with permanent magnets, the diaphragm would be attracted toward the poles of the electromagnets regardless of the direction in which the current flowed through the coils. Also, it would spring entirely back when the current ceased. It would apparently vibrate twice as fast if no permanent magnet were used which would interfere with the distinctness of the resulting sound.

289. Other Important Applications of Electromagnets are found in arc lamps, remote-control switches, circuit breakers, magnetic brakes and magnetic ore separators.

290. Magnetic Traction.—The *lifting power* or *portative force* of an electromagnet is due to the tension or pull always exerted along lines of force (Art. 65) whereby the lines tend to shorten themselves as do stretched rubber bands. For example, in Fig. 162,*I*, the lines of force flowing in the magnetic circuit traverse the path of least reluctance and must necessarily cross the air gap. If, however, a piece of iron *A* is placed across the air gap, as at *II*, nearly all of the lines will, because of the low reluctance of iron, flow through the iron piece. Then, the tension along the lines of force will draw the iron or armature up against the pole-pieces and hold it there.

* See THE ELECTRO MAGNET, Varley, p. 45.

291. The Factors That Determine the Lifting Power of a Magnet are numerous and are difficult to predetermine for all of the possible conditions. The tractive force (lifting power) of a given electromagnet depends not only on its magnetic strength but also on its form, the shape of its poles and on the form of the iron or armature that it attracts. Special treatises on electromagnets discuss in detail the characteristics of lifting magnets of certain forms and for certain purposes. However, if a magnet is assumed which has its poles in actual contact with the keeper, the problem is much simplified. For such a magnet the lifting power is proportional to just two factors: (1) It is directly proportional to the area of the pole faces which are in contact; (2) it is proportional to the square of the magnetic density (that is, to the square of the number of lines of force per square inch) in the minute gap between the pole face and the armature.

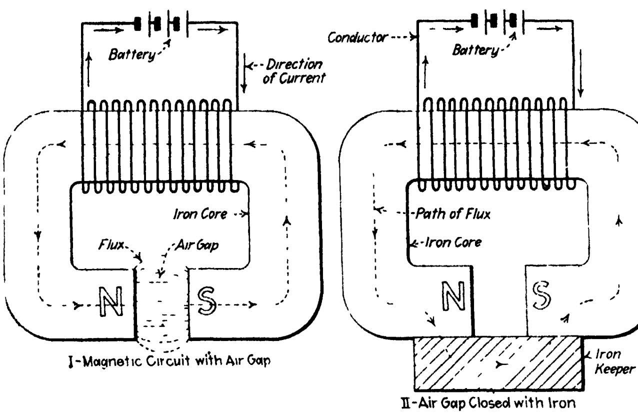


FIG. 162.—Illustrating the principle of magnetic traction.

292. The Lifting Power of a Magnet Is Proportional to the Square of the Flux Density in the joint between the pole faces and the keeper, as outlined in the preceding paragraph. That is, the lifting power depends not only on the number of lines of force flowing across the joint but also on the "closeness together" or density of these lines across the joint. This fact is illustrated by the values of Table 294, which were computed from the formula of Art. 295.

EXAMPLE.—Assume an electromagnet having a total pole area (Fig. 163,*I*) of 2 sq. in. and assume that the total flux, 5,000 lines, crosses each joint

between the magnet and the keeper. Obviously the flux density in each of these joints is 5,000 lines per sq. in. Now, from Table 294, the pull in pounds for each square inch pole area, with a density of 5,000 lines, is 0.34 lb. Hence the total pull of the magnet of *I* is: 2 sq. in. × 0.34 lb. = 0.68 lb. But if the magnetic circuit is rearranged, as at *II*, so that the total flux is 10,000 lines, the flux density will obviously be 10,000 lines per sq. in. in each of the joints. From Table 294, the pull for each square inch with this flux density is 1.4 lb. Therefore, the total pull is 2 sq. in. × 1.4 lb. = 2.8 lb. It is evident that by doubling the flux density the pull has been increased four times. This follows because the pull varies as the square of the flux density.

This fact (that the traction of a magnet increases as the square of the flux density) explains many apparently inconsistent phenomena relating to the lifting power of magnets. Where the surfaces of pole faces and keeper are exactly true and flat, the keeper may be held to the pole faces with considerably less force than if the pole faces are slightly convex. Furthermore, if the keeper of an electromagnet is shifted until only its sharp edge is in contact with the pole faces, it may be held to the poles with greater force than if it is placed fairly and squarely on them. Usually a magnet with pole faces that are slightly uneven will sustain a greater weight than one having surfaces that are absolutely true and smooth. The explanation for these odd conditions is that, when the area of contact is decreased, the flux density through the remaining contact surfaces is increased by the lines crowding into them. If the crowding is sufficient that the square of the density is increased more than the area is diminished, the lifting power is increased by reducing the area of contact.

293. To Produce a High Flux Density In the Joint so as to obtain great lifting power with minimum material in an electromagnet, the edges of the pole faces are frequently chamfered off as suggested in Fig. 164. With this construction, the flux density *in the joint* is high, but it is relatively low in the balance of the magnetic circuit. The following example illustrates the principle.

Example.—Consider the lifting magnet of Fig. 163,*III*, which is identical with that of *I*, except that the edges of the pole faces are chamfered off so that the area of each is ½ sq. in. Then the flux density in each joint is: 5,000 lines ÷ 0.5 sq. in. = 10,000 lines per sq. in. Now, from Table 294, the pull, per square inch area, with a flux density of 10,000 lines, is 1.4 lb. Then for the 1 sq. in. pole area of this problem the total pull will be: 1 sq. in. × 1.4 lb. = 1.4 lb. It is therefore evident that by chamfering off the edges of the poles the pulling power of the magnet has been increased from

0.68 lb. (in I) to 1.4 lb. (in II). A few more ampere-turns would be required to force the flux of 5,000 lines through the magnet circuit of II than through

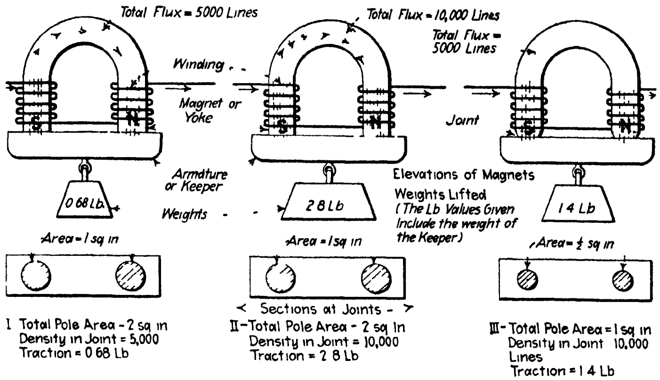


FIG. 163.—Illustrating the effect of flux density on lifting power.

that of I, but the increase in ampere-turns necessary would be very small and could probably be neglected in practice.

SECTION 14

MAGNETIC TRACTION AND LIFTING MAGNETS

294. Traction or Pull of Electromagnets at Different Flux Densities.—This table was calculated from the formula of Art. 295.

B Flux density, lines per sq. in. in joints between armature and magnet	t Traction, pull (in lb. per sq in pole area) between armature and magnet	B Flux density, lines per sq. in. in joints between armature and magnet	t Traction, pull (in lb per sq. in. pole area) between armature and magnet
5,000	0.34	75,000	78.0
10,000	1.40	80,000	88.7
15,000	3.10	85,000	100.0
20,000	5.50	90,000	112.0
25,000	8.70	95,000	125.0
30,000	12.50	100,000	138.0
35,000	20.00	105,000	153.0
40,000	22.20	110,000	168.0
45,000	28.10	115,000	183.0
50,000	34.60	120,000	199.0
55,000	41.90	125,000	216.0
60,000	49.90	130,000	234.0
65,000	58.50	135,000	252.0
70,000	67.90	140,000	272.0

295. The Formula for Computing the Pull of an Electromagnet is given below. It is derived by applying the higher mathematics but can be verified readily by experiment. The formula is strictly accurate only when there is no magnetic leakage, when the armature lies closely against the pole faces and when certain other ideal conditions are satisfied. However, it can be used without great error for solving nearly all practical problems—even if the armature and pole faces are some distance apart.

$$T = \frac{B^2 \times A}{72{,}134{,}000} \quad \text{(lb.)} \tag{89}$$

$$B = 8{,}494 \times \sqrt{\frac{T}{A}} \quad \text{(lines)} \tag{90}$$

$$A = \frac{72{,}134{,}000 \times T}{B^2} \quad \text{(sq. in.)} \tag{91}$$

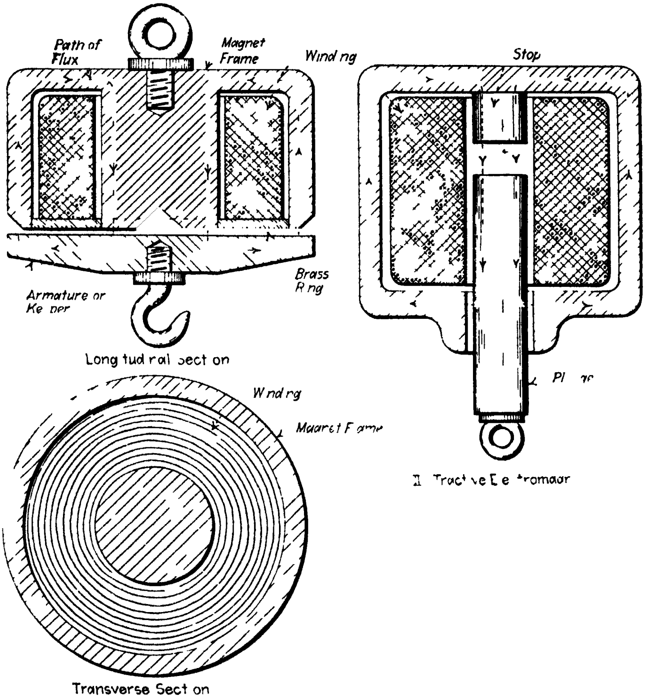


FIG. 164—Lifting and tractive electromagnets.

Wherein T = total pull in pounds between the armature and keeper and the magnet. B = flux density (in lines per square inch) in the joint between the pole faces and the armature. A = the cross-sectional area, in square inches, of the pole faces against which the armature is drawn.

296. Types of Lifting Magnets.—Probably the most effective design for a lifting magnet is that shown in Fig. 164,*I*. For maximum lifting power, the greatest possible number of lines should be driven across the joint between the magnet and the

armature. Furthermore, the area of the joint should be as small as feasible to produce a high flux density in the joint, for the reasons outlined above. The magnet case or core almost entirely encloses the winding and thereby protects it from damage. Fig. 165 shows the construction of a large lifting magnet of this type. The *coil* can be taken from the *case* by removing the *coil shield.* In Fig. 164,*II*, is shown a coil and plunger electromagnet. This kind of magnet provides a powerful pull over a short range.

297. Operating Voltages for Electromagnets.—In ordinary practice, 110 or 220 volts, direct-current, is used. Lower voltages can be used but higher voltages should not be. The counter e.m.f. of self-induction (Art. 464) induced in the coil of a magnet wound for a voltage in excess of 220 may be (at the instant the circuit feeding it is opened) considerable. It is likely to be so

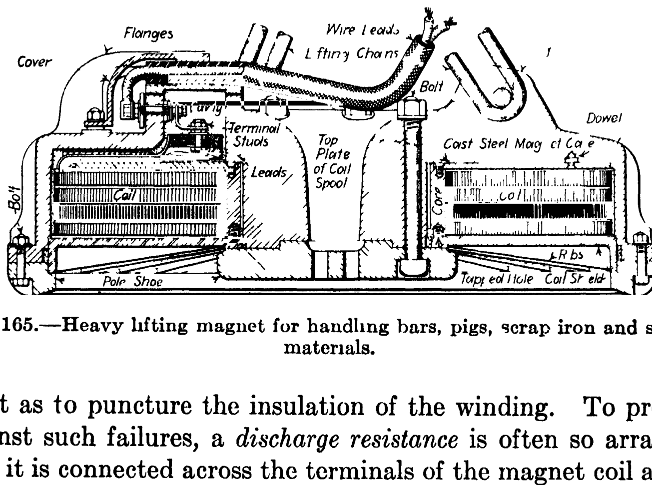


FIG 165.—Heavy lifting magnet for handling bars, pigs, scrap iron and similar materials.

great as to puncture the insulation of the winding. To protect against such failures, a *discharge resistance* is often so arranged that it is connected across the terminals of the magnet coil as the circuit is opened. The function of a discharge resistance is to afford a path wherein the energy of self-induction can be dissipated. It thereby protects the magnet coil.

298. To Make an Electromagnet Drop Its Load Instantly, a control switch is sometimes provided whereby a limited reverse current is passed through the winding. This neutralizes the effect of *residual magnetism* and causes the load to drop instantly. Such a control switch may have three positions: *Lift, Release* and *Off.*

299. Considerations Affecting the Design of a Lifting Magnet. —Where a magnet is to be designed for a given service and is to

have minimum weight and to involve minimum cost the following requirements should be satisfied: (1) The magnetic circuit should be as short as possible. (2) The cross-sectional area of the magnetic circuit should (except at the joint) be as nearly uniform as possible and should be as great as possible. (3) The iron used in the magnetic circuit should have a high permeability. (4) The flux densities should be, approximately, those outlined in Art. 301.

300. The Process of Designing a Lifting Magnet is substantially as follows. (It should be understood that it is impossible to design directly a lifting magnet to satisfy given conditions. It is necessary to make certain assumptions and calculate a design on this assumed basis. Then, if the tentative design does not work out satisfactorily, the assumptions are revised and the design is recalculated. This process must be repeated until the desired result is attained.)

PROCESS.—(1) Assume the magnetic densities, in accordance with the suggestions of Art. 301. Remember that the edges of the pole faces may be chamfered off to increase the density in the joint. (2) The densities that will be allowed being known, the areas of the pole faces and the cross-sectional area of the magnetic circuit can be determined. (3) Now select the overall dimensions of the magnet allowing ample space for the magnetizing coil. Several trials will probably be necessary before these dimensions can be decided. (4) Calculate the coil necessary (Art. 281) to supply the ampere-turns required for magnetizing the magnet. If this coil is too large or too small for the winding space allowed for it, the entire design must be revised accordingly.

301. Flux Densities in the Magnetic Circuits of Lifting Electromagnets (see also Art. 259).—For the reasons hereinbefore outlined, it is extremely desirable that the highest flux densities feasible be utilized in the joints between the magnet pole faces and the armatures. It is practically impossible to produce a flux density greater than 140,000 lines per sq. in. in ordinary steels. Frequently a density of 110,000 lines per sq. in. at the joint is taken as a maximum for ordinary steels because if an effort is made to develop a density much greater, the reluctance of the magnetic circuit is materially increased and consequently considerably more copper will be required in the exciting coil. The preceding data in this article relate only to the *density at the joint.*

In the remainder of the magnetic circuit reasonably high flux densities should be used. In the armature and magnet cores,

provided they are of cast steel, wrought iron or annealed steel, a density of about 100,000 lines per sq. in. may represent good practice. In cast-iron portions of the circuit, a density of about 60,000 should not be exceeded. Usually the entire magnetic circuit is of wrought iron, cast steel or annealed sheet iron. However, cast-iron armatures are sometimes employed. The principal portion of the magnetic circuit is seldom of cast iron because this material, assuming a given reluctance, occupies much more space than do the others.

302. Examples Illustrating the Methods Used in Designing Lifting Magnets are given in the following articles. They indicate the general principles involved. However, each problem has, usually, distinctive features.

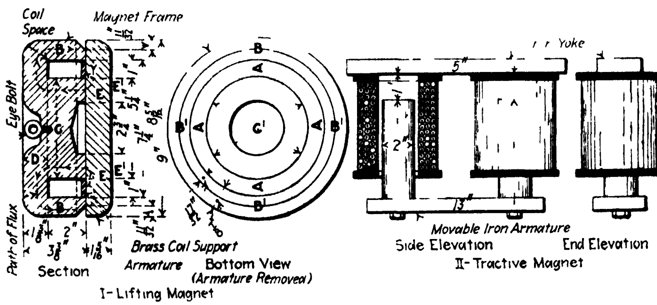


FIG. 166.—Examples in designing lifting magnets.

EXAMPLE.—What will be the lifting power of an electromagnet 9 in. in diameter, that is, what weight can a magnet of this diameter be made to support? Assume a flux density of 140,000 lines per sq. in. at the joint and a density of 100,000 lines in the remainder of the magnetic circuit. SOLUTION.—It will be assumed tentatively that the magnetizing coil will occupy a cross-section of 1 in. $\times$ 2 in., as shown in Fig. 166,*I*. This coil space *A* must be so located in the magnet frame that the sum of the sections *B* and *B*, outside of the coil, will equal, approximately, section *C* inside of the coil. The same number of lines will flow through the ring of iron outside of the coil as flows through the cylinder of iron inside of the coil. The diameters —determined by trial—indicated in the illustration provide areas that nearly satisfy these requirements. The area of the flat ring $B'B'B'B'$ is (0.7854 [$9 \times 9 - 7.25 - 7.25$]) 22.2 sq. in. The area of the circle C' is (0.7854 [5.25×5.25]) 21.65 sq. in. Therefore, the area $B'B'B'B'$ is, practically, equal to area C'.

With a density of 100,000 lines per sq. in., the total flux in the magnetic circuit will be: $100{,}000 \times 21.65$ sq. in. $= 2{,}165{,}000$ lines. The flux density at the joints is to be 140,000 lines. Hence, enough of the pole faces must be cut away that the area which remains will provide a density of 140,000 lines

per sq. in., the total flux being 2,165,000 lines. By proportion—100,000 lines : 140,000 lines : : x sq. in. : 21.65 sq. in. Then, x = (21.65 × 100,000) ÷ 140,000 = 15.5 sq. in. Each of the pole faces shown in the illustration has this area.

From Table 294, the pull (with a density of 140,000) exerted by every square inch pole area is 272 lb. Hence, the total pull that the magnet can exert is: 272 lb. × 31 sq. in. = 8,430 lb.

The armature should be sufficiently thick that the flux density in it will nowhere exceed 100,000 lines per sq. in. The area of maximum density in the armature of this problem will be an imaginary, cylindrical ring, 5¼ in. in diameter, passing through the armature from one of its faces to the other, as E–E'. The area of this cylindrical surface, through which the flux must pass, will equal: *the circumference of a 5¼-in. diameter circle × the thickness of the armature.* The circumference of a 5¼-in.-diameter circle is 16.5 in. Since an area of 21.6 sq. in. is required, the thickness necessary is, therefore: 21.6 sq. in. ÷ 16.5 in. = 1.32 in. or, say, 1⁵⁄₁₆ in. From this same calculation it is evident that the thickness D, at the upper part of the magnet, should also be 1⁵⁄₁₆ in., but a thickness of 1⅜ in. is adapted to allow more space for the eye bolt.

The ampere-turns necessary to produce the flux required and the amount and size of the wire to be used to develop these ampere-turns can be computed from directions given in Arts. 274 and 281. Magnet wire with a non-combustible insulation, such as asbestos or a similar material, can be used where the winding space is restricted. Wires covered with such materials can withstand very high temperatures without injury.

EXAMPLE.—How many amperes will be required to so excite the tractive magnet of Fig. 166,*II*, that it will lift 100 lb.? Plunger is 2 in. in diameter and each air gap is 1 in. long. SOLUTION.—The area of the magnetic circuit is: 2 in. × 2 in. × 0.7854 = 3.14 sq. in. A total load of 100 lb. is to be lifted. Hence, each of the two pole faces must lift: 50 lb. ÷ 3.14 sq. in. = 16 lb. From Table 294, a flux density of about 33,000 lines per sq. in. is necessary to lift a load of 16 lb. per sq. in. of pole face. Total flux will, therefore, be: 33,000 × 3.14 = 104,000 lines. Ascertain the ampere-turns necessary to produce this flux. Using the formula of Art. 254, there will be required *to drive this flux across the air gaps:*

$$(a) \qquad IN = \frac{\phi \times l}{\mu \times A} = \frac{104{,}000 \times 2}{3.19 \times 3.14} = 20{,}700 \text{ amp.-turns.}$$

To drive the flux through the remainder of the magnetic circuit, which is of wrought iron and has a cross-sectional area of 3.14 sq. in. and a length of approximately 18 in., there will be required (see Table 249 for flux density and permeability values):

$$(b) \qquad IN = \frac{\phi \times l}{\mu \times A} = \frac{104{,}000 \times 18}{5{,}100 \times 3.14} = 118 \text{ amp.-turns.}$$

Then the total number of ampere-turns required is: 20,700 + 118 = 20,818 *amp.-turns.* Note that practically all of the ampere-turns are required to drive the flux across the air gap and that comparatively few are necessary for the magnetization of the remainder of the magnetic circuit. Now assume that each coil of the magnet has 100 turns, giving a total of 200 turns, then the current that should flow in order that the weight of 100 lb. will be lifted is: 20,818 ÷ 200 = 104 amp.

SECTION 15

HYSTERESIS

303. Hysteresis is that quality of a magnetic substance (Art. 45) whereby energy is dissipated on the reversal of its magnetism. It may also be defined as a lagging of magnetization behind the force that produces it.

304. Explanation of Hysteresis.—Refer to Fig. 44 delineating how iron filings in a bottle may be magnetized with a permanent magnet and also to Fig. 43 which indicates the generally accepted theory of magnetization. Review the text relating to these illustrations. It will be readily conceived that work (Art. 158) is required to "line up" the molecular particles in a piece of iron to make it a magnet. This is true whether it becomes a permanent magnet or an electromagnet. It is also obvious that work will be required to shift around the molecules of a magnet if it is desired to demagnetize it. Furthermore, the expenditure of more work will be necessary if the magnet is now remagnetized in the other direction, that is, if a S pole is now developed at the end that was formerly a N pole. That this work is necessary is due to hysteresis. Hysteresis may then be thought of as being due to molecular friction. More work is necessary to reverse the magnetism in a piece of hard steel than is required to reverse the magnetism of a soft-iron piece of the same dimensions. The amount of work that is required increases as the reluctance of the material increases.

305. Hysteresis Loss is the loss of power (Art. 159) due to hysteresis and it can be expressed in watts—or in kilowatts. In any case hysteresis loss is relatively small. It is of no material consequence where a piece of iron is magnetized and demagnetized or has its magnetism reversed infrequently. However, in some electrical apparatus the flux in a magnetic circuit may be reversed many times a second (see following examples) and then hysteresis loss becomes of consequence, though it is usually very small as compared with the output of the apparatus. This power lost by hysteresis or molecular friction appears as heat but serves no useful purpose—it is a dead loss. It raises the temperature of that portion of the magnetic circuit in which the reversal of

magnetism occurs. In constructing magnetic circuits that are subject to hysteresis losses, great care is exercised to select therefor grades of iron that develop low hysteresis losses, that is, that have low *hysteresis coefficients* or *constants* so that the losses in them may be maintained at a minimum.

EXAMPLE.—In Fig. 167,*I* and *II*, is shown a cylinder of iron rotating in a magnetic field. The magnetism in the cylinder is reversed at each half revolution. At *I* the portion of the cylinder having the two slots cut in it has a S pole developed in it. Now, when it rotates a half revolution to position *II*, this portion becomes a N pole. Obviously, if this iron cylinder caused to rotate rapidly the flux in it will reverse very frequently and the hysteresis loss in it will be considerable. *Armatures* of generators and motors rotate in magnetic fields under conditions similar to those just described.

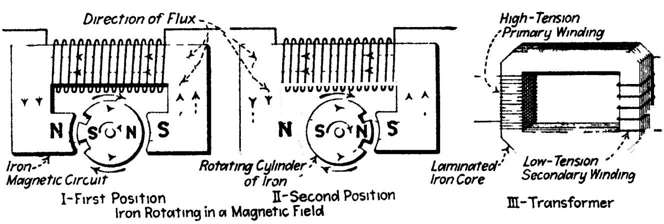


FIG. 167.—Illustrating hysteresis in iron.

EXAMPLE.—The elementary magnetic circuit of a transformer (Art. 821) is shown in Fig. 167,*III*. The alternating current (Art. 828) that magnetizes the core of a transformer changes in direction many times a second. Consequently, the flux in a transformer core reverses many times a second. This causes a hysteresis loss. The highest quality iron is used for transformer cores so that the hysteresis loss in them will be as low as possible.

306. Approximate Hysteretic Constants for Different Materials.

No	Material	Hysteretic constant	*Relative value
1	Best annealed transformer sheet steel (silicon steel)	0.001	0.25
2	Thin sheet iron, good quality	0.003	0.75
3	Wrought iron—ordinary sheet iron	0.004	1.00
4	Soft annealed cast steel	0.008	2.00
5	Soft machine steel	0.009	2.25
6	Cast steel	0.012	3.00
7	Cast iron	0.016	4.00

* The values in this column indicate how much greater or less is the hysteresis loss in metals as compared with wrought iron. Wrought iron is taken here as a standard because the values of Table 307 are for wrought iron.

307. Approximate Hysteresis Loss in Wrought Iron. The values are necessarily approximate because the loss varies with the quality of the material. The tabulated data is for good commercial soft wrought iron. The hysteresis loss in irons of other grades can be approximated by multiplying the result obtained from using the values here tabulated by the quantity given in the "*Relative value*" column of Table 306 for the grade of iron under consideration.

B Flux density, lines per sq. in.	Hysteresis loss, in watts wasted per cu. in. per cycle	*B* Flux density, lines per sq. in.	Hysteresis loss, in watts wasted per cu. in. per cycle
25,000	0.00378	75,000	0.0187
30,000	0.00523	80,000	0.0202
35,000	0.00630	85,000	0.0224
40,000	0.00750	90,000	0.0241
45,000	0.00888	95,000	0.0268
50,000	0.0140	100,000	0.0294
55,000	0.0115	105,000	0.0318
60,000	0.0133	110,000	0.0370
65,000	0.0149	115,000	0.0458
70,000	0.0165	120,000	0.0535

308. Hysteresis Loss Is Determined by Three Factors: (1) The number of cycles (Art. 681) per second, that is, the number of times the magnetism or flux is reversed in a second; (2) the maximum density, B, of the flux (Art. 246) in the magnetic material; and (3) the quality, that is, the hardness of the magnetic material.

309. Calculation of Hysteresis Loss.—All methods are more or less approximate unless detailed information relating to the properties of the specimen under consideration is available. The fundamental formula that applies, though it is quite simple, can not be included here because of certain mathematical operations that it involves. Approximate, practical calculations can be made by using the derived formula given below in connection with the values of Tables 306 and 307. Expressing the factors of Art. 308 in a working formula:

(92) $$P_H = p \times V \times \eta \qquad \text{(watts)}$$

(Note that the above formula as here given applies only to wrought iron. For the method of computing the loss in other

kinds of iron, see the following example.) Wherein P_H = the total hysteresis loss, in watts. p = the watts hysteresis loss, per cubic inch per cycle, as taken from Table 307. V = the volume of the iron, in cubic inches. η = the number of cycles, per second, or twice the number of magnetic reversals, per second.

EXAMPLE.—What will be the hysteresis loss in the iron cylinder of Fig. 167,*I* if it has a volume of 300 cu. in. and is turning at a speed of 1,000 r.p.m. in a field having a density of 50,000 lines per sq. in.? SOLUTION.—In a two-pole generator, one revolution is equivalent to 1 cycle or to 2 reversals of magnetism. Hence, with the generator indicated in the illustration, 1 revolution is equivalent to 1 cycle. Now, from Table 307, the hysteresis loss in 1 cu. in. of wrought iron, with a flux density of 50,000 lines is 0.014 watt per cycle. Then, substituting in the above formula (92): $P_H = p \times V \times \eta = 0.014 \times 300 \times 1{,}000 = 4{,}200$ *watts* = 4.2 *kw.*

EXAMPLE.—If the cylinder of the above example were of best annealed transformer steel, what would the hysteresis loss in it then be? SOLUTION.—From the column "Relative value" of Table 306, this best annealed steel shows a loss of 0.25 of that in wrought iron. Hence: 4,200 watts $\times$ 0.25 = 1,050 *watts*. Then, 1,050 watts, 1.05 kw., would be the loss were the cylinder made of best annealed steel.

EXAMPLE.—What would be the hysteresis loss in a block of wrought iron of 100 cu. in. if it were excited to a maximum flux density of 30,000 lines per sq. in. with a 60-cycle alternating current? SOLUTION.—With this flux density, from Table 307, the loss per cubic inch per cycle would be 0.0052 watt, then substituting in formula (92): $P_H = p \times V \times \eta = 0.0052 \times 100 \times 60 = 31.20$ *watts*.

SECTION 16

CONTACT ELECTROMOTIVE FORCES

310. How Electrical Energy May Be Developed Through the Contact of Dissimilar Substances.—There always exists between dissimilar substances in contact a voltage or difference of potential (Art. 96). But as outlined in Art. 316, if the dissimilar substances are arranged in a closed circuit to provide a path for a current, the voltages due to the contacts of the dissimilar substances will, under ordinary conditions, neutralize. The resultant e.m.f. will be zero. Hence, no current can flow. Therefore (Art. 181), no energy can be generated. However, if external energy be properly imparted to the dissimilar substances in contact the neutralization of the e.m.fs. will be deranged. An e.m.f. will then establish which will force current around the circuit. When the current flows energy will be developed (Art. 181).

Thus, this phenomenon is merely an example of the general principle outlined in Art. 179, that the "generation of electrical energy" really means the transformation of some other kind of energy into electrical energy.

311. The Amount of Energy that Can Be Developed Through the Contact of Dissimilar Substances Is Relatively Very Small.—The voltages developed are small (Table 315) and the currents are small. The method has no important *commercial applications* except for temperature measuring apparatus.

NOTE.—The e.m.f. of contact between metals and electrolytes is relatively large and advantage is taken of this fact by generating electrical energy by chemical action. See Art. 326.

312. In Generating Electrical Energy by Contact of Dissimilar Substances, the External Energy Imparted Is Usually Heat Energy.—Therefore, although, theoretically, energy might be generated by imparting heat at the point of contact of *any* two dissimilar substances, only dissimilar metals are ordinarily employed. Most other substances are either mechanically unsuitable or they can not withstand heat without damage.

313. Generation of E.m.f. by Contact of Unlike Substances.—It can be shown experimentally that when *any* two unlike substances are placed in contact there exists between them a differ-

ence of potential or voltage. Ordinarily, this voltage is very small except in the case of electrolytes in contact with metals. See Art. 358 under "Primary Cells." The value of this e.m.f. is in any specific case determined by: (1) *The substances;* (2) *the character of the contact surfaces;* (3) *the medium in which the contact occurs;* (4) *the conditions existing in the medium.*

NOTE.—THE ELECTRON THEORY EXPLANATION OF CONTACT ELECTROMOTIVE FORCES IS THIS:—Certain substances possess the property of readily giving up electrons to other substances when the two different substances are placed in contact. Thus, it has been demonstrated experimentally that zinc readily parts with electrons to copper, when the two metals are placed in contact. The consequence is that when two dissimilar substances are in contact, one acquires an excess of electrons and the other a deficit. Hence, one substance (the one which loses electrons) becomes positively electrified and the other (the one which gains electrons) becomes negatively electrified—a difference of potential or a voltage is thereby established between them.

For example, when (in air) a piece of zinc is placed in contact with a piece of copper, electrons pass from the zinc to the copper. The flow of electrons continues until the number of electrons, lost by the zinc and gained by the copper, is such that the voltage between the zinc and copper is just equal to the contact difference of potential (which has been determined by experiment, Table 315) which always exists between zinc and copper. It does not appear to be definitely known just why some substances part readily with their electrons to others. In other words, we do not know why the atoms of some materials hold their electrons more firmly than do those of other materials.

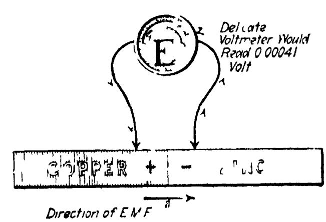


FIG. 168.—Showing contact e.m.f. developed between copper and zinc.

314. Table 315 Shows Values of E.m.f. Developed by Contact of Metals at ordinary temperatures. The order in which the names of the metals are arranged in the table indicates their relative abilities to develop e.m.fs. by contact. The greater the separation of any two metal names in the table, the greater will be the e.m.f. developed by the contact of the two metals. When two of the metals are in contact, the one the name of which appears first in the table will be positive (+), as related to the other which will be negative (−). The direction of the e.m.f. developed will be from + to −. The values and relations of the table were determined by experiment and calculation.

EXAMPLE.—If at ordinary temperatures a piece of copper is placed in contact with a piece of zinc (Fig. 168) there will be between them, from Table

315, a difference of potential or an e.m.f. of 0.00041 volt. Furthermore, the copper, since it precedes zinc in the columns of Table 315, will be positive and the zinc will be negative. That is, the direction of the e.m.f. will be from the copper toward the zinc.

315. Contact E.m.fs. Between Metals, in Volts.—The values given are the e.m.fs. developed between the different metals in contact at ordinary temperatures. As the temperature at the location of the contact increases, that is, as heat energy is imparted, the contact e.m.f. is increased; see Sec. 318. The values in the table are approximate. Different values are given by different authorities.

Metals +	Bismuth	Nickel	Palladium	Platinum	Aluminum	Tin	Lead	Gold
+ Bismuth	0	0.01591	0.01988	0.02108	0.02142	0.02149	0.02174	0.02181
Nickel	0.01591	0	0.00397	0.00517	0.00551	0.00558	0.00583	0.00590
Palladium	0.01988	0.00397	0	0.00120	0.00154	0.00161	0.00186	0.00193
Platinum	0.02108	0.00517	0.00120	0	0.00034	0.00041	0.00066	0.00073
Aluminum	0.02142	0.00551	0.00154	0.00034	0	0.00007	0.00032	0.00039
Tin	0.02149	0.00558	0.00161	0.00041	0.00007	0	0.00025	0.00032
Lead	0.02174	0.00583	0.00186	0.00066	0.00032	0.00025	0	0.00007
Gold	0.02181	0.00590	0.00193	0.00073	0.00039	0.00032	0.00007	0
Silver	0.02227	0.00636	0.00239	0.00119	0.00085	0.00078	0.00053	0.00046
Copper	0.02231	0.00640	0.00243	0.00123	0.00089	0.00082	0.00057	0.00050
Zinc	0.02272	0.00681	0.00248	0.00164	0.00130	0.00123	0.00098	0.00091
Cadmium	0.02303	0.00712	0.00315	0.00195	0.00161	0.00154	0.00129	0.00122
Iron	0.02524	0.00933	0.00536	0.00416	0.00382	0.00375	0.00350	0.00343
− Antimony	0.02793	0.01203	0.00806	0.00686	0.00652	0.00645	0.00620	0.00613

Metals +	Silver	Copper	Zinc	Cadmium	Iron	Antimony	(Negative) −
+ Bismuth	0.02227	0.02231	0.02272	0.02303	0.02524	0.02793	
Nickel	0.00636	0.00640	0.00681	0.00712	0.00933	0.01203	
Palladium	0.00239	0.00243	0.00248	0.00315	0.00536	0.00806	
Platinum	0.00119	0.00123	0.00164	0.00195	0.00416	0.00686	
Aluminum	0.00085	0.00089	0.00130	0.00161	0.00382	0.00652	
Tin	0.00078	0.00082	0.00123	0.00154	0.00375	0.00645	
Lead	0.00053	0.00057	0.00098	0.00129	0.00350	0.00620	
Gold	0.00046	0.00050	0.00091	0.00122	0.00343	0.00613	
Silver	0	0.00004	0.00045	0.00076	0.00297	0.00567	
Copper	0.00004	0	0.00041	0.00072	0.00293	0.00563	
Zinc	0.00045	0.00041	0	0.00031	0.00252	0.00522	
Cadmium	0.00076	0.00072	0.00031	0	0.00221	0.00491	
Iron	0.00297	0.00293	0.00252	0.00221	0	0.00270	
− Antimony	0.00567	0.00563	0.00522	0.00491	0.00270	0	

316. The Resultant E.m.f. of Metals All at the Same Temperature and Changed into a Closed Circuit Is Zero (0).—As in Fig.

169, *I*, the various e.m.fs. (from Table 215) at all of the different points of contact will always just neutralize one another and the resultant e.m.f. around the entire circuit will be zero (0), as indicated in the illustration. Hence, there will be no tendency toward the production of a current around the circuit. That is, no electrical energy can be generated by the mere contact of metals. Only e.m.fs. are generated by contact and, as shown, these ordinarily neutralize or cancel one another and are, therefore, practically ineffective. The foregoing statements in this paragraph are true only if energy from some external source is not imparted to the metals in contact. Where energy, heat for example, from an external source is imparted to a junction or junctions between dissimilar metals there will be a flow of current—hence the generation of electrical energy. How it is that this occurs will be shown in articles that follow.

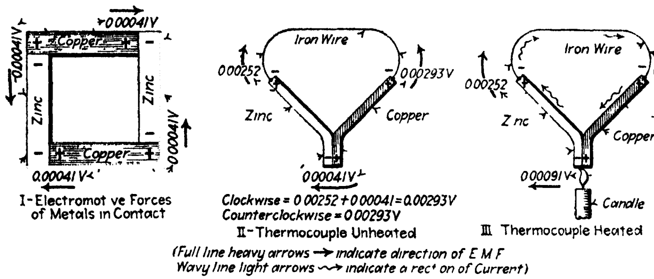


FIG. 169.—Illustrating generation of electrical energy by contact.

317. Performance of Metals in Contact When a Junction Between Two of Them Is Heated.—Consider Fig. 169,*II*. A piece of copper and one of zinc in contact and having their outer ends joined by a piece of iron wire comprise a closed circuit. Copper, zinc and iron are assumed to all be at the same temperature. An e.m.f. is developed at each of the points of junction between the dissimilar metals, as shown. The e.m.f. values are from Table 315. However, the resultant e.m.f. around the entire circuit is zero. There is a total e.m.f. of 0.00293 volt in a clockwise direction around the circuit. But there is also an e.m.f. of 0.00293 volt in a counterclockwise direction. Therefore, these e.m.fs. neutralize one another and there is no tendency to produce a current.

NOTE.—Using the voltage values of Table 315, the e.m.fs. around a circuit of metals in contact may not exactly cancel. This is because the values of

the table are not carried out to a sufficient number of decimal places. Were they carried out to say six decimal places, the resultant e.m.f. of contact, around a closed circuit, derived by using them would be zero (0) in every instance.

It can be shown experimentally that, if the junction of two dissimilar metals is heated, the contact e.m.f. at that junction is increased. Then, provided the remainder of the circuit is at a lower temperature than the heated junction, the contact e.m.fs. around the circuit no longer neutralize. Thus, if the junction of the copper and zinc is heated as at *III*, its contact e.m.f. is now greater than its contact e.m.f. at ordinary temperatures. Thereby a current is forced around the circuit. Another way of stating the same fact is: when the junction of two dissimilar metals is heated, as at *III*, an e.m.f. in addition to that of ordinary contact will be developed. This additional e.m.f. will force current around the circuit.

In *III* it has been assumed that the additional e.m.f. due to the heating of the junction is 0.00050 volt. This gives an e.m.f. of 0.00091 volt across the junction of the copper and the zinc. The current which flows around the circuit is due to this additional e.m.f. of 0.00050 volt. Obviously the current will be small.

318. The Value of the E.m.f. Developed Across the Junction of Dissimilar Metals by Heating Them is proportional to the difference between the temperature of the junction and the temperature of the remainder of the circuit. A considerable difference in temperature is necessary to produce an appreciable effective e.m.f. The e.m.f. value given in connection with Fig. 169,*III*, could not, probably, be obtained in practice with the apparatus shown. If the temperature of the junction between dissimilar metals is decreased below that of the remainder of the circuit, an e.m.f. is also produced but it is in the opposite direction to that developed when the junction is heated.

319. If External Energy is Imparted to Dissimilar Metals in Contact in a Closed Circuit, Electrical Energy Is Developed.— The energy imparted may be chemical energy as explained under "cells" in succeeding articles or it may be heat energy as suggested in the preceding article. As outlined in Art. 317, the mere contact of the zinc and the copper in Fig. 169,*II* did not develop energy. However, when energy—heat—was imparted from an external source an e.m.f. was developed which forced a current through the circuit. That is, by the application of heat energy, electrical energy was developed. What actually occurred was that some of the heat energy from the candle was transformed into electrical energy through this peculiar contact action of metals.

320. A Thermoelectromotive Force is one developed by the application of heat to the junction of dissimilar metals as described in preceding articles. *Thermoelectric currents* are the currents forced through circuits by these forces.

321. A Thermo-couple or Thermo-electric Couple consists (Figs. 169,*II*, 170) of two pieces of dissimilar metals. They are joined together or are in contact at one end while the other ends are electrically connected together by a conductor which completes an electrical circuit. When the joined ends are heated, an e.m.f. is developed as hereinbefore described, which forces a current through the conductor.

322. A Thermo-electric Pile or Thermo-pile consists of a series (Fig. 170) of thermo-couples so joined in series as to form a battery. The e.m.fs. generated by all of these couples in series are added to one another. Consequently, if a sufficient number of couples is grouped, a considerable voltage and current may be thus developed. Thermo-piles have never been applied successfully for the commercial generation of electricity, largely because it appears impossible to construct an arrangement of dissimilar metals that can continuously and successfully withstand the high temperatures and the differences in temperature upon which a thermo-pile must depend for its operation.

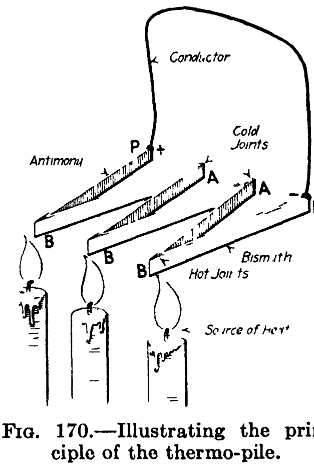


FIG. 170.—Illustrating the principle of the thermo-pile.

323. A Bolometer, Thermo-bolometer or Pyrometer is an arrangement, utilizing the principle of the thermo-couple, for measuring temperatures. This device represents the most important application of thermo-electricity. The value of the e.m.f. developed by a thermo-couple depends upon the difference between the temperature of the joint of the thermo-electric couple and the temperature of the remainder of the circuit. It follows that, if the remainder of the circuit is maintained at a constant temperature, the current forced through the circuit due to thermo-electric action will depend on the temperature of the thermo-couple joint.

If this current flows through an ammeter or galvanometer, the reading of the instrument will be proportional to the temperature at the joint. The instrument can be so calibrated that it will show directly the temperature of the joint. The joint can be arranged in any location—such as in a furnace or a pot of molten metal—the temperature at which it is desired to know. The registering instrument can be connected to the joint by conductors and can be mounted at any convenient point. Very high or very low temperatures can be measured and the instrument can be constructed so sensitively that, with a bismuth-antimony thermo-couple, a change in temperature of one-millionth of a degree can be readily measured. Platinum-rhodium thermo-couples are used for measuring high temperatures.

324. Peltier Effect.—It was discovered by Peltier that if a current is passed through a joint between dissimilar metals, it will heat the joint if it passes from positive to negative and that it will cool the joint if it flows in the opposite direction. This phenomena is Peltier effect. Do not confuse this with I^2R-loss heating effect which is an entirely different thing. Peltier effect and I^2R loss may occur in the same joint. Under ordinary conditions Peltier effect is inconsequential.

325. Thompson Effect.—Sir William Thompson discovered that if one portion of a conductor of pure metal had a temperature higher than another portion, an e.m.f. would be developed between these points. This phenomena is called Thompson effect. It is inconsequential for all ordinary conditions and is of theoretical interest only.

SECTION 17

THE PRINCIPLES OF PRIMARY CELLS

326. How Electrical Energy Is Generated by Chemical Action will be explained in some detail in the sections that follow. Briefly: An e.m.f. exists between a metal immersed in a chemical solution and the solution itself as outlined in Art. 328. If a circuit is provided, this e.m.f. will force current through the circuit and thus generate electrical energy. But as the current flows, the metal and the chemical solution are consumed—the chemical energy latent in them is transformed into electrical energy. Therefore, when the term "generation of electrical energy by chemical action" is used, the real meaning is that chemical energy is transformed into electrical energy.

327. Generation of Electrical Energy by Chemical Action Is Really a Specific Case of Generation by Contact of Dissimilar Substances.—As noted in Art. 310, when any two dissimilar substances are in contact an e.m.f. develops between them. Thus, an e.m.f. develops (Art. 328) between a metal and a chemical solution—dissimilar substances—when they are in contact. This e.m.f. will force current through a properly arranged circuit and thereby energy will be generated.

328. E.m.f. of Contact Between Metals and Liquids.—It can be demonstrated experimentally that there is always a difference of potential or an e.m.f. between metals and liquids in which they may be immersed. Even if two pieces of dissimilar metals have their lower ends immersed in water, it can be shown with delicate instruments that there exists between them a difference of potential. Obviously this follows from the information of Art. 313 (e.m.fs. of contact of metals in air) because the immersed metals are really in contact through the water. But experience shows that if the lower ends of two dissimilar metals (Fig. 171,*II*) are immersed in some chemical solution that combines actively with or attacks one of the metals, the difference of potential produced will be materially greater. Contact e.m.fs. between metals and the liquids that attack them are much greater

than the contact e.m.fs. between dissimilar metals or between metals and liquids which do not act chemically on the metals. Hence, practically all of the e.m.f. developed by a voltaic cell

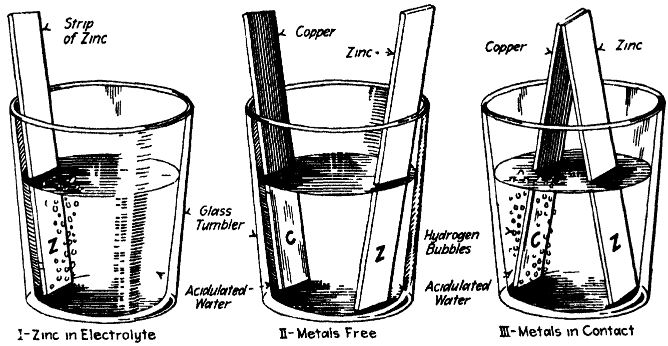


FIG. 171.—The elements of the voltaic or primary cell.

is generated at the area of contact between the electrolyte (Art. 335) and the positive electrode or anode (Art. 334).

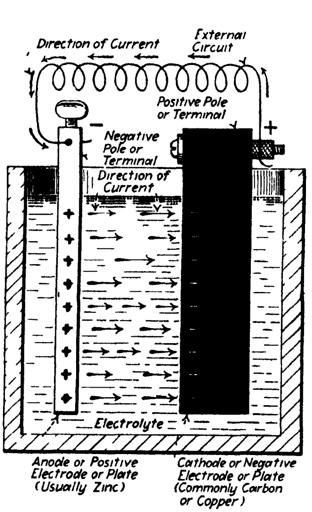


FIG. 172.—Names of components of simple voltaic cell.

329. Values of Contact E.m.fs. of Metals Immersed in Liquids.—Whenever a piece of some pure metal is immersed in an acid or alkaline solution of some certain chemical of a given strength and at a given temperature, a contact e.m.f. of a certain definite value will be developed. With this particular metal and solution and under these given conditions, the contact e.m.f. will be always the same. However, it changes with the kind of solution, the strength of the solution, the temperature of the solution and the temperature of the metal. Hence, it is impracticable to tabulate here the contact e.m.fs. between metals and liquids. Tabulations for certain special cases, similar to that of Table 346, may be found in most of the more complete electrical engineering handbooks.

330. A Primary Cell, sometimes called a *voltaic cell* (Fig. 172) comprises a combination of two different conducting materials

immersed in a liquid or electrolyte (Art. 335) which acts chemically on one of the materials more readily than it does on the other material. (The term *"voltaic cell"* should be used only to designate a cell using zinc, copper and sulphuric acid as shown in Table 358.) The materials must not touch one another in the electrolyte. Through the chemical action of the electrolyte on one of the materials or electrodes (Art. 334) an e.m.f. is maintained and an electric current can thereby be forced through an external circuit. The anode (zinc) and the electrolyte are consumed or used up in producing current. Usually, they can be replaced when they are exhausted—and then the cell is as good as new.

331. A Secondary Cell is a storage battery or accumulator as described at some length, commencing with Art. 367, in a different section of this book.

332. The Real Distinction Between a Primary Cell and a Secondary Cell is: *In a primary cell,* the anode (zinc) and the electrolyte are of such materials that electrolyte will act chemically on the anode when the external circuit is closed and thereby a current is forced through the external circuit. *In a secondary cell,* the electrodes and electrolyte are of such materials that there can be no chemical action between them until after current has first been forced through them.

333. The Distinction Between a Cell and a Battery is that the word "cell" denotes one unit or combination of materials for transforming chemical energy into electrical energy. A "battery" is a combination of cells.

EXAMPLE.—Two or more cells, so connected that they act in conjunction, constitute a battery. Broadly, a battery is any apparatus in which similar units are assembled to serve a common end; for example: "A battery of boilers," "a battery of artillery." Some writers use the words "battery" and "cell" interchangeably, but the practice is an incorrect one.

334. The Names of the Components of a Primary Cell are shown in Fig. 172. The terminal or pole from which the current flows through the external circuit, and therefore the one of higher potential, is called the *positive pole* or *terminal.* The other pole, that of the lower potential, is the *negative pole* or *terminal.* The plate from which the current flows through the external circuit is called the *negative plate, negative electrode* or *cathode,* because the current flows to it through the electrolyte. The other plate,

that of which the negative pole is a part, is the *positive plate, positive electrode* or *anode.* Note that current flows from the positive pole to the negative pole through the external circuit and from the positive plate (by virtue of the movement of ions, Art. 36A) to the negative plate through the electrolyte.

For Example.—In a zinc-carbon cell, the carbon is the positive pole and the negative plate but the zinc is the negative pole and the positive plate.

Note.*—The *cathode* is the name given to the negative or "leading-out" electrode. The "leading-in" electrode is called the *anode.* Some people find it difficult to remember which electrode is the anode and which the cathode. But if one thinks of the *electric current* (Art. 36C) entering and leaving, the words come in their alphabetical order, "a" before "c"—anode leading in, cathode leading out.

335. The Electrolyte in a Primary Cell is the exciting liquid. (The term electrolyte has another specific meaning as outlined in Art. 382.) This liquid, ordinarily an acid or alkaline solution, acts chemically on one of the metal plates of the cell—usually the zinc—and consumes it. It thereby maintains an e.m.f. and thus converts chemical energy into electrical energy.

336. The Function of a Primary Cell (sometimes called a battery) is to convert chemical energy directly into electrical energy. This it does by maintaining an e.m.f. across its terminals. This e.m.f. forces a current through the external circuit and thereby electrical energy is developed. In creating this electrical energy one of the metals of the battery is consumed, usually the zinc. A cell may, then, be considered as a sort of chemical furnace for generating electrical energy.

337. The Functions of the Components of a Primary Cell.—The complete cell constitutes a chemical furnace in which chemical energy is transformed into electrical energy (see Fig. 172). The anode (Art. 334), usually zinc, is the fuel which is consumed by the action of the electrolyte (Art. 335) in maintaining the e.m.f. which drives the current through the external circuit. The electrolyte promotes and makes possible the chemical action, that is, the "burning up" of the zinc. The cathode (Art. 334), often copper or carbon, usually acts merely as a collector for the ions and in most cases is in no way involved in the chemical action in the cell.

338. The Seat of Energy Development in a Cell is, in most cases, at the surface of the anode (Art. 334) or zinc, where the

* Charles R. Gibson, Scientific Ideas of Today, *Seeley, Service & Co*, *London.*

electrolyte combines chemically with the metal of the anode. It is here that the chemical action maintains the e.m.f.

339. The Symbol for a Cell is shown in Fig. 173. A long thin line represents the negative plate and positive pole while a short thick line represents the positive plate and negative pole. Fig. 172 and the accompanying text explain the names of the different parts of a primary cell.

340. What Determines the Voltage of a Cell.—Since the voltage of a cell depends on the contact e.m.f. between the solution or electrolyte (Art. 328) and the metals immersed in the electrolyte, for a given combination of metals and electrolyte the e.m.f. or voltage is always the same. Table 358 shows the voltages developed by the more important combinations of metals and electrolytes. The size of a cell, the volume or extent of surface of the metal plates or their distance apart have no effect one way or the other on the voltage of a cell. But these things do affect the internal resistance (Art. 351) of a cell, hence they determine in a measure the current that the cell can drive through a circuit. Hence, these things also determine, to an extent, the amount of effective energy that a cell is capable of developing. *The materials of the plates and the character of the electrolyte, and these things only, determine the voltage of a cell.*

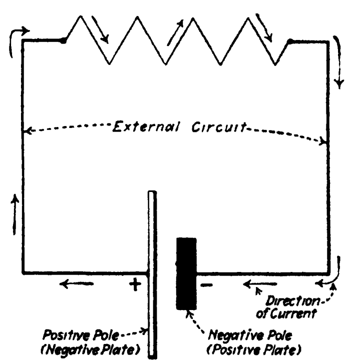


FIG. 173.—The symbol for a cell.

341. Maintenance of Difference of Potential or E.m.f. Between the Plates of a Cell.—There is normally a difference of potential between the plates or electrodes of a primary cell, which tends to force current through the external circuit if one is provided. However, if current is to be continuously forced through the external circuit, this difference of potential or e.m.f. must be maintained and the anode or zinc plate of the cell will be "burned up" in maintaining it as suggested in 337. What a primary cell does then is to carry the ions across between the two electrodes in the cell and to force electrons or particles of electricity to move around through the external circuit.

342. Chemical Action in Primary Cells.—It is beyond the scope of this book to discuss in detail the chemical reactions that

occur in the cells of the various types. As a general proposition, it may be stated that whenever there is chemical action there is electrical activity. This statement applies to all other chemical actions as well as to those in electric primary cells. In a perfect primary cell there would be no chemical action until the external circuit were completed, that is, until current flowed. However, commercial cells are not perfect, hence there is usually some chemical action in them even when the circuit is not completed. See 347, "Local Action."

NOTE.—When the external circuit of a cell is completed, the e.m.f. due to the contact of the electrolyte with the anode (334) forces a current around a path comprising the electrolyte inside of the cell, the electrodes and the external circuit (Fig. 172). This current in passing through the electrolyte, which always contains hydrogen, decomposes the electrolyte, and liberates hydrogen gas. Other components liberated from the decomposed electrolyte unite with the zinc to form new chemical compounds. Thus the zinc is "eaten up" or consumed. Chemical energy is transformed into electrical energy.

343. Typical Operation of a Primary Cell can be illustrated by the readily constructed apparatus of Fig. 171. This constitutes the simplest primary cell. However, the essential components are present: One of the electrodes is a piece of copper, the other a piece of zinc and the electrolyte is diluted sulphuric acid which acts more readily on the zinc than it does on the copper.

EXAMPLE.—Cut a strip of sheet zinc (Fig. 171,*II*) about 4 in. long and 1½ in. wide and a strip of sheet copper of the same dimensions. Scour the zinc with emery until it is bright. Place the strips in a glass tumbler two-thirds full of dilute sulphuric acid (two tablespoonfuls of sulphuric acid in a tumbler two-thirds full of water). On touching the strips together, which completes the external circuit, a shower of bubbles will rise from the copper strip. These bubbles are hydrogen gas. If the strips are not allowed to touch or if one of them is removed, the chemical action greatly diminishes.

If the zinc is now amalgamated (Art. 348) by rubbing some mercury on it, no hydrogen will be given off from its surface. However, if the two strips be touched together or if their upper ends are connected together with a conductor so as to form an external circuit, hydrogen will again be freely discharged from the copper. If the connection between strips is made with a non-conductor the action will cease. If the action be allowed to continue for some time, the zinc will waste away but the copper will not be altered. With an ammeter it can be shown that a current is forced through the external circuit by the e.m.f. maintained by the chemical action.

NOTE.—THE ELECTRON-THEORY EXPLANATION OF THE ACTION OF THE PRIMARY CELL.—When an electric current flows through a conductor,

there is merely a movement, in one direction, of the electrons between atoms, as is explained under Art. 104A. But, entirely different phenomena occur in the electrolyte of the primary cell when two metals (one of which is acted on chemically by the electrolyte) are immersed in it.

Consider as an example a cell comprising a strip of copper and a strip of zinc, having their lower ends dipping into a sulphuric-acid solution:— Now, sulphuric acid when in solution (that is when in water) has the property of breaking up into ions, Art. 36A. The chemical symbol of sulphuric acid is H_2SO_4. That is, a molecule of sulphuric acid contains two atoms of hydrogen, H_2, one atom of sulphur, S, and four atoms of oxygen, O_4. Then, when in solution, each molecule of acid breaks up into positive ions, H, and negative ions, SO_4. Furthermore, the zinc strip when submerged in the acid solution which acts upon it, tends at its surface to break up into zinc ions and electrons. Each zinc atom, thus disassociated at the surface of the zinc, breaks up into a positive zinc ion and a negative electron. Now the positive zinc ion is attracted to and combines with the negative, SO_4, ion of the electrolyte and forms neutral—neither positive nor negative—zinc sulphate, $ZNSO_4$. This zinc sulphate dissolves in the water of the cell and is of no further consequence or value in the operation of the cell.

The free electrons thus left by the zinc atoms (when they broke up into zinc ions and negative electrons, the negative zinc ions combining with the positive, $ZNSO_4$ ions) accumulate on the zinc plate. Then because of the great tendency of electrons to repel one another, Art. 4, these free electrons move up through the zinc plate and around the external conducting circuit to the copper strip, which is the other electrode of the cell. But note also that the positive ions, H, of the electrolyte are attracted through the solution to the copper strip. Here each positive, H, ion combines with a negative electron which has been transferred over from the zinc. The combining of an electron and a hydrogen ion constitutes an atom of hydrogen. Then these hydrogen atoms form into bubbles of hydrogen, rise to the surface of the electrolyte and dissipate into the atmosphere.

It is evident then that, in the electrolyte of the cell, there is a movement of hydrogen positive ions toward the copper plate and also a movement of negative, SO_4, ions toward the zinc plate. Hence, although there is an electron current in only one direction, through the external circuit, there are really two currents in the electrolyte. One is a "negative"—ion current toward the zinc plate and the other a "positive"—ion current toward the copper plate.

By the above-outlined process the zinc plate and the electrolyte are consumed as the cell "generates" energy. The rate of consumption of the zinc and that of the solution are obviously proportional to the energy delivered.

344. The Two Essential Laws of Chemical Action in a Primary Cell Are (these laws were proposed by Faraday):

I. The amount of chemical action, that is, the amount of metal displaced, in a cell is proportional to the quantity of electricity that passes through it. Quantity of electricity is represented by "coulombs" (Art. 122). Now, *amperes* $\times$ *seconds* = *coulombs.*

Hence the amount of zinc consumed in a cell is proportional to the current flowing multiplied by the time. This means that the greater the current that flows through a cell and the longer the current flows, the greater will be the consumption of the zinc. See Art. 345 for examples.

II. When a number of cells are connected in series to comprise a battery, the amount of chemical action is the same in each cell. This follows from the preceding law because the current in each unit of a series-connected group is the same. If the current through each cell is the same the chemical action, from *I*, in each must be the same. If 0.1 oz. of zinc is consumed in an hour by one cell of a series-connected group there will also be 0.1 oz. of zinc consumed by every other cell in that group. This law assumes that there is no local action (Art. 347).

345. Electrochemical Equivalents.—From the laws of Art. 344 it is evident that 1 amp. flowing for 1 hr., from a given metal and into an electrolyte in which the metal is immersed will displace (eat away) a definite amount of that metal. The specific quantity of a substance which is thus consumed per ampere-hour is the *electrochemical equivalent* of that substance. The character of the electrolyte does not affect the amount of metal displaced per ampere-hour. In some cells the metal that is carried away from the anode by the chemical action is deposited on the cathode. In other cells the metal thus carried away is dissolved in or, speaking more properly, combines chemically with, the electrolyte and remains in solution. If a current be driven through a voltaic cell in a direction opposite to that in which the cell normally forces current, the electrolytic process will be reversed and metal will be recovered from the electrolyte and deposited on the anode. The following numerical examples illustrate these principles of electrochemical action:

EXAMPLE.—If a current of 1 amp. flows through a cell for 1 hr., zinc weighing 0.0027 lb. (from Table 346) will be dissolved in the acid and 0.00008 lb. of hydrogen will be liberated.

EXAMPLE.—If a large battery of cells (any type of cell using zinc as an anode) delivers a current of 20 amp. for a period of 10 hr., how much zinc will be consumed in the process? SOLUTION.—The ampere-hours will be: $20 \times 10 = 200$ amp.-hr. From Table 346, 0.0027 lb. of zinc is dissolved per amp.-hr. Therefore, for 200 amp.-hr., the consumption of zinc would be: $200 \times 0.0027 = 0.54$ lb. That is, about ½ lb. of zinc would be consumed.

346. Electrochemical Equivalents of Chemical Elements. (Foster's Electrical Engineers' Pocket Book)

Element	Symbol	Electrochemical equivalents	
		Pounds per ampere-hour	Ampere-hours per pound
Aluminum	Al	0.000,743	1,346.0
Antimony	Sb	0.003,299	303.1
Carbon	C	0.000,246	4,064.5
Copper, cupric	Cu	0.002,614	382.6
Copper, cuprous	Cu	0.005,228	191.3
Gold	Au	0.005,404	185.1
Hydrogen gas	H	0.000,083	12,063.6
Iron, ferric	Fe	0.001,535	651.5
Iron, ferrous	Fe	0.002,302	434.4
Lead	Pb	0.008,506	117.6
Mercury, mercuric	Hg	0.008,222	121.6
Mercury, mercurous	Hg	0.016,444	60.8
Nickel	Ni	0.002,413	414.4
Nitrogen gas	N	0.000,384	2,603.8
Oxygen gas	O	0.000,658	1,520.1
Platinum, platinic	Pt	0.004,006	249.7
Platinum, platinous	Pt	0.008,012	124.8
Silver	Ag	0.008,873	112.7
Tin, stannic	Sn	0.002,446	408.8
Tin, stannous	Sn	0.004,892	204.4
Zinc	Zn	0.002,688	372.0

347. Local Action is the electrochemical action which occurs in primary cells—usually on the anode or positive plate—that contributes nothing to the current in the external circuit. All commercial zinc contains particles of carbon, iron and various other metallic impurities. When a piece of commercial zinc is immersed in an acid solution, these impurities at the surface of the zinc in combination with the zinc itself constitute small primary cells. For example, consider the particle (magnified many times) of carbon, *a*, Fig. 174, imbedded in the surface of the zinc. The acid in combination with the zinc maintains an e.m.f. which forces a local current around a "short-circuit:" (1) from the surface of the zinc, (2) through the electrolyte, (3)

through the carbon and (4) through the zinc back again to the surface of the zinc.

NOTE.—This local action and these local currents will, unless corrected, divert materially from the legitimate current output of the cell. Furthermore, they cause a great waste of chemicals and of zinc. Local action continues whether the external circuit is open or closed. With homogeneous, chemically pure zinc there can not be local action. Alternate hard and soft spots in zinc can cause local action. If the zinc is amalgamated, 348, local action is almost entirely prevented.

EXAMPLE.—If a strip of ordinary commercial zinc be placed in an acid solution (two tablespoonsful of sulphuric acid in a tumbler two-thirds full of water) as at *I*, Fig. 171, a continuous series of bubbles will collect on the metal and break away from it, rising to the surface. These are bubbles of hydrogen gas. The zinc will soon become black because of the minute particles of carbon and other impurities exposed to view by the rapid wasting away of the zinc. If the action is allowed to continue the zinc will, finally, be entirely dissolved. This wasting away is due to local action.

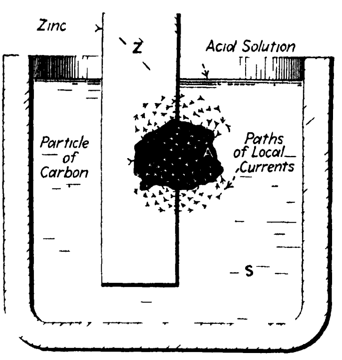


FIG. 174.—Illustrating local action.

348. Amalgamation, which is the coating of a metal with mercury, minimizes or eliminates local action, (Art. 347). When mercury is applied to a clean metallic surface, it dissolves a portion of the metal of that surface and forms a semi-liquid alloy. For example, when applied to commercial zinc, the mercury brings pure zinc to the surface and coats the particles of foreign matter. Furthermore, and probably most important, it forms a smooth surface so that a thin film of hydrogen gas clings to it. It is thus protected from chemical action except when the external circuit is closed. Amalgamated zinc behaves electrically as if it were pure zinc. Amalgamation of pure zinc is not necessary or desirable (see "Local Action," 347). In most commercial battery zincs, the mercury is melted in with the zinc before it is cast. About 1 part mercury to 16 parts zinc, by weight, is a proportion that has been used. Zinc in which mercury is thus combined is very brittle.

EXAMPLE.—Withdraw the zinc strip of *I*, Fig. 171, from the acid solution and while it is still wet with the acid (which now constitutes a cleanser) rub some mercury over its surface. The mercury will spread over the zinc

and cause it to assume a clean, smooth silvery appearance. The zinc has thus been amalgamated. If now the strip is replaced in the acid in the tumbler, no bubbles will rise: amalgamation has eliminated local action.

349. Polarization in a primary cell is the collection of a gas—hydrogen in practically every case—on the surface of the cathode or negative electrode, which is usually of carbon or copper. Note that *local action*, Art. 347, occurs on the anode, while *polarization* occurs on the cathode. The hydrogen evolved from the electrolyte by the chemical action in a primary cell is carried across with the current from the anode (zinc) and tends to collect on the cathode (carbon or copper). This is polarization. Hydrogen gas has a very high resistance, hence if permitted to form a film on the cathode, the internal resistance of the cell is very materially increased. Since the e.m.f. of any given cell is constant, Art. 340, this increase in internal resistance decreases the possible current output of the cell.

NOTE.—Furthermore, polarization tends to decrease the e.m.f. of a cell because there is a reverse e.m.f. developed by the contact of the hydrogen with copper or carbon. See Art. 352 for method of measuring the effect of polarization. This counter e m.f., though small as compared with that developed by the contact of the zinc with the electrolyte, opposes the zinc-electrolyte e.m.f. If uncorrected, polarization would render most cells useless for many practical purposes. However, it can be corrected with depolarizers as will be shown.

EXAMPLE.—If a strip of amalgamated zinc, Art. 348, is immersed (*II*, Fig. 171) in a tumbler of dilute sulphuric acid solution in which there is also a strip of copper of the same dimensions, there will be no action. If now the upper extremities of the strips be placed in electrical contact as at *III*, a circuit is completed and the e.m.f. developed by the contact of the zinc with the acid solution or electrolyte drives a current through this circuit. Hydrogen bubbles will now be deposited on the copper plate or cathode. Some of the bubbles rise to the surface of the electrolyte but many remain on the cathode and polarize it.

If the upper ends of the plates be connected through an ammeter so as to form an external circuit, the readings of this instrument will show that a considerably greater current flows in the external circuit when the circuit is first closed than after the cathode has had time to become polarized by the hydrogen bubbles. If the bubbles are brushed from the cathode with a feather or a stick of wood, the current through the external circuit will immediately become greater as indicated by the ammeter readings. However, the current will again decrease if the cathode is allowed to become polarized.

350. A Depolarizer is a medium whereby an accumulation of hydrogen bubbles on the cathode of a primary cell is prevented. The methods of depolarization may be divided into three classes:

(1) *Mechanical,* (2) *Chemical* and (3) *Electrochemical.* Descriptions of cells employing these different methods are given in Table 358 and in the following articles.

IN MECHANICAL DEPOLARIZATION, the hydrogen bubbles are brushed or forced from the cathode. This may be effected by a current of air that agitates the electrolyte or by a constant movement of the cathode in the electrolyte. If the cathode has a roughened surface, as in the Smee cell, where it is coated with granular platinum, depolarization is partially effected. The bubbles can not cling as readily to a rough surface as to a smooth one. Mechanical depolarization is expensive, is not generally used and is not very effective nor successful.

IN CHEMICAL DEPOLARIZATION, some chemical substance for which the hydrogen has an affinity and with which it will readily combine, is provided on or near the cathode. Then, the hydrogen that is liberated unites with this substance to form a new chemical compound and is thus prevented from collecting on the cathode. Bichromate of potash, chlorine and nitric acid are substances with which hydrogen readily combines, hence they are frequently used as depolarizers. Chemical depolarization is effective, economical, and much used.

IN ELECTROCHEMICAL DEPOLARIZATION, the electrodes and electrolyte are of such materials that the chemical action within the cell liberates some metal, usually copper, at the cathode instead of hydrogen. Thus the formation of hydrogen at the cathode is entirely prevented and polarization is an impossibility.

351. The Internal Resistance of a Cell depends on the area of the plates that is exposed to the electrolyte, the distance between the plates and the temperature and—strength or density of the electrolyte. All cells have some internal resistance. The internal resistance of a cell is actually the resistance of the volume of the electrolyte or other material through which the current flows in passing from one plate to the other. Obviously, this internal resistance is an exceedingly variable quantity. It may vary considerably in two cells of exactly the same size and construction and which appear to be precisely alike. The internal-resistance values given in Table 358 are averages for commercial cells of the construction and sizes in ordinary use.

352. It Is Difficult to Calculate the Internal Resistance of a Cell, but it can be determined experimentally as indicated in Art. 352. Polarization (Art. 349), although it is a phenomenon entirely different and distinct from internal resistance, tends to decrease the current output of a cell in much the same way as does internal resistance. For a given cell, internal resistance is reasonably constant while polarization may vary considerably.

353. Determination of the Internal Resistance of a Cell.—The e.m.f. of a cell on open circuit is always greater than the effective e.m.f. when the cell is on closed circuit, that is, when current is flowing through it, as shown in *I* and *II*, Fig. 175. The drop in voltage is due to two causes: (1) *Internal-resistance drop*, and (2) *polarization* (Art. 349).

354. Internal-resistance Drop is the loss or drop in voltage due to the current flowing through the internal resistance of the cell. There is always a drop in voltage when current flows through a resistance. Polarization, as explained in Art. 349, is a combined resistance and opposing e.m.f. effect. Both of these effects combine to form a counter e.m.f. effect which can be measured in volts and which combine to decrease the effective voltage of the cell when current is flowing through it. These facts are utilized in determining internal resistance, as will be shown.

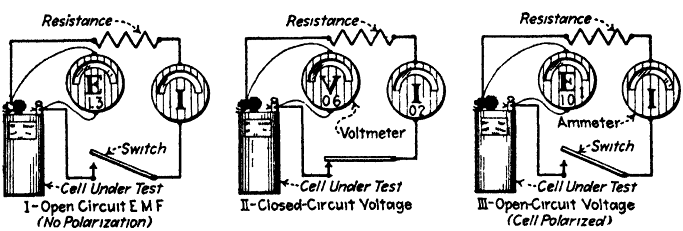


FIG. 175.—Determination of the internal resistance of a cell.

355. Determination of the Apparent Internal Resistance of a Cell.—With this method, the combined effect of internal resistance and polarization is termed the *apparent internal resistance* of the cell. Suppose, as at Fig. 175,*I*, a voltmeter is connected across a cell on open circuit and that it reads 1.3 volts. Now suppose the cell to be connected to a closed circuit as at *II*. The ammeter now indicates 0.2 amp. and the voltmeter 0.6 volt. There has been a drop of: 1.3 *volts* − 0.6 *volt* = 0.7 *volt* due to polarization and internal resistance combined. Now, since we know the current in the circuit, we can by applying Ohm's law (Art. 134) determine the resistance that would cause this 0.7 volt drop. Thus: $R = E \div I = 0.7$ *volt* $\div$ 0.2 *amp.* = 3.5 *ohms*. This 3.5 ohms is the apparent internal resistance of the cell. These operations can be expressed in a formula thus:

$$R_P = \frac{E - V}{I} \qquad \text{(ohms)} \tag{93}$$

Wherein R_P = the apparent internal resistance of the cell in ohms. E = the e.m.f. of the cell, unpolarized and on open circuit, in volts. V = the voltage reading across the cell when the current I flows as shown in Fig. 175,*II*.

EXAMPLE.—What is the apparent internal resistance of the cell of Fig. 175? Its e.m.f. on open circuit, as at *I*, is 1.3 volts; the voltage reading with a current of 0.2 amp. flowing is, as shown at *II*, 0.6 volt. SOLUTION.—Substitute in the formula (93): $R_P = (E - V) \div I = (1.3 - 0.6) \div 0.2 = 0.7 \div 0.2 = 3.5$ *ohms.*

356. Determination of the Actual Internal Resistance of a Cell.—The method of Art. 355 does not give the *actual* internal resistance of the cell because the cell was not polarized when the reading of *I* (Fig. 175) was taken but it was polarized when that of *II* was read. If now, however, after the cell has been on closed circuit and is polarized, the voltage reading *II* be taken and then the circuit be opened and an open circuit reading taken quickly as at *III* (before depolarization can take place) the error due to polarization will be eliminated. The reason for its elimination is that the counter e.m.f. due to polarization will be effective in the readings of *II* and also in those of *III* and in both cases it will decrease the effective voltage of the cell by the same amount. This method and the formula below give the actual internal resistance of a cell, thus:

$$(94) \qquad R_1 = \frac{E_I - V}{I} \qquad \text{(ohms)}$$

Wherein R_A = actual internal resistance of the cell in ohms. E_I = the voltage across the cell measured, as shown at *III*, immediately after the circuit has been opened and before depolarization can take place. V = the voltage reading across the cell when current I, in amperes, flows as shown at *II*.

EXAMPLE.—What is the actual internal resistance of the cell shown in Fig. 175? The voltmeter indicates 0.7 volt across it when a current of 0.2 amp. flows, as at *II*, and the voltmeter indicates 1.0 volt on open circuit as at *III* while the cell is still polarized. SOLUTION.—Substitute in the formula (94): $R_A = (E_I - V) \div I = (1.0 - 0.6) \div 0.2 = 0.4 \div 0.2 = 2$ *ohms.*

NOTE.—The following data relates to the cell and circuit shown in Fig. 175? E.m.f. = 1.3 volts. Internal resistance = 2 ohms. Counter e.m.f. of polarization = 0.3 volt; this counter e.m.f. is effective only when the cell is polarized, at which time the effective e.m.f. of the cell is: 1.3 *volts* − 0.3 *volt* = 1.0 *volt*, as shown at *III*. The resistance of the external circuit is 3 ohms.

SECTION 18

TYPES AND CONNECTIONS OF PRIMARY CELLS

357. Different Types of Primary Cells are almost numberless. Many have been invented, lived their day and are now forgotten. Table 358 lists the most important, either historically or commercially, and shows their characteristics. Art. 372 suggests the suitable applications for cells of different types. Descriptions

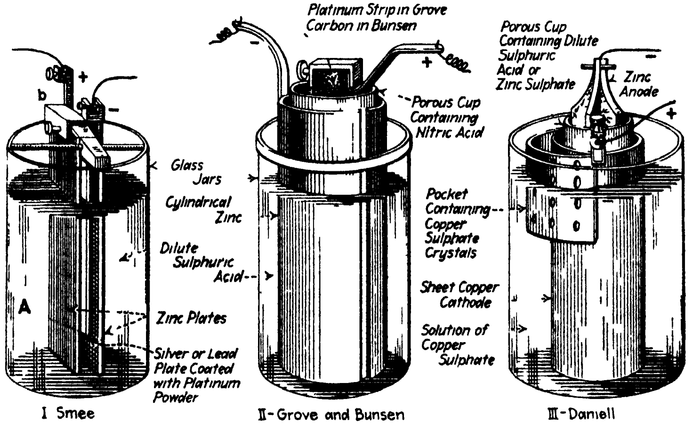


FIG. 176—Older types of cells

of some of the cells and directions for their setting up and maintenance are given in following articles.

358. The Smee Cell (Fig. 176,*I*), so-named from its originator, comprises a plate of silver or lead suspended between two zinc plates. All dip into a glass jar containing dilute sulphuric acid. As a remedy for polarization, the silver or lead plate is coated with finely divided or granulated platinum. The remedy is not very effective. Smee cells, though simple and rugged, are seldom used now because better cells are available.

359. Characteristics of Primary Cells.

Name	Anode or positive electrode	Electrolyte or excitant, solution of	Separator	Depolarizer
Cells employing mechanical depolarization				
Volta	Zinc	Sulphuric acid	None	None
Wollaston	Zinc	Sulphuric acid	None	None
Smee	Zinc	Sulphuric acid	None	None
Law	Zinc	Sal-ammoniac	None	None
Hercules	Zinc	Sal-ammoniac	None	None
Carbon cylinder	Zinc	Sal-ammoniac	None	None
Cells employing chemical depolarization				
Fuller	Zinc	Sulphuric acid or common salt	Porous cup	Bichromate
Grenet	Zinc	Sulphuric acid	None	Bichromate
Poggendorf	Zinc	Sulphuric acid	None	Bichromate
Bichromate	Zinc	Sulphuric acid	None	Bichromate
Grove	Zinc	Sulphuric acid	Porous cup	Nitric acid
Bunsen	Zinc	Sulphuric acid	Porous cup	Nitric acid
Leclanche	Zinc	Sal-ammoniac	Porous cup or bag	Manganese dioxide
Lelande	Zinc	Caustic potash	None	Cupric oxide
Edison-Lelande	Zinc	Caustic potash	None	Cuprous oxide
Sampson	Zinc	Sal-ammoniac	Carbon cylinder.	Carbon and manganese
Fitch	Zinc	Sal-ammoniac		
Papst	Iron	Iron chloride	....	Iron chloride
Dry cell	Zinc	Sal-ammoniac and zinc chloride	None	Carbon and manganese peroxide
Cells employing electrochemical depolarization				
Daniell	Zinc	Zinc sulphate	Porous cup	Copper sulphate
Gravity or crowfoot	Zinc	Zinc sulphate	None	Copper sulphate
Medinger	Zinc	Zinc sulphate	....	Copper sulphate
Minotto	Zinc	Zinc chloride	....	Copper sulphate
De la Rue	Zinc	Zinc chloride	....	Silver chloride
Marié Davy	Zinc	Zinc sulphate	....	Mercurous sulphate
Latimer Clark (standard)	Zinc	Zinc sulphate	....	Mercurous sulphate
Weston (standard)	Cadmium	Cadmium sulphate.	....	Mercurous sulphate
Von Helmholtz	Zinc	Zinc chloride	....	Mercurous chloride

Cathode or negative electrode	* Open circuit e.m.f., volts	* Internal resistance, ohms	Remarks, services for which cells are fitted
Copper	1.0	1.0 to 0.5	For experimental work
Copper	1.0	1.0 to 0.5	For experimental work
Platinized silver	1.0	1.0 to 0.5	Polarizes rapidly—seldom used now
Carbon	1.3	2.0 to 0.8	For open-circuit work, bells and signals
Carbon	1.3	2.0 to 0.8	For open-circuit work, bells and signals
Carbon	1.3	2 0 to 0.8	Open-circuit work, bells and signals
Carbon	2 0	4 0 to 0.5	For telephone work—open or closed circuit
Carbon	2 0	4.0 to 0.5	For experimental work, open or closed circuit
Carbon	2 0	1 0 to 0 5	For experimental work, open or closed circuit
Carbon	2 0	1.0 to 0.5	For experimental work, open or closed circuit
Platinum	1 9	0.2 to 0.1	Closed circuit—seldom used
Carbon	1 9	0 2 to 0.1	Closed circuit—seldom used
Carbon	1 5	3.0 to 1.0	Open circuit—bells and signals
Carbon or iron	0 8	1.5 to 1 0	Closed circuit—seldom used
Cuprous oxide	1 0	0 9 to 0 2	Closed or open circuit—signal work
Carbon	1 5	0 2 to 0 1	Open circuit—bells and signals
Carbon	1 1		Seldom used
Carbon	0 4		Seldom used
Carbon	1 6	0 5 to 0 1	Open circuit, bells, signals, telephones
Copper	1 1	6 0 to 2 0	Closed circuit—seldom used in U. S
Copper	1 1	2 0 to 0 7	Closed circuit—telegraph and signals
Copper	1 1		Seldom used
Copper	1 1		Seldom used
Silver	1.4		Seldom used
Carbon	1 4		Seldom used
Mercury	1.434	0 5 to 0 3	Laboratory standard
Mercury	1 083		Laboratory standard
Mercury	1 0		Seldom used in U S

* The values in the "e m f." and the "Internal resistance" columns are necessarily approximate for reasons outlined in preceding sections of this book. Internal resistance in particular is subject to considerable variations with cells of the same materials but of different dimensions. The internal resistance values given are typical of those of cells of the dimensions and construction ordinarily used in the United States and are the *apparent internal resistances* (Art 355).

360. The Carbon-cylinder Cell (Fig. 177) has a zinc anode, a carbon cathode and a sal-ammoniac electrolyte. Ordinarily, no depolarizer is employed. Therefore, the cell polarizes even more rapidly than does the Leclanche cell (Art. 364). Its recovery is also more sluggish. It will give fair service for

residence door bells and annunciators or where it will be used infrequently and for short intervals. It polarizes too rapidly for telephone or severe signal work. Its e.m.f. is lower than that of Leclanche cells because it uses no peroxide of manganese. Sometimes manganese is mixed into the carbon cathode composition, providing a depolarizer. This arrangement minimizes polarization to some extent and increases the e.m.f. of the cell.

TO SET UP AND MAINTAIN CARBON-CYLINDER CELLS.—The directions given under "Leclanche Cells" may be followed, substantially as there outlined.

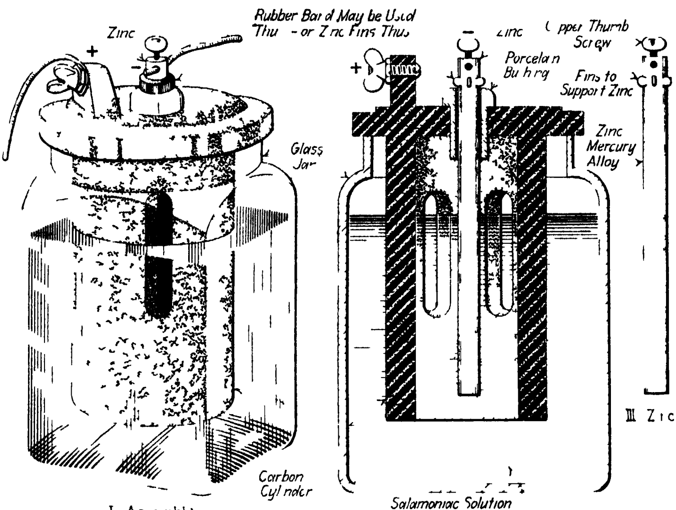


FIG. 177.—Carbon-cylinder (sal-ammoniac) cell.

361. The Fuller Cell (Fig. 178) is really a modification of the bichromate cell, as suggested in Sec. 362. However, in the Fuller cell, the bichromate solution is not mixed with the electrolyte but is separated from it by the porous cup. The result is that the zinc is not consumed appreciably on open circuit. The e.m.f. and chemical action are about the same as those of bichromate cells. The bichromate or depolarizing solution surrounds the carbon cathode in the outer portion of the cell. The existing liquid, which may be a weak solution of either sulphuric acid, ammonium chloride (sal-ammoniac) or sodium chloride (common salt), is placed in the porous cup surrounding the zinc anode.

Fig. 178 shows the type of Fuller cell used in America for telephone work. It can stand on open circuit for several months continuously without appreciable deterioration. The zinc is cast cone-shaped with a copper wire leading from it. Its diameter is made smaller at its top to facilitate uniform consumption of the metal along the entire length of the zinc. The consumption of this cone-shaped zinc is quite uniform over its entire length. Fuller cells with several carbon plates in multiple, or with a cylindrical carbon for cathodes can be purchased; such cells have very low internal resistances.

THE FULLER CELL IS SET UP AS FOLLOWS.—Mix the electrolyte by adding 6 oz. of potassium bichromate and 17 oz. of sulphuric acid to 56 oz. of soft water. Pour this mixture into the glass jar. Into the porous cup put one

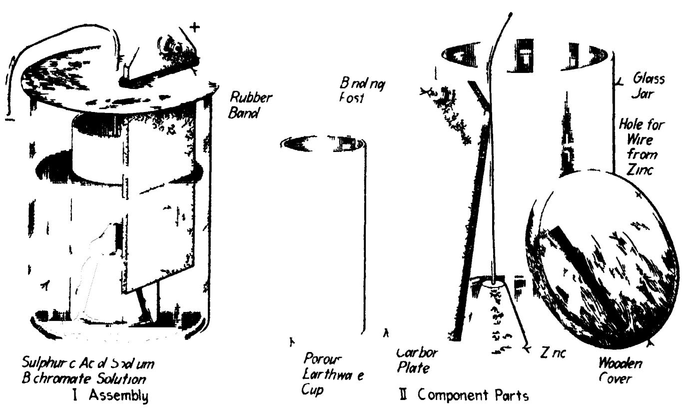


FIG. 178. Fuller cell—telephone type.

teaspoonful of mercury and two teaspoonfuls of common salt (sodium chloride). The mercury provides for the constant amalgamation of the zinc. Place the porous cup, now containing the zinc anode, in the glass jar and fill the porous cup to within 2 in. of its top with soft water. Put on the cover, insert the carbon cathode and the cell is ready for use. When the cell is in proper working order the color of the solution is orange.

362. Bichromate or Chromic Acid Cells. Grenet Cell. Plunge Batteries (Fig. 179).—There are many different forms. The Fuller cell (Art. 361) is really a modified bichromate cell. In these cells a zinc anode is so suspended between two carbon-plate cathodes that it does not touch them. Sulphuric acid and water is the electrolyte and bichromate of potassium is the depolarizer. When hydrogen is liberated, by the action of the

sulphuric acid on the zinc, it combines with the bichromate, which is rich in oxygen, preventing polarization. These cells have a high e.m.f. and low internal resistances, hence are capable of furnishing relatively large currents. However, the electrolyte soon becomes exhausted so the cells are not applicable to many commerical uses. They are very convenient for laboratory work and probably there find their widest application. The *Grenet Cell* (Fig. 179,*II*) has its zinc anode mounted on a rod so that it can be easily removed from the electrolyte when the cell is not in use. A *plunge battery* (Fig. 179,*I*) consists of a series of bichromate cells, so arranged that the electrodes can all be raised from the electrolyte at will by turning a crank. A large plunge battery may develop a considerable voltage and a large current.

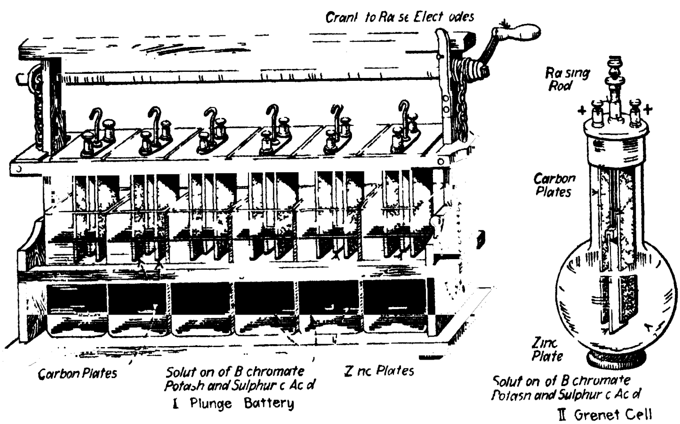


FIG 179 —Bichromate or chromic acid cells.

ELECTROLYTE FOR BICHROMATE CELLS may be made in accordance with any one of a number of different receipts The following, which is sometimes called *Electropoian Solution* will give good results 1 gal of water, 1 lb of bichromate of potash crystals and from ½ to 1 pt of sulphuric acid, according to the energy of action desired A small amount of nitric acid added to the solution increases the constancy of the battery A *Chromic Acid Solution* which is used where an intense current is desired is chromic acid, 10 parts, water, 10 parts, sulphuric acid 4 parts—all by weight.

363. Grove and Bunsen Cells are shown in Fig. 176,*II*. The Grove cell comprises a glass jar containing dilute sulphuric acid in which are immersed: a cylindrical zinc and a porous cup in which is a strip or plate of platinum. The porous cup contains strong nitric acid. The hydrogen, evolved by the action of the

sulphuric acid on the zinc, combines with the nitric acid and water is formed, so there is no free hydrogen. A brownish-red, poisonous and corrosive gas is evolved by this process. The Bunsen cell differs from the Grove only in that it uses a block of carbon instead of a strip of platinum for the cathode. Grove and Bunsen cells are seldom used now.

364. The Leclanche Cell (Figs. 180, 181 and 182) is adapted only to intermittent work such as that of bell, signal and telephone installations. Although a depolarizer is provided, the cells soon polarize on closed circuit. However, they recover quickly and are very satisfactory for the services indicated. In action, the sal-ammoniac (ammonium chloride) electrolyte attacks the zinc anode, forming zinc chloride and liberating

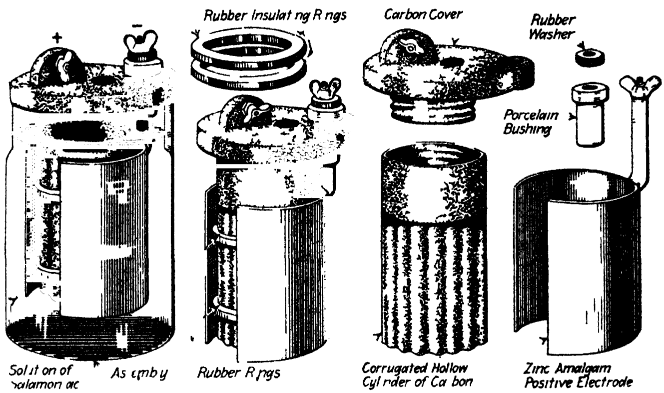


FIG 180 —Carbon porous cup (sal-ammoniac) cell.

hydrogen and ammonia gas on the surface of the carbon cathode. The peroxide of manganese, which is usually in small lumps, is rich in oxygen. This oxygen combines with the liberated hydrogen to form water. Thus polarization is largely avoided.

NOTE —The peroxide of manganese is, however, more than a depolarizer. Because of its contact e m f (Art. 310) with the electrolyte, it increases the voltage of the cell Note from Table 358, that a zinc-carbon-sal-ammoniac cell (the Leclanche) employing peroxide of manganese has a greater e.m f. than a carbon-cylinder cell which uses the same elements but which does not employ the manganese The powdered carbon is mixed with the manganese to give greater conductivity and a greater surface to the carbon electrode. The forms of Leclanche cell most used in America are described below.

365. Different Forms of Leclanche Cells are shown in Figs. 180, 181 and 182. *The porous-cup type* (Fig. 182,*I* and *II*)

was at one time used widely but it has now been superseded largely by the carbon-cup type. The zinc is a rod or pencil. The carbon in the unglazed pottery porous cup is packed around with peroxide of manganese and carbon. *The canvas-bag type* (Fig. 182,*III*) differs from the porous-cup type only in that the manganese and carbon are retained in a bag of heavy cloth instead of in an earthenware porous cup. A rubber ring on the lower end of the pencil zinc prevents its contact with the bag. *The carbon-porous-cup type* (Figs. 180 and 181) uses practically the same materials as the others but the corrugated carbon cylinder itself forms the porous cup. The manganese and powdered carbon are within, in it. The cylindrical zinc offers a large surface.

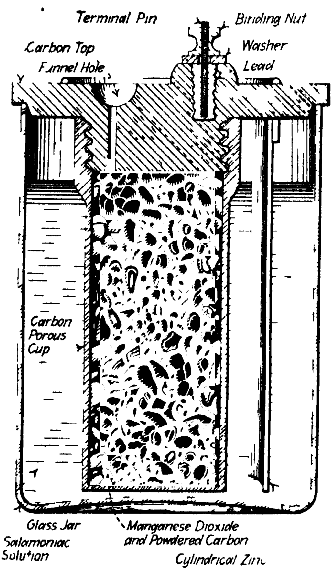


FIG. 181.—Section of a carbon porous-cup cell.

TO SET UP A LECLANCHE CELL. – Place 4 oz.—not more—of white powdered sal-ammoniac in the jar. If too much sal-ammoniac is used, "creeping salts" will give trouble. Fill the jar one-third full of water and stir until the

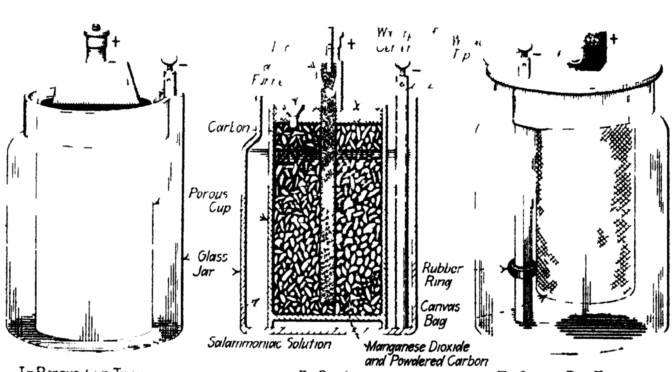


FIG. 182.—Leclanche cells.

sal-ammoniac is entirely dissolved. Put the carbon and zinc electrodes in position. Some water poured in the vent hole or funnel of the porous cup will accelerate the initial action of the cell.

MAINTENANCE OF A LECLANCHE CELL.—Water should be added to supply the loss due to evaporation. Where a cell fails, examine the terminals for poor connections. If the zinc is badly eaten, replace it with a new one. If the cell is not improved by the new zinc, discard the old solution and pour in a new one. If now, the cell does not work properly, the porous cup or carbon cathode should be soaked in warm water to dissolve out any crystalline salts. If this does not afford relief, the entire cathode element should be replaced. It seldom pays to merely disassemble the element and replace the depolarizer.

366. The Edison-Lelande or Edison Cell (Fig. 183) is suitable for either open- or closed-circuit work. The mechanical construction of this cell is particularly good. The cathode is a plate of compressed oxide of copper, the surfaces of which are reduced to metallic copper to improve the conductivity. This form of plate also acts as a depolarizer. The anode is of pure zinc homogeneously amalgamated by adding mercury when the casting is made. The electrolyte is a solution of caustic soda. The top of the solution is covered with a heavy mineral oil to prevent evaporation. Several different types are manufactured, hence for complete information the manufacturer should be consulted.

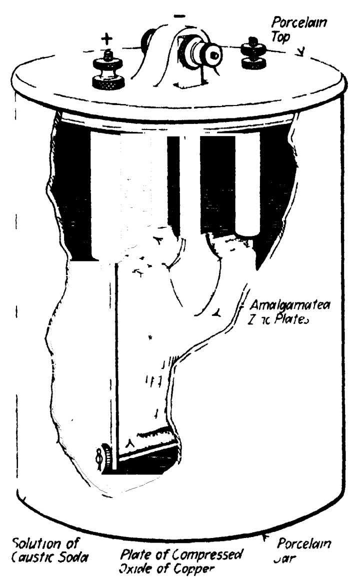


FIG. 183—The Edison-Lelande or Edison cell.

367. Dry Cells (Fig. 184) have in America, for open-circuit work, almost entirely superseded cells of other types. The term "dry cell" is in a sense a misnomer because, although the cells are dry externally, the compositions within them must be moist to insure the propulsion of electric current. In the dry cell, which is merely a modification of the Leclanche cell, only enough water is added to the electrolyte to moisten the blotting-paper, cloth or paste lining that separates the zinc-casing anode from the carbon-and-manganese-dioxide cathode. Nearly all American dry cells are 6 in. high and 2½ in. in diameter. The illustration indicates typical construction. The zinc chloride is in-

troduced to minimize the rapid deterioration that would otherwise occur on open circuit.

Note.—The e.m.f. of all new dry cells is about 1.5 or 1.6 volts. A good dry cell may stand on open circuit for many months with a decrease in e.m.f. of only about 0.1 volt. A good cell should have an open-circuit e.m.f. of at least 1 5 volts. Internal resistance increases with age. Thus, while a good cell has an internal resistance of about 0 1 ohm, this may increase in 10 or 12 months to 0 5 ohm, even if the cell has been only on open circuit. A new cell of good manufacture should give a short-circuit current, through an external resistance of not more than 0.01 ohm, of at least 15 amp. and of not more than 25 amp. A cell giving more than 25 amp. will, probably, polarize rapidly. One giving less than 15 amp. is, likely, made with inferior materials.

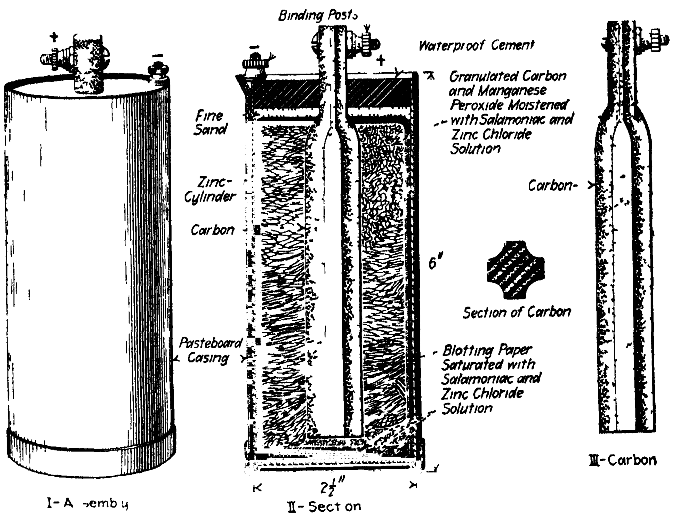


Fig. 184.—Construction of typical American dry cell.

After it has been stored on open circuit for a year, a good new cell should give a short-circuit current of about 10 amp. Cells not in use should be stored in a cool place; this prevents rapid deterioration.

368. The Daniell Cell is shown in one of its numerous forms in Fig. 176,*III*. The glass jar contains a saturated solution of copper sulphate (blue vitriol). The porous cup contains dilute sulphuric acid (about 10 per cent. by volume) or zinc sulphate or both. The anode is amalgamated zinc. The cathode is a cylindrical copper plate. A pocket of sheet copper arranged near the top of the jar contains blue vitriol crystals whereby the saturation of the solution is maintained. In Europe, portable Daniell cells are used wherein a layer of sand or sawdust is util-

ized instead of the porous cup. The internal resistance of these sawdust cells may be as high as 30 ohms, whereas the resistance of a glass-jar cell, 7 in. high, similar to that shown, is about 3 ohms. The zinc and copper sulphate combine to form copper and zinc sulphate. This copper is deposited on the copper cathode of the cell and the zinc sulphate dissolves in the solution surrounding the zinc.

369. The Gravity or Crowfoot Cell, Fig. 185, is merely a modification of the Daniell cell. It is a cheap, simple, reliable cell and is extensively used in America for closed-circuit telegraph, telephone and signal work. The zinc sulphate solution (formed as described under "The Daniell Cell," Art. 368) is lighter than

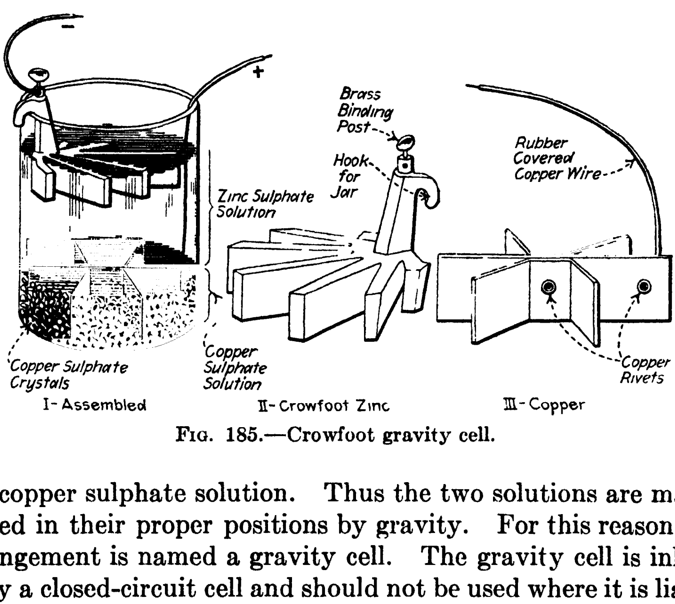


FIG. 185.—Crowfoot gravity cell.

the copper sulphate solution. Thus the two solutions are maintained in their proper positions by gravity. For this reason the arrangement is named a gravity cell. The gravity cell is inherently a closed-circuit cell and should not be used where it is liable to stand on open circuit for considerable periods. The internal resistance of the cell is about 2 or 3 ohms.

IN SETTING UP THE GRAVITY CELL, place the copper cathode in the bottom of the jar and pour in over it about 3 lb. of copper sulphate (blue vitriol) crystals. Now hang the zinc anode on the top of the jar. Fill the jar with water to cover the zinc. To the water add a tablespoonful of sulphuric acid. Cover the electrolyte with a layer of pure mineral oil, which should be free from naphtha or acid and have a flash point above 400 deg. F. This prevents evaporation and creeping. Where oil is not used, creeping can be prevented by dipping the edge of the jar in hot paraffin. After being set up, the cell

should be short-circuited for a day or so, so that zinc sulphate will be formed which will protect the zinc. Such a preliminary run will also reduce the internal resistance.

MAINTAINING THE GRAVITY CELL.—Its temperature should be kept above 70 deg. F., since the internal resistance increases very rapidly with a decrease in temperature. A blue color in the bottom of the cell denotes a good condition, but a brown color shows that the zinc is deteriorating. When renewing the copper sulphate, empty the cell and set it up with an entirely new electrolyte. The blue line, which marks the boundary between the copper sulphate and the zinc sulphate, should stand about halfway between the electrodes. If the blue line is too close to the zinc, some of the copper sulphate can be siphoned out or the cell can be short-circuited. This will produce more zinc sulphate. If the blue line is too low, some water and copper sulphate crystals should be added.

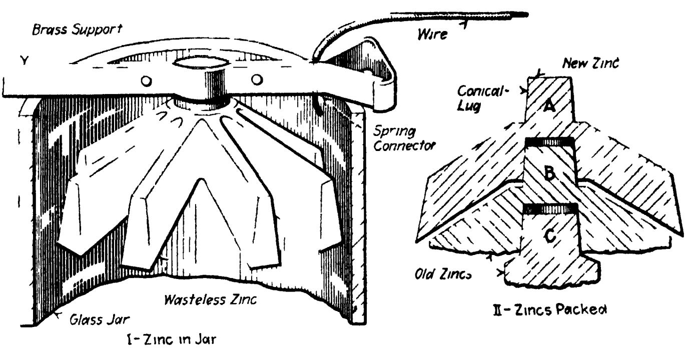


FIG 186 —D'Infreville wasteless zinc.

370. Wasteless Zincs for Gravity Cells (Fig. 186) are used by the telegraph companies and other concerns that employ large batteries.

NOTE.—With a crowfoot zinc like that of Fig. 185,*II*, there is a certain amount of waste. After such a zinc has been in service for some time it is so eaten away that only a stub remains. The surface offered to the electrolyte by this stub is so small that the internal resistance of the cell is greatly increased and it is thereby rendered ineffective. To correct conditions the old zinc stub must be discarded and a new zinc substituted for it. With zincs of the wasteless type shown in Fig. 186, a conical lug *A* cast on the top of each zinc fits into a corresponding cavity also cast in each zinc. Thus, partly consumed zincs are packed together as at *II* and there is no waste. The brass support, shown at *I*, is so arranged that it also serves as a terminal or connector for the zinc anode. The bared connecting wire is clamped between the two spring-brass strips, which affords a good electrical contact.

371. Standard Cells (Fig. 187) are used in laboratory work as standards of e.m f. and are utilized in measurements where a

voltage of known and fixed value is required for comparison or for calibrating instruments. Large currents are not required from standard cells, hence they may and do have high internal resistances—frequently as high as 500 to 1,000 ohms. Sometimes a very high external resistance is connected in series with standard cells to limit the current.

TYPES OF STANDARD CELLS are numerous. *The Daniell standard cell,* made as shown in Fig. 176 was used during the pioneer days of the electrical science. It has an e.m.f. of about 1 volt and is still used as a rough and ready standard. However, it deteriorates rapidly and is otherwise not well adapted for accurate laboratory work. *The Clark standard cell* (Fig. 187,*I*) was adopted as the standard of e.m.f. by the International Electrical Congress of Chicago in 1893. At 15 deg. C. its e.m.f. is 1.434 volts. The e.m.f. decreases as the temperature increases but correction can be made for this by applying certain formulas. *The Weston standard cell* has an e.m.f. of 1 083

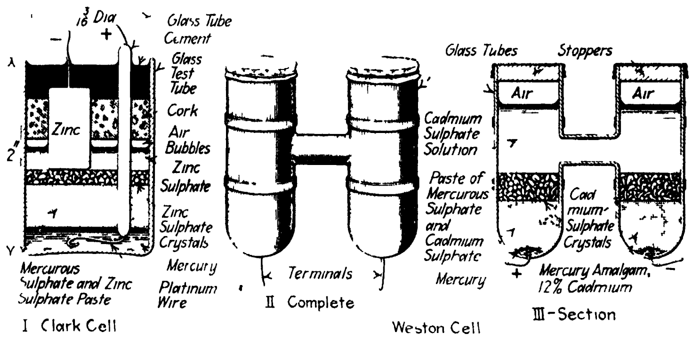


FIG. 187.—Standard cells.

volts. Its e m.f. is constant at all ordinary temperatures and remains constant for years, provided no current greater than 0.0001 amp. is allowed to pass through the cell. Because of its constancy under various conditions, the Weston has largely superseded the Clark cell as a working standard. Other types of standard cells are manufactured.

372. Selection of Cells for Given Services.—As a general proposition, a cell that is suitable for open-circuit work is not fitted for closed-circuit service. However, there are exceptions to this maxim. The following notes give information relating to the proper applications of cells of different types:

CARBON-ZINC-SAL-AMMONIAC OR LECLANCHE CELLS are best suited for light signal work, such as residence, door-bell-and-annunciator applications, where long periods of rest occur between operations. These cells, if well made are also satisfactory for local telephone work. Large cells of the

Leclanche type will provide heavy currents for short periods and require little attention. Hence, they are sometimes conveniently used in experimental work. However, regardless of their size, they soon polarize temporarily, thereby decreasing the current output. After a period of rest they recuperate and are then as powerful as before until polarization again occurs. Carbon-zinc-sal-ammoniac cells with cylindrical zincs are more powerful than those with rod or pencil zincs and hence should be used for important bell and annunciator work. Cells with pencil zincs are satisfactory for residence bell and annunciator work.

COPPER-ZINC-COPPER SULPHATE CELLS can be used for electric bells but are best fitted for service, such as that of closed-circuit telegraph and burglar-alarm installations, where current flows through them continually.

BICHROMATE PLUNGE CELLS, the Grenet cell for example, are suitable for service where large currents are required but where it is convenient to dismantle the cell when it is not in use. Bichromate cells will not operate for long periods without attention as do copper-sulphate cells. The disadvantage of the bichromate cells is that they should be cleaned and disassembled when not in use. In spite of this, they are largely used in experimental work because of the large currents that can be propelled by them until the solution becomes exhausted.

LELANDE CELLS.—Those using copper oxides and caustic potash are suitable for the heaviest closed-circuit work for which chemical cells can be applied, such as operating motors, lamps and induction coils. They are probably better adapted than any of the other cells for supplying large currents at infrequent intervals. They are excellent for signal work but are expensive hence have not been much used for such service except for railway block and fire-alarm signaling.

373. The Application of Primary Cells as Generators of Electrical Energy is, in general, limited to conditions for which a relatively small current is required intermittently. Cells are used widely for ringing bells, operating signals, for telephone and telegraph work and for electrical testing. They are not commercially applicable where large amounts of energy are required because electrical energy generated by chemical action is much more expensive than that developed by dynamos or generators. See Art. 509. Even for telegraph and signal work it is, in some cases, more economical to generate with dynamos than with cells or batteries.

374. The Cost of Electrical Energy Generated with Primary Batteries varies with the type of cell but it is always much greater than the cost of energy produced with dynamos or generators.

EXAMPLES.—If electrical energy is developed with the bichromate cell (Art. 362), which probably produces energy as cheaply as any primary cell, the cost of the materials alone would be about 30 cts. per h.p.-hr. This cost assumes that the materials would be purchased in large quantities. With

dynamos driven by steam engines, a horse power-hour of energy can be developed readily for 4 cts., while under favorable conditions, it can be developed for less than 1 ct. Using a silver-chloride battery, the cost of materials is in the neighborhood of \$140 per h.p.-hr. The cost of the materials per horse power-hour can be estimated for a cell of any type by using the electrochemical equivalents of Table 346.

375. How to Compute the Current Propelled by a Battery or Cell in a Given Circuit.—The voltage developed by the cell will be the e.m.f. that will drive the current through the circuit. This voltage can be ascertained from Table 358 or from a similar table. The resistance of the entire circuit will be the sum of the external-circuit resistance and the internal resistance (Art. 351) of the cell. An average approximate value for this internal resistance can also be obtained from Table 358. Then, to ascertain the current, use Ohm's law (Art. 134), thus: current = *voltage* ÷ (*internal resistance* + *external resistance*).

EXAMPLE.—What current will flow through the electric-bell circuit of Fig. 188,*I*, when the button is pressed? SOLUTION.—A Leclanche cell is

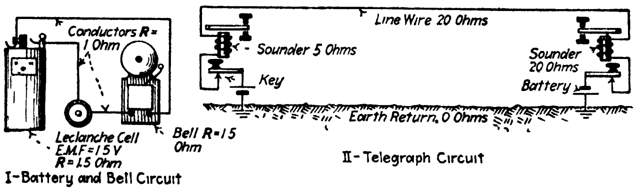


FIG. 188.—Examples of currents impelled by batteries.

used, the e.m.f. of which, from Table 358, is 1.5 volts. The bell has a resistance of 1.5 ohms and the remainder of the external circuit has a resistance of 1 ohm. Hence, the total resistance of the entire circuit is: cell, 1.5 ohms + conductors, 1 ohm + bell, 1.5 ohms = 4 ohms. Then, using the Ohm's law formula (9):

$$I = \frac{E}{R} = \frac{1.5}{4} = 0.4 \ amp.$$

EXAMPLE.—What current will four gravity cells connected in series drive through the telegraph circuit of Fig. 188,*II*. Each sounder has a resistance of 5 ohms and the 1 mile of No. 10 iron line wire has a resistance of 20 ohms. SOLUTION.—The earth return has, for all practical purposes, zero (0) resistance. The keys have no appreciable resistance. Each gravity cell has, from Table 358, an internal resistance of about 4 ohms. Then the resistance of the entire circuit is: 4 batteries @ 4 ohms = 16 ohms + 2 sounders @ 5 ohms = 10 ohms + line wire, 20 ohms = 46 ohms. From Table 358, each gravity cell has an e.m.f. of 1.07 volts. Hence, the combined voltage of the four cells is: $4 \times 1.07 = 4.28$ volts. Now using the Ohm's law formula (9):

$$I = \frac{E}{R} = \frac{4.28}{46} = 0.9 \ amp.$$

376. Methods of Varying the Current Output of a Battery by Changing the Arrangement of Its Cells.—The current through any electric circuit can be increased in two ways: (1) *By decreasing the resistance;* (2) *by increasing the e.m.f.* Where several cells are available, the e.m.f. may be increased by joining the cells in series as in Fig. 189. The internal resistance can be decreased by connecting the cells in parallel as in Fig. 190. Obviously, by

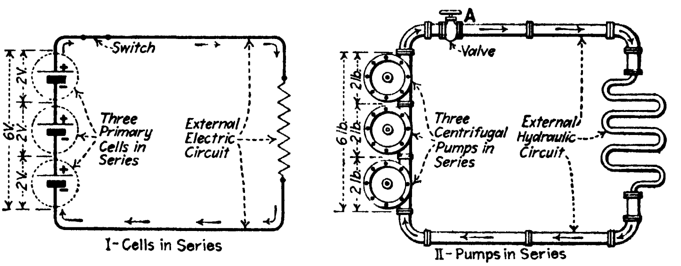


FIG. 189.—Connection of primary cells and centrifugal pumps in series.

changing the connections of a group of cells, the current output of the group can be altered. The different methods of arrangement and their effects on current output will be discussed in following articles.

377. Current Output When Cells Are Connected in Series.—For a series connection, the positive pole of one cell joins the negative pole of its neighbor, the cells being arranged in tandem as shown in Fig. 189,*I*. See Art. 187, "Series Circuits."

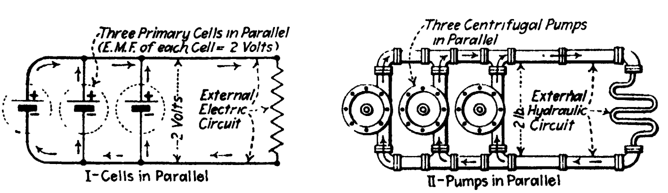


FIG. 190.—Connection of primary cells and centrifugal pumps in parallel or multiple.

A HYDRAULIC ANALOGY TO CELLS CONNECTED IN SERIES is shown in Fig. 189,*II*. Three small centrifugal pumps of the construction of that of Fig. 93,*I* are coupled in series in a hydraulic pipe circuit which is full of water. Since each pump can develop a pressure of 1 lb. per sq. in., the three in series develop a pressure of 3 lb. per sq. in. when the valve *A* is closed to prevent water from flowing. If, however, the valve *A* is opened, thereby allowing the pumps to force water through the hydraulic circuit, there will be a small drop or decrease in the pressure imposed on the circuit by the pumps.

This loss in pressure when water flows is caused by the friction of the water against the interior of the pump. This internal pump friction is analogous to the internal resistance of a primary cell. When the pumps are coupled in series, their internal resistances are added together. The greater the number of pumps coupled in series the greater is the total hydraulic pressure developed but the greater is the internal pump friction.

The e.m.f. of a group or battery of cells connected in series is the sum of the e.m.fs. of the component cells. In Fig. 189,*I*, three cells are joined in series and since each has an e.m.f. of 2 volts, the e.m.f. of the battery is: 2 *volts* × 3 *cells* = 6 *volts*. Where cells are connected in series, the internal resistance of the battery is equal to the sum of the internal resistances of the component cells. Stating these facts in a formula which follows from Ohm's law (Art. 134):

$$I = \frac{S \times E}{R_x + (S \times R_I)} \quad \text{(amp.)} \tag{95}$$

Wherein I = current, in amperes in the external circuit. S = number of cells in series in the battery. E = e.m.f. in volts of each cell, from Table 358, for example. R_x = resistance, in ohms, of the external circuit. R_I = internal resistance in ohms of each cell, from Table 358, for example.

EXAMPLE.—What current will a battery of six cells, connected in series as in Fig. 191,*I* and *II*, drive through a circuit having an external resistance of (*a*) 0.1 ohm, (*b*) 500 ohms? Each cell has an e.m.f. of 2 volts and an internal resistance of 0.5 ohm. SOLUTION.—Substitute in the formula (95): (*a*) With 0.1 ohm external resistance:

$$I = \frac{S \times E}{R_x + (S \times R_I)} = \frac{6 \times 2}{0.1 + (6 \times 0.5)} = \frac{12}{0.1 + 3} = \frac{12}{3.1} = 3.87 \; amp.$$

(*b*) With 500 ohms external resistance:

$$I = \frac{S \times E}{R_x + (S \times R_I)} = \frac{6 \times 2}{500 + (6 \times 0.5)} = \frac{12}{500 + 3} = \frac{12}{503} = 0.024 \; amp.$$

378. Current Output When Cells Are Connected in Parallel.—With the parallel or multiple method of connection (see Art. 191) all of the negative or − poles are joined together and in combination constitute one terminal. All of the positive or + poles are likewise connected and in combination constitute the other terminal as shown in Fig. 190.

A HYDRAULIC ANALOGY TO CELLS CONNECTED IN PARALLEL is shown in Fig. 190,*II*. With the pumps connected in this way, their pressures are not added but the pressure imposed by the group or battery is the same as the pressure developed by any one pump. However, the internal resistance of

the group of three pumps in parallel is less than the internal resistance of any one pump. Similar conditions exist when primary cells are connected in parallel.

The e.m.f. of a group or battery of cells connected in parallel is the same as that of each member of the group. (Cells of different types, having different e.m.fs. should not be connected in parallel.) Therefore, the e.m.f. of the battery of Fig. 190,*I* is 2 volts. However, the internal resistance of the battery decreases as the number of cells in it is increased. If there are two cells in the battery its internal resistance will be one-half that of one cell; with three cells the internal resistance will be one-third that of one cell and so on. Connecting cells in parallel is equivalent to increasing the areas of the plates. Each cell contributes an equal share to the output of the battery. Stating these facts in a formula:

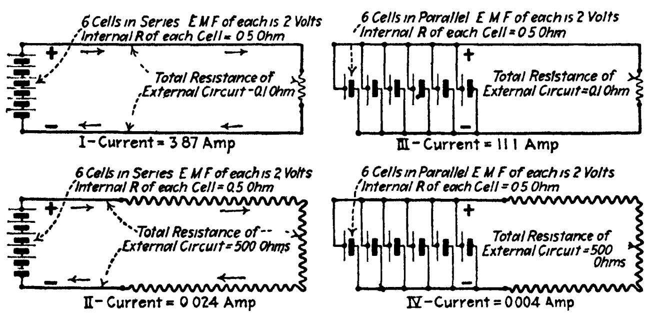


FIG. 191.—Varying current output by changing arrangement of cells.

(96) $$I = \frac{E}{R_x + \left(\frac{R_I}{S}\right)} \quad \text{(amp.)}$$

Wherein each letter has the same meaning as in equation (95).

EXAMPLE.—What current will a battery of six cells, connected in parallel as in Fig. 191,*III* and *IV* drive through a circuit having an external resistance of: (*a*) 0 1 ohm, (*b*) 500 ohms? Each cell has an e m.f. of 2 volts and an internal resistance of 0.5 ohm. SOLUTION.—Substitute in the formula (96): (*a*) With 0.1 ohm external resistance:

$$I = \frac{E}{R_x + \frac{(R_I)}{S}} = \frac{2}{0.1 + \frac{0.5}{6}} = \frac{2}{0.1 + 0.08} = \frac{2}{0.18} = 11.1 \; amp.$$

(*b*) With 500 ohms external resistance:

$$I = \frac{E}{R_x + \frac{(R_I)}{S}} = \frac{2}{500 + \frac{0.5}{6}} = \frac{2}{500 + 0.08} = \frac{2}{500.08} = 0.004 \; amp.$$

379. Applications of the Series and the Parallel Methods of Grouping Cells.—Tabulating the values of the examples of Arts. 377 and 378 and Fig. 191 we have:

Arrangement of cells	Current	
	External circuit of 0.1 ohm resistance	External circuit of 500 ohms resistance
6 cells in series...	3.87 amp.	0 024 amp.
6 cells in parallel ...	11 1 amp.	0.004 amp.

The table shows that when the resistance of the external circuit is large the greatest current will be obtained by connecting the cells in series. Also, when the external resistance is small, the greatest current will be obtained by connecting the cells in parallel. It can be shown that, to secure maximum current

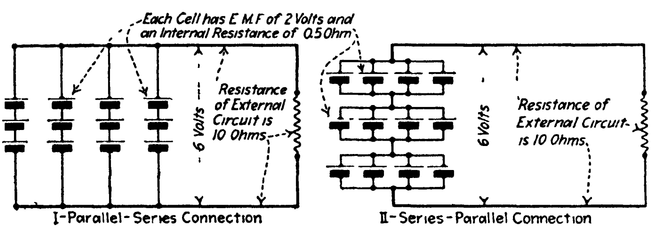


FIG. 192.—Series and parallel combinations of cells.

output, the cells should be so arranged that the internal resistance of the battery will be equal to (or as nearly so as possible) the resistance of the external circuit.

NOTE.—From a consideration of Ohm's law it will be apparent that where the external resistance is small nothing can be gained by increasing the e.m.f. of the battery by joining the cells in series because this also increases the internal resistance of the battery. Hence, the current will be little changed one way or the other. If, however, the external resistance is large, the increase in internal resistance caused by the joining of the cells in series is more than offset by the corresponding increase in e.m.f. It follows that, where the external resistance is large, the greater the number of cells in series the greater will be the current.

380. Current Output of Series and Parallel Combinations of Cells.—See Art. 200 for definitions of the words *parallel-series* and *series-parallel* and note that with primary cells arranged into a battery the series-parallel grouping of Fig. 192,*II* is

equivalent to the parallel series grouping of I. Following the reasoning of the two preceding sections and applying Ohm's law it can be shown that, for any parallel-series combination of cells:

$$I = \frac{P \times S \times E}{(P \times R_x) + (S \times R_I)} \quad \text{(amp.)} \tag{97}$$

Wherein I = current, in ampere in the external circuit. S = number of cells in series in each parallel group. P = number of groups of cells in parallel. E = e.m.f., in volts, of each cell. R_x = resistance, in ohms, of the external circuit. R_I = internal resistance, in ohms, of each cell.

EXAMPLE.—What current will the battery of Fig. 192,*I* drive through the external circuit? There are four parallel groups each containing three cells in series. The e.m.f. of each cell is 2 volts and each has an internal resistance of 0.5 ohm. The resistance of the external circuit is 10 ohms. SOLUTION.—Substitute in the formula (97):

$$I = \frac{P \times S \times E}{(P \times R_x) + (S \times R_I)} = \frac{4 \times 3 \times 2}{(4 \times 10) + (3 \times 0.5)} = \frac{24}{41.5} = 0.58 \; amp.$$

SECTION 19

ELECTROLYSIS

381. Electrolysis is the chemical decomposition of a conducting substance caused by the flow of current through it. One specific form of electrolysis is the "eating away" or corrosion of underground metallic structures (Art. 383) due to the passage of stray electric currents from them. The other important form of electrolysis is the decomposition of electrolytes by electric currents; electroplating (Art. 384) and electrotyping (Art. 385) are practical examples of this. If a current passes from one conductor (positive plate or anode, Fig. 193,*I*) through an electrolyte to another conductor (negative plate or cathode), the electrolyte will be decomposed and the anode may, under certain conditions, also be decomposed.

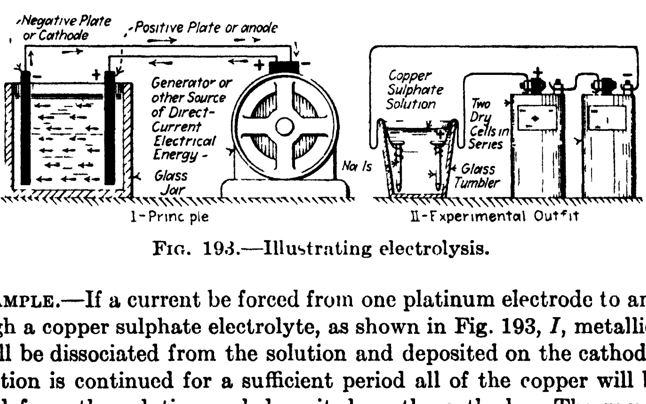


FIG. 193.—Illustrating electrolysis.

EXAMPLE.—If a current be forced from one platinum electrode to another through a copper sulphate electrolyte, as shown in Fig. 193, *I*, metallic copper will be dissociated from the solution and deposited on the cathode. If the action is continued for a sufficient period all of the copper will be extracted from the solution and deposited on the cathode. The remaining components of the copper sulphate will unite with the water in the solution to form sulphuric acid. This illustrates how an electric current can decompose electrolytes. This principle can be readily demonstrated (Fig. 193,*II*) by using a couple of dry cells as a source of energy, a couple of iron nails as electrodes and a solution of blue vitriol (copper sulphate—a crystal the size of a walnut in a tumbler full of water) for an electrolyte and a glass tumbler for a jar.

382. Electrolytes are solutions in water of acids, bases (alkalies) and salts. They are decomposed when an electric current passes

through them. The exciting solutions in primary and secondary cells (Art. 332) and the solutions used in electroplating (Art. 384) and electrotyping are examples of electrolytes.

383. Electrolysis of Underground Metallic Structures is illustrated in Fig. 194. Direct-current railway systems practically always use the track as a return conductor. The return current leaks from the track, which is always in contact with the earth, and often seeks a route of minimum resistance through water mains, cable sheaths and other metallic underground structures. Where these leakage currents enter the buried metallic systems (*A*, Fig. 194) there is no trouble. But at locations (*B*), where the stray currents leave, electrolysis—wasting away of the metal—occurs.

NOTE.—The action is similar to that of an electroplating process (Art. 384). The water main in the illustration at *B* is the anode. The chemical salts in the earth in combination with the moisture in it constitute an elec-

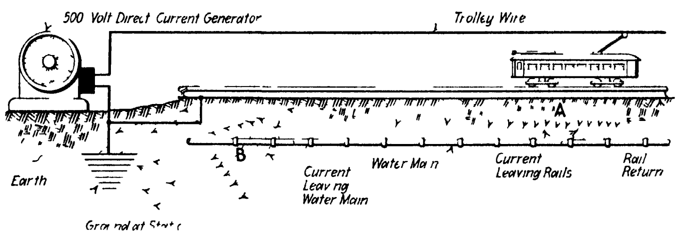


FIG. 194.—Illustrating cause of the electrolysis of a water main.

trolyte. The ground plate or connection at the generating station is the cathode. With large railway systems the leakage may be very great, in which case the consequent eating away of underground metals is extensive. In some cases, a pipe may be eaten entirely through in a month. The most effective method of correcting such electrolytic action is to minimize the tendency for its occurrence. This can be effected by connecting the station ground directly with the rails at various locations with heavy copper return conductors.

384. Electroplating (Fig. 195).—Whenever a current passes through a solution of a salt of a metal the metal will be extracted electrically from the solution and deposited on the positive plate or cathode. Fig. 193 illustrates a sort of electroplating process. Electroplating consists in coating by electrolysis a baser metal with copper, gold, silver, nickel or almost any other metal.

EXAMPLE.—Fig. 195 shows a silver plating outfit. In modern commercial outfits low-voltage, direct-current generators are practically always used to

propel current instead of primary cells. The process is about as follows: The surface of the object to be plated is thoroughly cleaned of all fatty matter. The object is connected to the negative pole of the source of energy. The object thus constitutes a cathode. The electrolyte is a solution of some chemical salt of the metal to be deposited. For silver, cyanide of silver is used; for copper, copper sulphate, etc. To maintain the strength of the solution a piece or anode of the metal to be deposited is attached to the positive pole of the electricity source. The current in flowing through the solution deposits the metal of the solution on the cathode. Certain metals, such as iron, steel, zinc, tin, and lead, cannot be plated with certain other metals, such as gold and silver, until after they have first been given a thin plating of copper.

385. Electrotyping is an electrolytic process, similar to electroplating, whereby wood cuts, type and like objects can be repro-

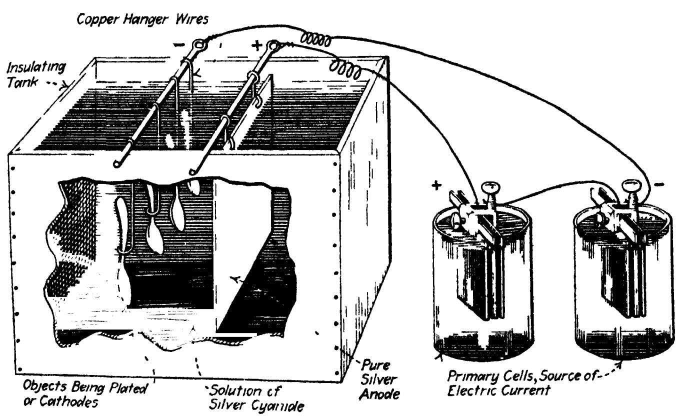


FIG. 195.—A silver electroplating outfit.

duced in metal, usually copper. An impression of the object to be reproduced is taken in wax, plaster of Paris or similar moulding material. The surface of the mould thus made is thinly coated with some fine metallic substance, such as powdered graphite, to render it conducting. The mould is then immersed in a copper sulphate solution bath and its conducting surface is so connected to the positive pole of a source of electrical energy that it constitutes a cathode. It is then treated much like any other object to be plated. When the copper coating on it has become about the thickness of a visiting card, it is removed and reinforced by pouring molten metal on its back. If it is to be used for printing it is backed so as to be the same height as type.

386. In the Electrolytic Refining of Metals the process is somewhat similar to electroplating (Fig. 195). The impure metal to be refined is suspended as an anode in a solution of one of its salts (for copper, copper sulphate solution is used). Current is forced through the solution from these anodes to cathodes. Pure metal only is deposited on the cathode. The impurities in the anodes fall to the bottom of the tank as the pure copper is extracted from them. The electrolytic refining of copper is a very important process commercially.

SECTION 20

STORAGE BATTERIES

387. A Storage or Secondary Cell comprises two relatively inactive plates or electrodes of metals or metallic compounds immersed in an electrolyte which will not act on the plates until after an electric current has been forced through the electrolyte from one plate to the other. The action of the current changes the chemical relations of the elements of the cell and it will then, when connected to an external circuit, force current through that circuit. In practice, several plates are electrically connected

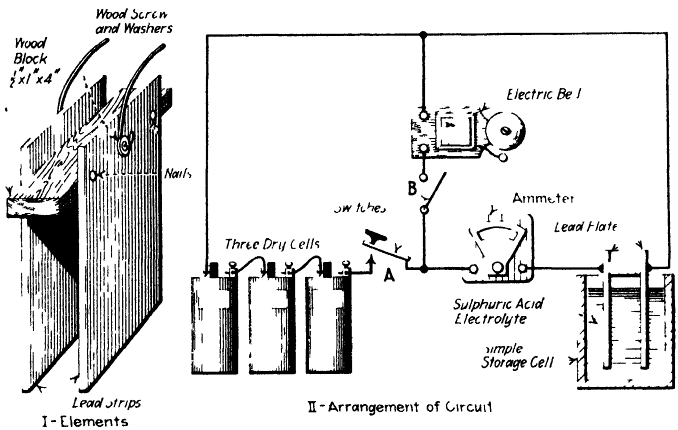


FIG. 196.—Apparatus for illustrating the principles of the lead storage cell.

together so as to, in reality, form one positive plate and several other plates are assembled so as to form one negative plate. A combination of a multiple negative plate and a multiple positive plate made as described and suspended in a glass jar or other container having in it electrolyte, constitutes a commercial storage cell. *A storage battery, secondary battery or accumulator* comprises two or more storage cells so connected together as to act in conjunction.

388. The Distinction Between a Primary Cell and a Secondary Cell is discussed in Art. 332. The information there given relat-

ing to primary cells should be reviewed before the following sections are read.

389. Any Primary Cell Is Essentially a Storage Cell.—When the positive plate (Art. 334), usually zinc, of a primary cell is almost consumed, it is replaced with a new one to rehabilitate the cell. However, it would be theoretically possible to redeposit on the zinc the metal (zinc) electro-chemically dissolved in the electrolyte by driving a suitable current, from an outside source, through the cell from the carbon to the zinc. By this process the cell would be renewed by electrolysis (Art. 381) and would of itself be again capable of forcing current through an external circuit. It has, therefore, become a storage cell. Although all primary cells are, fundamentally, storage cells they are not used as such in practice because of certain mechanical and chemical difficulties and because they would be very uneconomical as storage cells.

390. A Storage Cell Does Not Store Electricity.—It stores only energy (Art. 169). When a cell is being charged the electrical energy imparted to it is transformed into chemical energy which is stored in the cell. Then, when the cell discharges, that is, when an external circuit is completed through which current can be forced by the e.m.f. of the cell, the stored chemical energy is reconverted into electrical energy.

391. Charging a Cell.—When current is forced through a cell to store chemical energy in it the cell is being "charged." The charging current must be forced in against the e.m.f. of the cell, hence the voltage of the charging current must be greater than the e.m.f. of the cell by an amount sufficient to overcome the internal resistance of the cell. That is:

$$\text{(98)} \qquad \textit{Charging voltage} = (\textit{E.m.f. of cell}) + (\textit{IR drop in cell})$$

Furthermore, the e.m.f. of a cell is always greater when it is charged than when discharged because the electrolyte becomes more dense on charging and hence is more powerful. The charging voltage is always greater than discharging voltage. The graphs of Figs. 197 and 198 show the voltages of cells on charge and on discharge.

392. Discharging a Cell.—When a storage cell is connected to an external circuit its e.m.f. forces a current through the circuit and, as the current flows, the chemical energy of the cell is reconverted into electrical energy. When this condition occurs the

cell is said to be "discharging." The voltage of a cell that is discharging is always less than the e.m.f. of the cell because of the voltage drop due to its internal resistance. That is:

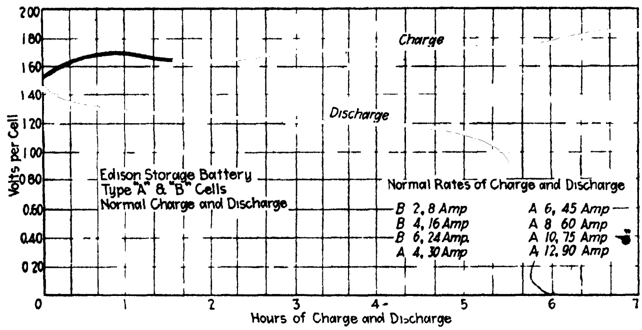


FIG 197—Graph showing charge and discharge characteristics of the Edison storage cell

(99) *Discharging voltage* = (*E.m.f. of cell*) − (*I* × *R drop of cell*)

The electrolyte becomes less dense as the cell discharges and this also tends to decrease the "voltage on discharge." See the graphs of Figs. 197 and 198.

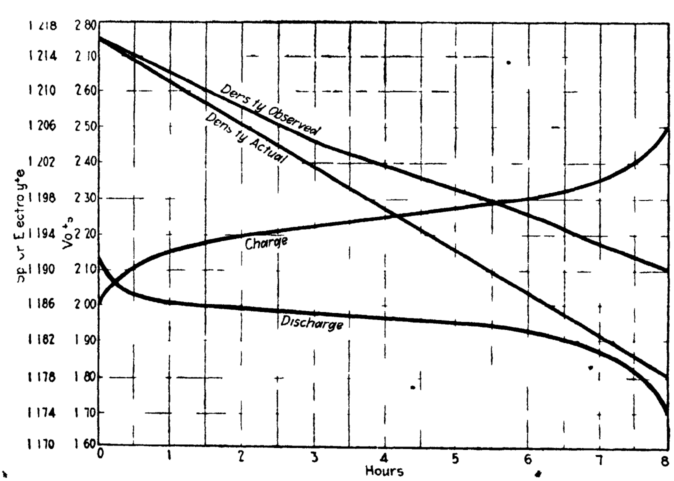


FIG 198 Graph showing typical characteristics of lead storage cells.

393. The Principle of the Storage Cell can be readily illustrated with the simple apparatus shown in Fig. 196. The storage cell there shown is not capable of storing any great amount of

energy but will indicate the general operation as suggested in the following example:

EXAMPLE.—If two strips of sheet lead be mounted on a block of wood as shown at Fig. 196,*I* and immersed in a tumbler of electrolyte consisting of 1 part sulphuric acid and 10 parts distilled or rain water, they will constitute a simple storage cell. This cell can be charged in about 30 min. by three dry cells arranged as shown—with switch *A* closed and *B* open. The ammeter, which should be of the "zero center" type so that it will indicate the current regardless of the direction in which it flows, will show the current flowing as the cell is charging. When the storage cell is charged, if switch *A* be opened and *B* closed, the cell will discharge. That is, it will force current through the bell thereby causing it to ring.

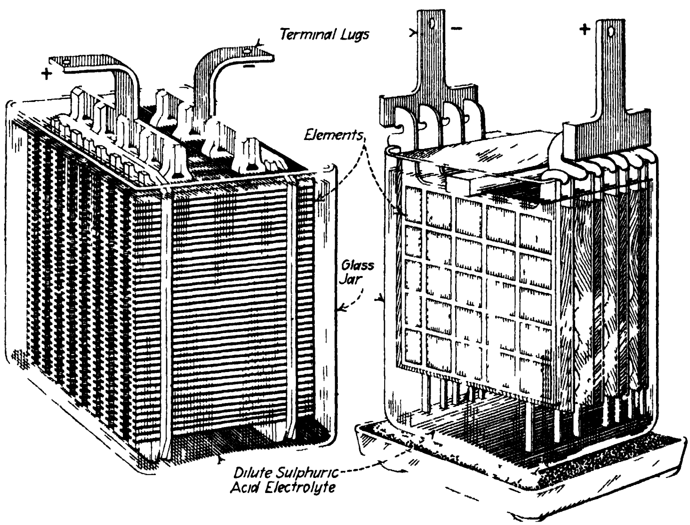


FIG. 199.—Examples of storage cells having plates of the two types.

As the storage cell described above is charged, the color of its anode changes from the gray natural-lead color to brown. This is caused by a coating of lead peroxide, a compound of lead and oxygen formed on the anode by the passage of the current. When the cell is allowed to discharge through the external circuit it really is then one form of primary cell. When discharging, the lead peroxide is reconverted into the metal lead. It will impel current as a primary cell until all of the lead peroxide has been converted into metallic lead. There are other chemical actions that occur in lead storage cells but the one indicated above is probably the most important.

394. Efficiency of a Storage Cell.—A storage cell resembles every other mechanical or electrical device in one respect, in that it is impossible to get out of it all of the energy imparted

to it. With a lead storage cell or battery, only about 75 per cent. of the energy imparted to it on "charge" can be recovered on "discharge." That is, the efficiency of the lead storage battery (Art. 400) is about 75 per cent. The efficiency of the Edison battery (Art. 405) is said to be 60 per cent. The loss of energy in the cell is largely caused by the $I^2 \times R$ loss (Art. 167) due to the passage of the current through the electrolyte both in charging and in discharging.

EXAMPLE.—If a storage battery, which has an efficiency of 75 per cent., is charged with 1,000 kw.-hr. of energy, the energy that can be recovered on discharge will be: $1{,}000 \times 0\ 75 = 750$ *kw.-hr.*

395. The Unit of Capacity of Any Storage Cell Is the "Ampere-hour" and the capacity is usually based on the *normal* or 8-hour rate of discharge.

EXAMPLE.—A 100-amp.-hr. battery will provide a continuous current of 12.5 amp. for 8 hr. Theoretically the cell should give a discharge of 25 amp. continuously for 4 hr. or 50 amp. for 2 hr. Actually, however, the ampere-hour capacity decreases as the rate of discharge increases.

396. What Determines the Capacity of a Storage Cell or Battery.—The capacity of a cell is proportional to the area of the plates exposed to the electrolyte and it depends on the quantity of active material on these plates. It follows that the capacity of a battery depends on the exposed area and the number of plates in parallel, their character, the rate of discharge and also on the temperature. With the standard 8-hr. rate of discharge and a temperature of 60 deg. F., the capacities which obtain in American practice are from 40 to 60 amp.-hr. per sq. ft. of positive plate surface = *number of positive plates in parallel* $\times$ *length* $\times$ *breadth* $\times$ 2.

397. The Voltage of a Storage Cell is determined solely by the character of the electrodes, the density of the electrolyte and the condition of the cell. It is independent of the size of the cell. In these respects, the e.m.f. of a storage cell depends on the same factors that determine the voltage of a primary cell. See Art. 340. See Figs. 197 and 198 for graphs showing the voltages of storage cells under different conditions of charge and discharge. The voltage of a lead-sulphuric-acid storage cell when being charged is from 2.0 to 2.5 volts, while on discharge it varies from 2.0 down to 1.7 volts. The voltage of the Edison cell on charge varies from about 1.5 volts to 1.8 volts while on discharge it decreases from 1.5 volts down to 1.0 volt.

398. To Obtain High Battery Voltages with Storage Cells a number of cells of suitable ampere-hour capacity are connected in series. Thus for a 100-volt circuit, approximately 50 cells connected in series would be required.

399. There Are Two General Classes of Storage Cells In Use.—*The lead-sulphuric-acid cells* (Fig. 199), described more in detail in Art. 400 and *the nickel-iron* or *Edison cell* (Fig. 200), treated in Art. 405.

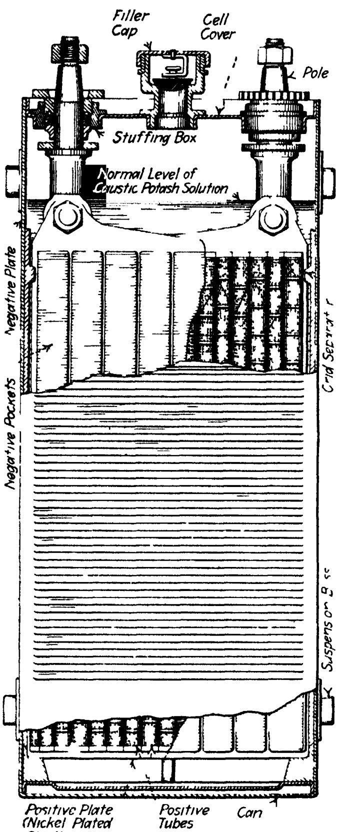


FIG. 200.—Showing general construction of Edison storage cell.

400. Lead Storage Cells. The Two-General Types.—In all lead-sulphuric-acid cells the active materials are lead peroxide and sponge lead and they are immersed in a dilute sulphuric-acid electrolyte. The lead peroxide is on the positive electrode and the sponge lead on the negative electrode. These active materials are poor conductors and are soft, hence they are supported in a frame or grid—usually of lead-antimony alloy—which provides mechanical strength and conductivity. The electrolyte does not attack this alloy, hence there is no local action. A graph indicating typical characteristics of lead storage cells is shown in Fig. 198. The two types of plates are:

1. THE PLANTÉ OR FORMED TYPE (Fig. 202).—In this type of plate the active materials are formed by chemical processes out of and on the lead surfaces of the plates themselves. The Gould cell is an example of one that uses Planté plates

2. THE FAURE OR PASTED TYPE (Fig 203) —Plates of this type are made by applying the active material by some mechanical process, such as mixing it into a paste and spreading it on the surface or into the interstices of the grid or plate. The pasted material has in it some substance which causes it to set or harden. The Chloride Accumulator is an example of a cell using pasted plates.

401. The Essential Differences Between the Planté and Pasted Plates are: For a given output Planté plates are more

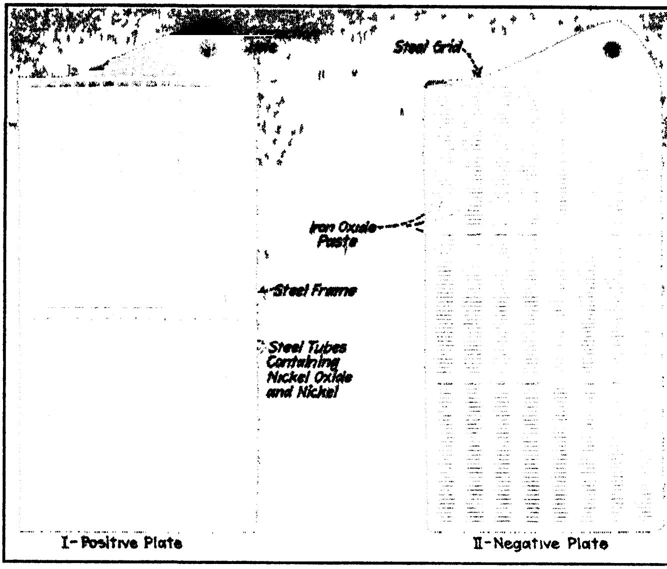


FIG. 201.—Plates of the Edison or nickel-iron storage cell

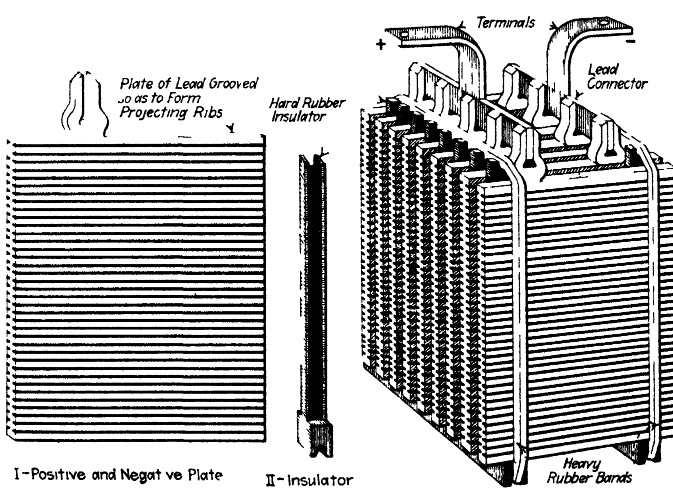


FIG. 202.—Elements of a lead storage cell using Planté or formed plates.

expensive, more bulky and heavier than the equivalent pasted plates. They are also more easily injured by impurities in the

electrolyte. They are, however, capable of standing more rapid charging and discharging rates without injury. They are less liable to lose their active material and be injured by the accumulation of sediment in the bottom of the cells. They are more durable and have longer life. In general they are a more dependable type of plate than the pasted. The pasted plates, however, for a given output, are cheap, light and occupy a smaller space. They also are not so badly damaged by impurities in the electrolyte. The efficiency of pasted cells is lower at high current rates than that of the Planté type.

402. Application of Planté and Faure or Pasted-type Cells.—Each has applications for which it is best suited. For work such as motor-car propulsion the pasted cell is better adapted than the Planté because of its light weight and low cost. (The nickel-

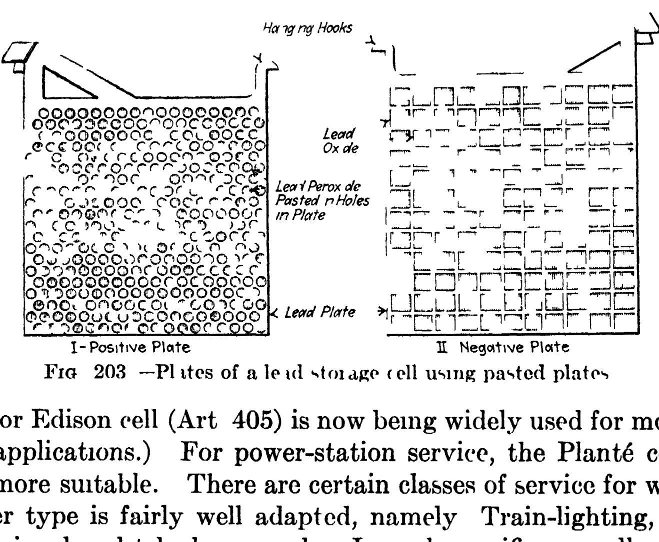


FIG. 203.—Plates of a lead storage cell using pasted plates.

iron or Edison cell (Art. 405) is now being widely used for motor-car applications.) For power-station service, the Planté cell is the more suitable. There are certain classes of service for which either type is fairly well adapted, namely Train-lighting, railway-signal and telephone work. In each specific case all of the conditions, commercial as well as technical, should be considered before the type that is most suitable for the requirements is determined.

403. To Distinguish the Electrodes of Lead Storage Cells.—The electrode which is the cathode (Art. 334) when the cell is being discharged is the positive plate. The other electrode is the negative plate. The positive plate always has one less grid than the negative. Furthermore, the positive plate is reddish brown in color while the negative plate is dark gray.

404. Maintenance and Operation of Lead-sulphuric-acid Storage Cells.—It is not possible to give here complete instructions because each manufacturer has specific directions for his make of cell. Any manufacturer will furnish complete data on request or will answer specific questions. The most important points are these:

ELECTROLYTE.—Be sure the electrolyte is free from injurious impurities. Keep electrolyte well above tops of plates. Maintain the specific gravity of the electrolyte at the density specified by the manufacturer of the battery. Do not let the density of the electrolyte in any cell differ from the standard density more than 0.005. Thus a cell having normal density of 1.200 must register above 1.205 and below 1.195 when fully charged. Test each cell with hydrometer once a week at least.

CELLS.—Keep cells cleaned out and remove sediment when it has deposited metal near the lower edges of the plates. Be sure separators are all in place and in good order. Note any evidences of tank leakage and correct at once. Maintain insulation of cells from ground and from each other.

CHARGING AND DISCHARGING.—Begin charge immediately after the end of discharge or as soon thereafter as practicable. Do not continue charge after the negative plates begin to give off gas, except the occasional "boiling" to be mentioned later. Never let charging current fall below the 8-hr. rate except toward the end of charge, and stop discharge when the battery potential falls to 1.75 volts per cell with the normal current; 1.70 volts per cell discharging at the 4-hr. or 1.60 volts per cell discharging at the 1-hr. rate.

PLATES.—Watch the colors of the plates and if they begin to grow lighter treat at once for removal of sulphate.

OVERCHARGE.—Give the battery a prolonged overcharge about once a month. This over-charge should continue at about 60 per cent. of the 8-hr. rate until free gassing of the negative plates has continued for 1 hr.

405. The Nickel-iron or Edison Storage Cell (Fig. 200) is the result of an effort to overcome certain of the undesirable features of lead-sulphuric-acid cells. It is a radical departure therefrom in every detail of construction. The positive plate, *I*, Fig. 201, consists of hollow, perforated, sheet-steel tubes filled with alternate layers of nickel hydrate and metallic nickel. The hydrate is the active material. The nickel, which is made in the form of microscopically thin flakes, is added to provide good conductivity between the walls of the tube and the remotest active material. The negative plate, *II*, Fig. 201, is made up of perforated, flat, sheet-steel boxes or pockets loaded with iron oxide and a small amount of mercury oxide. The oxide is added to provide conductivity. The grids which support these tubes and pockets are punchings of sheet steel. The cell terminals and container are likewise of steel and all metallic parts are heavily

nickel plated. The electrolyte is a 21 per cent. solution of caustic potash containing also a small amount of lithium hydrate. All separators and insulating parts are of rubber.

406. The Chief Characteristics of the Edison Storage Cell are: ruggedness, due to its solid, steel construction; low weight, because of its stronger and lighter supporting metal; long life, because of the complete reversibility of chemical reactions and the absence of shedding active material; and low cost of maintenance, due to its freedom from the troubles such as sulphation, so commonly encountered in storage-battery practice, and from the necessity of internal cleaning and plate renewals. The importance of these features should be considered in each proposed installation. The efficiency of the Edison cell is about 60 per cent. as against 75 per cent. for the lead cells; however, this disadvantage of lower efficiency is more than offset in certain services because of its reliability, light weight, ruggedness and low maintenance cost. The Edison cell has attained its chief prominence in electric-vehicle propulsion and in train-lighting service. See Fig. 197 for charge and discharge graph.

407. The Chemical Action in the Edison Storage Cell is as Follows.—The charging current causes an oxidation of the positive plate and a reduction of the negative. That is, oxygen is added to the + plate and taken away from the − plate. On discharge these operations are reversed. The electrolyte acts merely as a medium and does not enter into combination with any of the active material as it does in the acid cells. Its specific gravity remains practically constant throughout the complete cycle of charge and discharge. The charge and discharge graphs are shown in Fig. 197.

408. In Maintaining Edison Storage Cells, the attention necessary is of the simplest character. The principal requirement is that the electrolyte be replenished from time to time with distilled water so that the plates will be entirely immersed and that the outsides of the cells be kept clean and dry, for if the exteriors of the cells are not kept clean there will be current leakage and consequent corrosion of the containers by electrolysis.

409. Charging Storage Batteries.—The arrangements that are desirable for properly charging large storage batteries, such as those used in central-station and railway service are relatively complicated and can not be adequately discussed in the space available here. For information concerning them, the reader is

referred to the books on storage-battery engineering. The voltage of the charging source must always be greater than the discharging voltage of the battery being charged, as outlined in Art. 391. The charging current should be so controlled that the charging will be at the normal, 8-hr. rate as indicated in Art. 404. Charging current must always flow through the battery from the positive pole to the negative pole, that is, in a direction opposite to that in which the discharge current flows:

DIRECTIONS FOR CHARGING SMALL BATTERIES.—Alternating current can not be used directly. When it only is available it must be converted to direct current by means of motor-generators, rotary converters, or mercury-vapor converters. Connections are shown in Fig. 204 for charging small storage batteries from direct-current mains. An ammeter in the circuit is convenient but not absolutely necessary and lamps or a rheostat (Fig. 204,

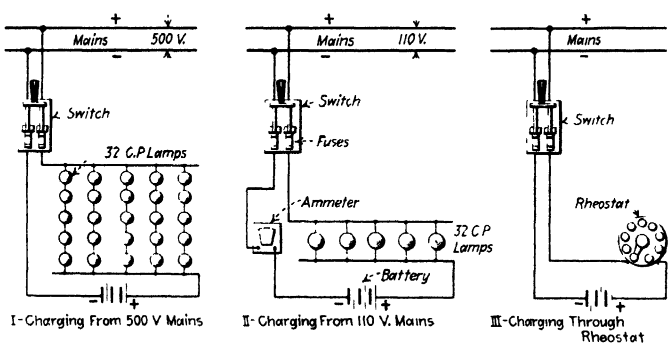


FIG. 204.—Connections for charging small storage batteries.

III) are used to vary the current. A 16-c.p., 110-volt, carbon-filament lamp has about 220 ohms resistance and will carry 0.5 amp.; a similar lamp of 32-c.p. rating has about 110 ohms resistance and will carry 1 amp. Therefore, the charging current from 110-volt mains (Fig. 204,*II*) can be limited to, say, 5 amp. by connecting five 32-c.p. lamps in parallel, or from 500-volt mains (Fig. 204,*I*) by connecting in parallel five series of lamps, each series containing five 32-c.p. lamps. In both cases, two 16-c.p. lamps in parallel can be used in place of each 32-c.p. lamp.

410. Applications of Storage Batteries.—The following tabulation from the AMERICAN HANDBOOK for ELECTRICAL ENGINEERS, published by Wiley and Sons, indicates the principal applications of storage batteries:

Central stations

1. Emergency reserve or "stand-by" service.
2. Load or voltage regulation.
3. Taking peaks.
4. Day load on small systems.
5. Exciter reserve.
6. Remote-control switch operation.

Isolated plants

1. Mine hoists, steel mills and other heavy motor regulation.
2. Carrying entire load during certain hours of light load.
3. Load and voltage regulation in office buildings and hotels where electric elevators are in service.
4. Giving 24-hr. service in residences.
5. Operation of drawbridges.

Miscellaneous Applications of Storage Batteries:

1. Regulation of voltage on long feeders of trolley systems.
2. Propulsion—trucks, street cars, submarine boats, launches, etc.
3. Gas-engine ignition.
4. Railway car lighting—train lighting.
5. Railway signalling.
6. Telephone and telegraph.
7. Portable and small-stationary lamps.
8. Fire and burglar-alarm systems.
9. Electrotyping.
10. Dental and other surgical work.
11. Source of constant potential and current in laboratory work.
12. For changing voltage by charging the cells in series and discharging them in parallel—or the reverse.
13. For providing the different voltages for a multi-voltage system, as for a three-wire system or a five-wire system.
14. For automobile starting and lighting.

411. Storage Batteries on Systems Having Fluctuating Loads.—A central station supplying a load that is subject to sudden and extreme variations (Fig. 205) must, unless a storage battery is used have a generating capacity sufficient to supply the maximum. However, if a storage battery is installed the generating capacity need be only sufficient for the average demand. The battery is so arranged, somewhat as indicated in Fig. 206, that it will be charged when the load demand is below the average and will discharge when the load demand is above the average. Thus, the battery maintains a practically constant load on the generating equipment. This minimizes wear and tear on the machinery and permits the operation of the machinery at a constant load that will insure high efficiency.

412. Storage Batteries for Carrying Part of the Load at Times of Heavy Demand, that is, *at times of peak load:* With certain

classes of central station service, the load for some portion of each 24 hr. is always greater than during the balance of each daily period. For example, with electric-lighting stations there is always a "peak" in the evening from about 5:00 P.M. to 10:00 P.M., as shown in the graph of Fig. 207,*II*. Where storage batteries are not used, sufficient generating capacity must be provided to carry the peaks. But if a storage battery is installed, as suggested in Fig. 206, of the proper capacity, it in conjunction with the generators can handle the peak. The battery is charged during periods of *light load* and discharges during the peak period.

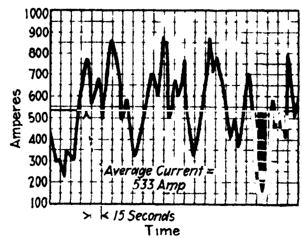


FIG. 205.—Graph indicating fluctuation of current of a railway load.

NOTE.—By this arrangement the load on the generating machinery is maintained reasonably constant. The generating equipment will then

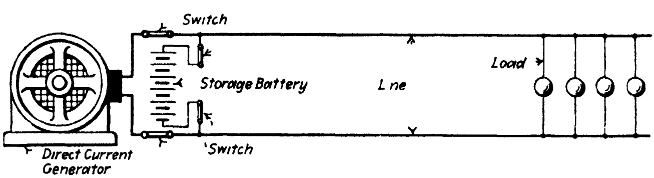


FIG. 206.—Elementary diagram showing how a storage battery is applied for "stand-by" or peak-load service.

operate at good economy and first cost will be less than if a storage battery is not used. However, the storage battery will also involve investment and operating costs. Hence, a comparison should always be made for each spe-

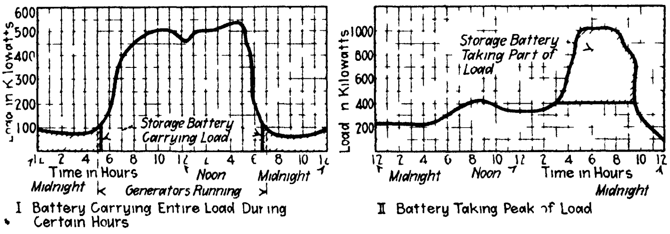


FIG. 207.—Graphs indicating applications of storage cells.

cific case to ascertain whether, in the long run, it will be more economical to install a storage battery to assist during the peak or to install additional generating equipment to handle the peak.

413. Storage Batteries for Carrying the Entire Load During Certain Hours are arranged as diagrammatically indicated in Fig. 206. In certain cases, in isolated plants particularly, the load may be so small during certain hours, as shown in Fig. 207,*I*, that it is very expensive to operate generating equipment to supply it. In such cases a storage battery may be installed. The battery can be charged while the generators are in operation. When the load becomes light the generators can be shut down and the battery switched on the line to carry the entire load. The graph of Fig. 207,*I*, illustrates this situation.

414. Storage Batteries Are Well Adapted for Certain Classes of Laboratory and Testing Work because the e.m.fs., hence the currents, supplied by them are perfectly steady—they are not subject to the variations that obtain when current is propelled by generators driven by engines or other prime movers. Furthermore, a variety of voltages may be obtained from one battery by using different series and multiple arrangements of the cells.

SECTION 21

ELECTROMAGNETIC INDUCTION

415. Relation Between Magnetism and Electricity.—In preceding sections (Art. 204) it has been demonstrated that whenever a current (of electrons) flows through a conductor, magnetism—a flux of lines of magnetic force—develops in the neighborhood of the conductor. That is, *a current of electricity or electrons in motion produces magnetism.* Static electricity or electricity at rest does not produce magnetism. It will now be shown, in the sections immediately following, that this process is, in a measure, reversible. That is, a flux of magnetism can, by proper manipulation, be made to produce a current of electricity.

Invariably, the first requirement if a current of electricity is to be made to flow is the production of an e.m.f. (Art. 123). It follows, then, that if it is desired to circulate a current of electricity by means of a magnetic flux, the first step must be to generate an e.m.f. by means of that flux. How this can be done will be shown. Then, if the e.m.f. thus generated be impressed on a closed circuit, a current of electricity will be driven around the circuit. In this way a flux of magnetism can be made to move electrons, that is, to propagate an electric current.

416. Electromagnetic Induction is that phenomenon, discussed in detail in following articles, whereby an e.m.f. is induced in any conductor that cuts across or is cut by a magnetic flux. Do not confuse "electromagnetic induction" with *electrostatic induction* or *magnetic induction* (Art. 64) which are entirely different, but related, phenomena.

417. Importance of the Principle of Magnetic Induction.—This principle is of great commercial importance because on it depends the operation of dynamos or generators (Art. 509). As suggested elsewhere (Art. 374), electrical energy can be developed with generators more cheaply than by any other means—at much lower cost than by primary cells (Art. 330). If magnetic induction had never been discovered and it were

now necessary for us to depend on primary cells for the generation of electrical energy, it is obvious that the use of electricity would be exceedingly restricted. The numberless present-day electrical applications and the modern wonderfully extensive development would be out of the question. Furthermore, the long-distance transmission of electrical energy would not be commercially possible were it not for the principle of magnetic induction because it is on this principle that transformers (Art. 826) also depend for their operation. Without transformers long-distance electrical transmission would not be feasible.

418. Whenever a Conductor Cuts Across Lines of Force an E.m.f. Is Induced in That Conductor, or, what amounts to the same thing, whenever a flux of lines of force cuts across a conductor an e.m.f. is induced in the conductor. Michael Faraday discovered these important facts in 1831. If the conductor in which the e.m.f. is induced forms part of a closed circuit, it (the e.m.f.) will drive a current of electrons through the circuit. Such a current is an *induced current* or an *induction current.*

NOTE.—Instead of the statement, "Whenever a conductor cuts across lines of force an e.m.f. is induced in that conductor" the following synonymous statements are sometimes used: (1) "When a magnetic flux within a loop of wire varies with the time, an e.m.f. is induced in the loop." (2) "Whenever the number of magnetic lines linking a circuit is changed an e.m.f. is developed in the conductor forming the circuit."

The idea of "*cutting of lines*" appears preferable to that of "variation of flux" or "linking of flux" because it seems that the idea of cutting is the more readily understood. Furthermore, a complete loop or circuit is not necessary for the development of an e.m.f. An e.m.f. can readily be induced in a short straight piece of wire not forming a closed circuit and not connecting to anything. However, no current can flow unless the e.m.f. thus induced is impressed on a closed circuit.

Relative to this situation, Carl Hering has made experiments* which tend to prove that an e.m.f. can not be induced by a magnetic flux unless the flux actually cuts the conductor or unless the flux is actually cut or sheared by the conductor.

419. An E.m.f. Is Also Induced in a Non-conductor When It Cuts Lines of Force† or when lines of force cut it. This has been proven by experiment. But since current can not flow in a non-

* TRANS. A. I. E. E., Vol. 27 (1908), part 2, p. 1341. A list of other references, bearing on this matter, is given in Karapetoff's THE MAGNETIC CIRCUIT (McGraw-Hill, 1911) at the foot of p. 57.

† For additional information see PHIL. TRANS., Royal Society of London, Vol. 204A, p. 121, 1905. Also PROC. ROY. SOC., Vol. 73, p. 490, 1904. Also Fleming's PROPAGATION OF ELECTRIC CURRENTS, p. 56, Van Nostrand, N. Y.

conductor, this method of generating an e.m.f. is of little commercial importance and will, therefore, not be discussed further.

420. Why It Is That an E.m.f. Is Induced in a Conductor When the Conductor Cuts or Is Cut by Flux Is Not Known.—The statement must be accepted as that of a fact that can be readily demonstrated by experiment.

NOTE.—We know that when a conductor, forming part of a closed circuit, is moved through flux that the electrons in the conductor are thereby forced to move, thus creating an electric current. Even if the circuit is open, and part of it cuts through flux, there is a tendency toward making the electrons move—an e.m.f. is established. That is, if electrons—those in the conductor—are moved through a magnetic field there is always a tendency to push them out of the field sidewise.

421. The Flux Whereby an E.m.f. Is Induced May Be Produced by Any Possible Method.—It may be produced by either a permanent magnet, or an electromagnet or it may be the circular flux enveloping a conductor carrying current. Providing the resulting flux has the same number of lines, an e.m.f. can, theoretically, be induced with equal facility with the flux produced by any of the methods. A flux of lines of force has the same general properties regardless of the method of its production. However, in the practical generation of e.m.fs., a strong flux can be produced more readily by electromagnets than by other means. Hence, electromagnets are utilized in all practical e.m.f. generators.

EXAMPLE.—An e.m.f. can be induced in a wire by swinging the wire (which is connected into a closed circuit through a galvanometer) through the flux due to the earth's field (Art. 66). Whenever it is swung so as to shear through the flux, a deflection of the galvanometer will indicate the existence of an induced e.m.f. When the wire is not cutting flux, no e.m.f. will be induced.

422. An Example of the Generation of an E.m.f. by a Conductor Cutting a Flux is illustrated in Fig. 208. The flux of magnetic lines of force flows out of the N pole face of the permanent magnet across the air gap and into the S pole face—since flux always flows from N pole to S pole. Assume that a piece of any conductor is first held in the position AA' and then moved downwardly through position BB' to final position CC'. In moving from AA' to CC' it will cut through all of the lines comprising the flux between the two pole faces. While the conductor is cutting lines an e.m.f. will be induced in it. Obviously the conductor

can cut no lines unless it is in motion; hence, no e.m.f. can be generated (the pole faces and flux being stationary) unless the conductor is moved and thereby cuts lines.

If the conductor were held stationary at AA' and the pole faces and their flux were moved upward (at the same rate of speed as

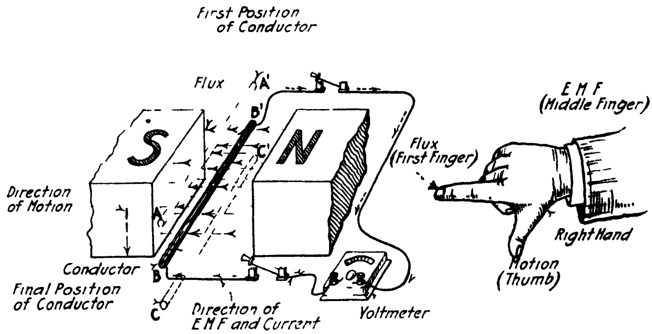


FIG. 208.—Generation of E.M.F. by cutting lines of force.

that at which the conductor was moved downward) until the conductor assumed position CC' in relation to the poles, the effect would be precisely the same as if the conductor were moved downward through the flux. An e.m.f. is induced in the con-

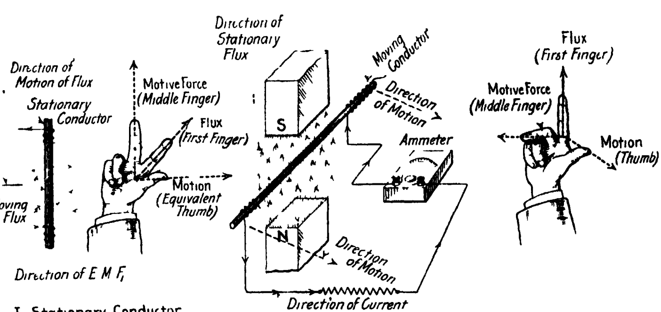


FIG. 209.—Illustrating application of right-hand rule for determining the direction of an induced E.M.F.

ductor whenever a flux cuts a conductor or the conductor cuts a flux. The e.m.f. is induced in the conductor whether the circuit is open as shown in Fig. 208, or closed as in Fig. 209,*II*. However, when the circuit is open no current can flow, but the existence of the e.m.f. can be detected with a voltmeter—as for instance

by closing the switches of Fig. 208. An electrostatic voltmeter, which does not permit current to flow through it, can be used.

423. If an E.m.f. Is to Be Induced in a Conductor, Lines of Force Must be Sheared or Cut Through by the Conductor.—This may be accomplished either by pushing the conductor through the lines so that it shears some or all of them or by moving the flux of lines so that the lines cut through the conductor. If a conductor is moved in a magnetic field in a direction parallel to the direction of the lines, no lines will be sheared and hence no e.m.f. can be developed.

424. The Effect on a Magnetic Field When It Cuts or Is Cut by a Conductor.—It might be inferred that the generation of e.m.fs. and consequent production of currents by electromagnetic induction (Art. 416) would weaken or change the intensity of the magnetic field, the lines of which were cut or sheared to induce the e.m.f. Such, however, is not the case. When a flux cuts a conductor, either by the movement of the conductor or of the flux, the strength of the field is in no way affected. It is obvious that, if current is forced through a conductor by an induced e.m.f., electrical energy is generated. The question then arises: How does this energy originate? This is answered in Art. 437.

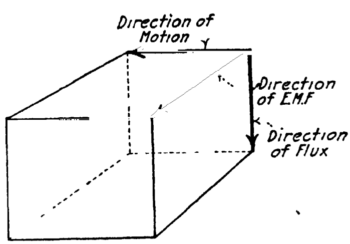


FIG. 210.—Relative directions of motion, electromotive force and flux.

425. The Relation of the Directions of Motion, E.m.f. and Flux, where the e.m.f. is developed by a conductor cutting lines of force, is indicated in Fig. 210. The directions are always at right angles to one another and are always in the relation shown. If the flux moves to cut the conductor, the direction of *motion of the flux* will be opposite to the direction of *motion of the conductor* which is shown in Fig. 210; however, in this case the *equivalent* direction of motion of the conductor will be that shown in the illustration.

426. What Determines the Direction of an E.m.f. Induced in a Conductor When It Cuts or Is Cut by Lines of Force.—If the conductor of Fig. 208 is moved downward shearing through the flux from AA' to CC', it will be found (if an ammeter is connected in the external circuit) that the current driven through the cir-

cuit by the induced e.m.f. will be from B toward B'. When the conductor is moved upward from CC' to AA' the e.m.f. is in the opposite direction or from B' to B. If the conductor were made to oscillate between positions AA' and CC' the induced e.m.f. would be first in one direction and then in the other.

Furthermore, if the N and S poles of the illustration were transposed, making the direction of the flux from left to right, this also would reverse the direction of the induced e.m.f. It is obvious, then, that there is a relation between: (1) the direction of motion of the conductor, (2) the direction of the flux, and (3) the direction of the induced e.m.f. This relation is explained in following articles:

427. A Hand Rule for Determining the Direction of an E.m.f. Induced in a Conductor was proposed by Fleming. It is the most serviceable of all of the rules for this purpose. It is: Use the *right hand.* Extend its thumb, first finger and middle finger so that they are at right angles to one another as shown in Fig. 209. Then turn the hand into such a position: (1) *that the thumb points in the direction of the motion or equivalent motion of the conductor and* (2) *that the first finger points in the direction of the magnetic flux. Then, the middle finger will point in the direction of the induced e.m.f.* If the directions of any two of the factors are known, the direction of the other can be determined by applying this hand rule.

NOTE.—This rule can be remembered by associating the sounds of the words "thumb-motion," "first finger-flux" and "middle finger-motive force." If the flux moves to cut the conductor, the thumb must point in the direction of *equivalent motion of* the conductor, that is, in a direction opposite to that in which the flux is moving.

428. A Rule for the Determination of the Direction of an E.m.f. Induced in a Coil is: Look through the coil in the direction of the lines of force, that is, look along the lines of force from the N to the S pole; then a decrease in the flux enclosed by the coil induces an e.m.f. in a clockwise direction around the coil. Conversely, an increase in the flux enclosed by the coil induces an e.m.f. in a counterclockwise direction around the coil.

NOTE.—This rule follows from the hand rule already given in Art. 427, as will be shown. Refer to Fig. 211 and note the direction of the e.m.f. induced in the length of conductor LM. First, assume LM to be part of coil $LMOP$; then, if this coil is moved up (in direction AB) it will enclose additional flux and the e.m.f. induced in it will be in a counterclockwise direc-

tion. That is, in section LM it will be from right to left, which is also the same e.m.f. direction as that indicated by the hand rule for an upward movement of LM.

Now if it be assumed that LM is a part of coil $QRLM$, then if this coil is moved upward (in direction AB) the flux enclosed by it will be decreased,

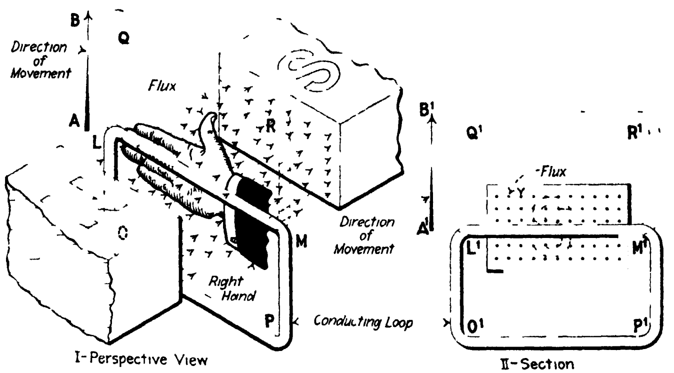


FIG. 211—Showing how the right hand rule is consistent with the "increase or decrease of flux" law

inducing an e.m.f. in it in a clockwise direction, which again will be from right to left in LM. Thus, the "change-of-flux" rule is in agreement with the "hand" rule. If a coil is moved in a direction parallel to the direction of flux, or if it is moved parallel to itself in a *uniform* magnetic field, the flux enclosed by it does not, obviously, change. Hence, under these conditions, no e m.f. is induced.

NOTE.—There is another "right-hand" rule, illustrated in Fig. 212, which can frequently be more conveniently applied than that just recited. It is: Extend the fingers of the right hand along the conductor so that: (*a*) the thumb points in the direction of motion of the conductor and (*b*) the palm receives the flux lines; then the fingers point in the direction of the induced e.m.f.

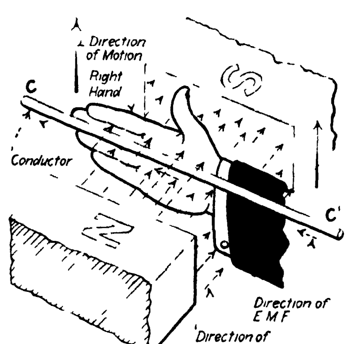


FIG. 212.—Illustrating application of second "right-hand" rule.

429. An Alternating E.m.f. (Art. 680) could be readily produced by continuously oscillating the conductor of Fig. 208 back and forth between positions AA' and CC'. That is, the e.m.f. thus induced would alternate in direction. It would first tend to impel a current in one direction—and then in the other. If, while being oscillated, the conductor BB were connected into a

closed circuit, it would force an *alternating current* (Art. 678) through the circuit. The contents of this paragraph suggest, in a rough way, the principle of operation of the alternating-current generator.

430. When an Induced E.m.f. Forces Current Through a Circuit.—If a conductor that is moved in a magnetic field so as to cut flux, as in Fig. 209,*II*, is connected to a closed circuit, the e.m.f. induced in it (as described in Art. 422) will force current through the circuit. The intensity of the current thus produced will be determined by the resistance of the circuit and the e.m.f. or voltage induced in the conductor. The value of this current can be readily determined by Ohm's law (Art. 134) if the voltage induced is known, thus:

EXAMPLE.—If a conductor moved in a magnetic field generates an e.m.f. of 12 volts and is connected in series in a circuit having a resistance of 6 ohms

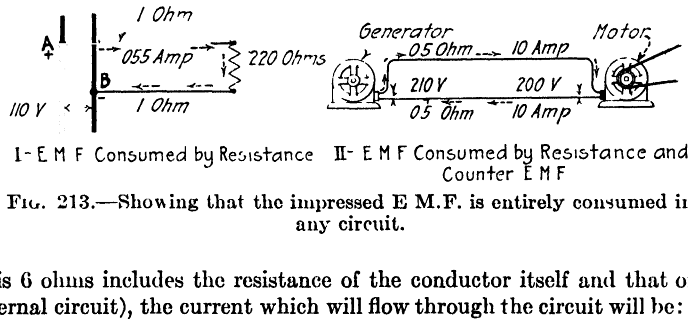


FIG. 213.—Showing that the impressed E M.F. is entirely consumed in any circuit.

(this 6 ohms includes the resistance of the conductor itself and that of the external circuit), the current which will flow through the circuit will be: $I = E \div R = 12\ volts \div 6\ ohms = 2\ amp.$ This current of 2 amp. will flow through the conductor and the external circuit, inasmuch as they are connected in series.

431. The E.m.f. Impressed on Any Electric Circuit must at every instant be entirely consumed in resistance, in counter e.m.f. or in both. The sum of the forces must at every instant equal zero.

EXAMPLE.—In Fig. 213,*I*, 110 volts is impressed on the circuit. By Ohm's Law (Art. 134): $I = E \div R = 110 \div (1 + 1 + 200) = 0.55\ amp.$ flows. But again, by Ohm's law, the resistance drop in the circuit is: $E = I \times R = 0.55 \times 202 = 110\ volts.$ A voltage or e.m.f. of 110 is impressed and 110 volts is consumed, which verifies the above-stated law.

In Fig. 213, *II*, the generator impresses 210 volts on the circuit. The voltage drop or the volts consumed in the line conductors is: $E = I \times R = 10 \times (0.5 + 0.5) = 10\ volts.$ This leaves a voltage of: $210 - 10 = 200\ volts$ available at the motor terminals. This 200 volts is entirely consumed by

the counter e.m.f. (Art. 631) and the internal resistance of the motor. Thus, the entire 210 volts impressed on the circuit is consumed in the circuit.

432. No Force Is Required to Induce an E.m.f. in a Conductor Unless the E.m.f. Propagates a Current, that is, unless current flows in the conductor. *No force is required to move a conductor (of a non-magnetic material) through a magnetic field unless the conductor comprises part of a closed circuit. In other words, an e.m.f. *can* be induced in a conductor without the expenditure of energy. But if the conductor is connected in a circuit so that the e.m.f. generated in it by induction can force a current through itself and the circuit, then (when a current is established) force is necessary to move the conductor and work must be expended in moving it. Why this is true will be explained in following articles.

EXAMPLE.—No force or work would be necessary to move the conductor of Fig. 208 through the flux there shown, because the e.m.f. induced in the conductor could not, the switches being open, originate a current—except infinitesimal internal eddy currents (Art. 504). An e.m.f. would be induced in the conductor when it sheared the flux but as the conductor is not connected in a closed circuit this e.m.f. could not impel a current, current (displacement current Art. 105 excepted) in an open circuit being an impossibility.

However, if the switches in Fig. 208 were closed, an e.m.f. induced in the conductor by moving it across the flux could impel or originate a current, and then, with current flowing through the conductor, force would be required to push the conductor through the flux lines. Similarly, it would require force to push the conductor of Fig. 209,*II* through the flux there shown because the conductor is connected in a closed circuit. As soon as the conductor (Fig. 209,*II*) is pushed through the flux lines, it cuts them and an e.m.f. is induced in it. Immediately, this e.m.f. impels a current through the circuit.

433. There Is No Reason Why Force Should Be Required to Move an Isolated Conductor of Copper or Other Non-magnetic Material in a Magnetic Field† because there is no attraction or repulsion (Art. 65) between non-magnetic substances (which are not carrying current) and magnetic poles. It is a fact, readily verified by experiment, that no force is necessary to move a

*NOTE.—This statement is not strictly true because eddy currents (Art. 504) are induced in any conductor when it is so moved in a field as to cut lines and a small force is required to produce these currents. However, the eddy currents are always very small relatively and the force required for their production is correspondingly small. Hence eddy currents and the force required to produce them will be disregarded in the present discussion.

†NOTE.—See "eddy-current" footnote under Art. 432.

piece of copper wire—not connected in a closed circuit—through the strongest field obtainable.

434. The Reason Why Force Is Required to Move a Conductor Generating an Induced E.m.f. and a Current Through the Flux, whereby the e.m.f. is induced, is suggested in Fig. 214. Around every conductor carrying a current, a circular cylindrical flux of lines of force establishes (Art. 204). When such a conductor (one carrying current) is pushed across, that is, through a flux, the circular lines around the conductor combine with the lines of the flux on one side of the conductor—see the illustration. On the other side of the conductor, the circular force lines of the conductor and the lines of the flux oppose each other. This tends to crowd the lines of the flux to one side of the conductor as shown and to distort them. The lines of force are like taut rubber bands (Art. 56) and the distorted ones tend to straighten. Hence they exert a force tending to expel the conductor from the field. This expelling force must be overcome if the conductor is to be moved through the flux. If the current-carrying conductor so lies in the field that the conductor assumes a position parallel to the direction of the field, there is no force tending to expel the conductor.

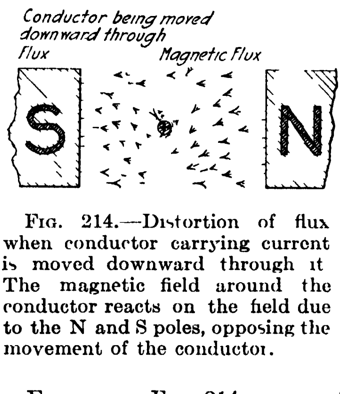


FIG. 214.—Distortion of flux when conductor carrying current is moved downward through it The magnetic field around the conductor reacts on the field due to the N and S poles, opposing the movement of the conductor.

EXAMPLE.—Fig. 214 represents conditions when the conductor of Fig 208 is pushed down through the flux, cutting lines of force. As it shears through the lines, an e.m.f. is induced in it and the e.m.f. is in such a direction, Art. 427, that (it is now assumed that the conductor is connected in a closed circuit) the current impelled thereby is away from the reader. This current produces a field around the conductor as shown (see hand rule, Art. 427).

The lines of the magnet's field and those of the conductor's field combine as indicated. The bent or distorted lines tend to straighten and to push the conductor upward—out of the field. Hence, force must be exerted to push it downward to make it shear through the lines of the flux and generate an e.m.f. Obviously, if movement of the conductor is stopped, no lines are cut by it, no e.m.f. is induced, no current flows and there is no force tending to push the conductor from the field.

435. Lenz's Law: Electromagnetically Induced Currents Always Have Such a Direction That the Action of the Magnetic

Fields Set Up by Them Tends to Stop the Motion Which Produces Them.—In other words, the direction of an induced current is always such that it tends to stop the motion which produces it. Fig. 214 and the text accompanying it indicate one proof of the truth of this law. Lenz's law also accounts for the condition described in following articles: *That energy must always be expended to produce an electric current by electromagnetic induction but that, theoretically, no energy (practically, very little) is necessary to induce merely an e.m.f.* Any induced counter e.m.f.—or current—is always in such a direction as to oppose any change in the current producing it.

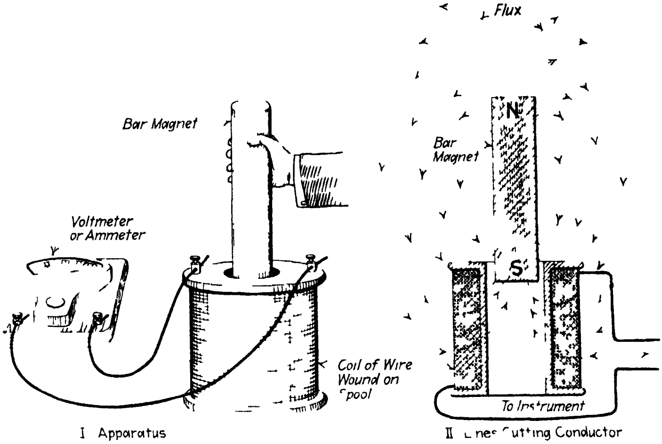


FIG. 215.—Flux of a moving bar magnet cutting conductor of stationary coil and inducing therein an e.m.f.

EXAMPLE.—When the permanent magnet shown in Fig. 215 is thrust into the coil of wire there shown, an e.m.f. is induced in the coil. This e.m.f. forces a current through the circuit connected to the coil. It will be noted if careful observations are made, that no force is required to thrust the magnet into the coil if the external circuit connected to the coil is open.

But if this external circuit is closed, force—work—will then be necessary to push the magnet into the coil. The reason is this: When the circuit is closed, current is forced through the coil by the e.m.f. induced when the bar magnet is pushed into the coil. The direction of the current (by Lenz's law above and by the hand rule of Art. 427) is such that when the bar magnet is thrust into the coil, the upper end of the coil has, due to the flow of the induced current, the same polarity as the lower end of the magnet. They tend to repel each other magnetically, hence force is necessary to push down the magnet.

When the magnet is being pulled out of the coil, the induced current through the coil is in the opposite direction and the upper end of the coil

now has a polarity opposite to that of the lower end of the magnet. They attract each other magnetically. This attraction tends to prevent the withdrawal of the bar magent. Force—work—must be exerted to withdraw it. Thus Lenz's law is again verified.

When the circuit connected to the coil is open, no current can flow through the coil. Hence, its ends can not develop polarities and the conditions described above can not occur.

435A. An Induced Current Is Always in Such a Direction That Its Field Opposes Any Change in the Existing Field is another (and an important one) form of Lenz's law.

436. What Determines the Power Necessary to Produce an Induced Current Through the Agency of Magnetic Induction.—From the preceding it is evident that when no current is impelled by an induced e.m.f. no work—or energy—is required in inducing the e.m.f. That is, unless energy is generated by the e.m.f. no energy is necessary to produce it. Similarly, when an induced e.m.f. does impel a current, energy is required to produce the e.m.f. The amount of power (Art. 159) required to maintain a given e.m.f. is proportional to the current propagated by the e.m.f.

The current—the number of amperes—propagated is determined by the resistance of the circuit through which the current is forced by the e.m.f. (Ohm's law, Art. 134). The greater the resistance of a circuit, the less the current, and *vice versa*—assuming that the e.m.f. is constant. It follows then that the *resistance—or opposition—of a circuit* through which any e.m.f. generator impels current determines the power required by or consumed by the generator for impelling the current. The power required to drive or the power taken by any generator of a certain e.m.f. at any instant is, then, determined by the characteristics—resistance—at that instant, of the circuit through which the generator forces current, rather than by the characteristics of the generator itself.

NOTE.—If a generator impresses an e.m.f. on a circuit having such low resistance that a current, larger than the internal conductors of the generator can carry without excessive heating (Art. 609), will be forced through the circuit and generator, the generator conductors will heat excessively and the machine may be ruined. Hence, generators for developing large amounts of power must have internal conductors of large cross-sectional area to carry the correspondingly large currents without excessive heating. Generators for developing small amounts of power will, in general, carry relatively small currents; therefore, the cross-sectional areas of their internal conductors may be proportionately small.

EXAMPLE.—What power will be required to drive a generator developing at its terminals a pressure of 110 volts, if this e.m.f. is impressed on an external circuit having a resistance of 2 ohms? What power if the resistance of the external circuit is 0.05 ohm? SOLUTION.—From Ohm's law (Art. 134) the current is 110 volts. ÷ 2 ohms = 55 amp. Since $P = E \times I$ (Art. 164), the power required will be: $110 \times 55 = 6{,}050$ *watts* = 6.05 *kw.* The horse power will be (Art. 160) $6.05 \div 0.746 = 8.1$ *h.p.*

With a resistance of 0.05 ohm: $I = E \div R = 110 \div 0.05 = 2{,}200$ *amp.* $P = E \times I = 110 \times 2{,}200 = 242{,}000$ *watts* = 242 *kw.* *H.p.* = $242 \div 0.746 = 325$ *h.p.*

Both of the above examples could have been solved directly by using formula (25) of Art. 134, thus: $P = E^2 \div R = (110 \times 110) \div 2 = 6{,}050$ *watts.* Also, $P = (110 \times 110) \div 0.05 = 242{,}000$ *watts.*

437. How Electrical Energy Is Generated by Electromagnetic Induction.—Frequently the statement is made that: "electrical energy is generated by electromagnetic induction." The real meaning of this is that the mechanical energy, necessary to force the inductor conductor to shear through the flux (or the flux to shear through the conductor) to produce an induced current, is transformed into electrical energy. This is accomplished through the agency of the phenomena which is called electromagnetic induction.

438. The Intensity of the E.m.f. Induced Depends on the Rate of Cutting Lines of Force.—That is, the greater the number of lines cut by a conductor in a given time, the greater will be the e.m.f. induced in the conductor. The intensity of the induced e.m.f. does *not* depend on the total number of lines cut in an indefinite time. Any means whereby the *rate of cutting* lines is increased will therefore correspondingly increase the e.m.f. induced. Note that **rate** is the important word in this paragraph.

EXAMPLE.—To induce 1 volt, lines must be cut at the rate of 100,000,000 (10^8) per sec. (Arts. 226 and 444).

439. Factors Affecting the Value of the E.m.f. Induced in a Conductor When It Cuts Lines.—Since the value of the e.m.f. or voltage induced in a conductor is determined by the rate of cutting, that is, by the number of lines cut per second, it is evident that there are several factors each of which in a measure, determines the value of the e.m.f. induced. These factors may be specified thus: (1) *Speed with which conductor moves through flux or with which flux moves through conductor.* (2) *Strength of the field through which the conductor cuts.* (3) *Angle of direction of conductor with respect to direction of field.* (4) *Length of con-*

ductor which cuts lines. Each of these factors will be discussed in a following article.

440. The Speed with Which a Conductor Cuts Lines Affects the Value of the E.m.f. Induced.—The greater the number of lines cut in a second by a conductor, the greater will be the e.m.f. induced in the conductor. Obviously, the faster the inductor conductor moves through a field, the greater will be the number

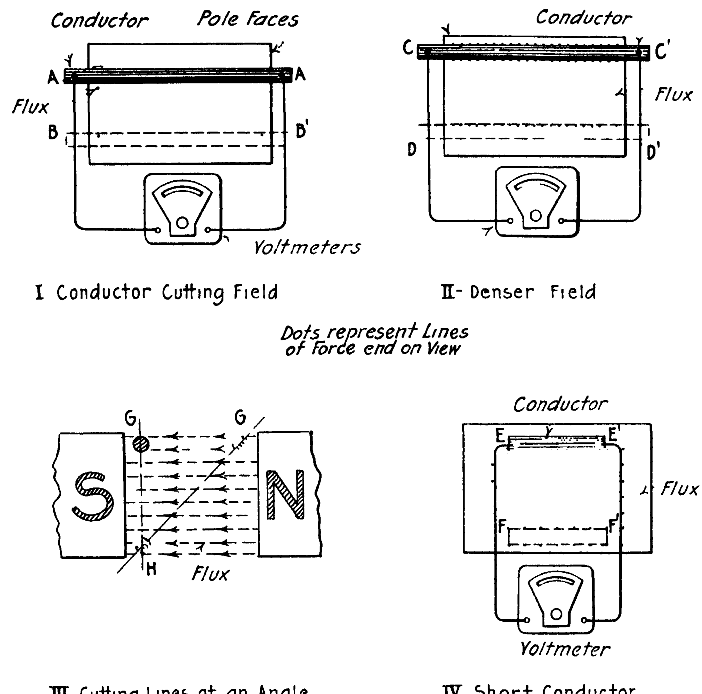


Fig. 216.—Illustrating the factors affecting the value of an induced e.m.f.

of lines cut per second—therefore the greater the voltage, and conversely. This same idea can be expressed thus: The greater the rate of movement of the conductor and magnetic field with respect to each other, the greater the induced e.m.f.—and *vice versa.*

Example.—If the conductor *AA'* (Fig. 216,*I*) be moved down, through the field shown to position *BB'* in 1 sec., a certain number of lines will be cut *per second* and a certain e.m.f., say for example 4 volts, will be induced in it. If, however, it be moved from *AA'* to *BB'* in 0.5 sec., that is with twice the

speed, twice the number of lines will be cut *per second* and twice the e.m.f., or 8 volts, will be induced in it. If it be moved from AA' to BB' with half the speed, that is in 2 sec., one-half the number of lines will be cut *per second* and one-half the e.m.f., or 2 volts, will be induced.

441. The Density of the Field Which the Conductor Cuts Affects the Value of the E.m.f. induced in the conductor. The density, B, of a field can be expressed by the number of lines per square inch in the field (Art. 246). It is obvious then that more lines of force thread through a strong field than through a weak one. It follows that a greater e.m.f. will be induced in a conductor forced in a given time through a strong field than through a weak one.

EXAMPLE.—If the conductor of Fig. 216,*I* be pushed down through the field there shown from position AA' to position BB' in, say, 1 sec., it will cut through the lines of the field and there will be induced in it an e.m.f. of say 4 volts. If now, however, the conductor be forced down through the field of *II*, which is four times as dense (that is, it has four times as many lines) from CC' to DD' in 1 sec., four times as many lines will be cut. Then, the number of lines cut *per second* will be four times as great and if the e.m.f. induced in *I* were 4 volts, that induced in *II* will be: $4 \times 4 = 16$ *volts.*

442. The Direction of Motion of the Cutting Conductor with Respect to the Direction of the Field Affects the Value of the Induced E.m.f. If a conductor being moved with uniform speed cuts through the lines of a flux or field at right angles to the lines it will cut more of them in a second—or in any other unit of time—than if it cuts through the lines at an angle. Hence the maximum e.m.f. is generated in any field in a given time when the conductor cuts through the lines at right angles.

EXAMPLE.—If the conductor of Fig. 216,*III* were moved with uniform speed from G to H it would cut at right angles a certain number of lines. Assume that it moved from G to H in 1 sec. and that the e.m.f. induced in it was 4 volts. If now it were moved at an angle from G' to H, at the same speed as before, it would cut the same number of lines but since it has moved a greater distance (but at the same speed) it has required a longer time, say 1.25 sec., to cut this same number of lines. Therefore the e.m.f. induced would be smaller than in the first case.

If the e.m.f. induced in the first case were 4 volts, that in the second case would be: $4 \div 1.25 = 3.2$ *volts.* The field in both cases is assumed to be the same. The number of lines cut *per second* is smaller in the second case than in the first, hence the e.m.f. in the second case is the smaller.

443. The Length of the Conductor Which Cuts Lines Affects the Value of the Induced E.m.f.—The longer the conductor being moved in a field, the more lines it will cut per second, provided

of course that the cutting conductor always lies wholly within the field. Therefore, the longer the conductor, the greater the e.m.f. induced and *vice versa*.

EXAMPLE.—If the conductor of Fig. 216,*I* were moved from *AA'* to *BB'* in 1 sec., and the e.m.f. induced in it were 4 volts, the e.m.f. induced in the conductor of *IV* in moving from *EE'* to *FF'*, in 1 sec. would be only 2 volts. The field of *IV* is the same as that of *I*. The voltage induced in *IV* is only one-half of that induced in *I* because the length, in the field, of the conductor of *IV* is only one-half of that of *I*. Hence, the number of lines cut per second in *IV* is only half of that cut in *I* and the induced e.m.f. is decreased accordingly.

444. To Determine the Value of an Induced E.m.f.—To induce an e.m.f. of 1 volt in a conductor, flux must be cut by the conductor at the rate of *one hundred million* (100,000,000) *lines per second*. This follows from the definition of a flux line, Art. 226. If flux is cut by a conductor at the rate of 200,000,000 lines per sec., the e.m.f. induced in the conductor will be 2 volts, and so on.

NOTE.—The value "100,000,000 lines" is frequently stated as 10^8 lines which means exactly the same thing. This method of expression is termed *index notation*. The expression $10^2 = 10 \times 10 = 100$, likewise $10^3 = 1{,}000$. Similarly, 10^8 is a short hand method of expressing the value 10 multiplied by itself eight times, or 100,000,000.

The above suggested law can be stated as a formula thus:

(99*a*) *E.m.f. induced* = *flux lines cut per second* ÷ 100,000,000

or

$$E = \frac{\phi}{100{,}000{,}000 \times t} \quad \text{(volts)} \tag{100}$$

and

$$t = \frac{\phi}{10^8 \times E} \quad \text{(sec.)} \tag{101}$$

and

$$\phi = 10^8 \times t \times E \quad \text{(lines)} \tag{102}$$

Wherein E = average e.m.f., in volts, induced in conductor. ϕ = total number of lines of force or the total flux cut by the conductor. t = the time, in seconds, consumed while the conductor cuts the flux ϕ. (If the rate at which the conductor cuts flux is uniform or the same at all times, the induced e.m.f. will be correspondingly uniform or constant. If the rate of cutting varies from instant to instant, the induced e.m.f. will also vary from instant to instant).

EXAMPLE.—If the flux between the N and the S pole faces of Fig. 208 is 800,000,000 lines and the conductor is moved from position AA' to position CC' in 1.5 sec., what e.m.f. will be induced in the conductor? SOLUTION.—Substitute in the formula (100):

$$(a) \qquad E = \frac{\phi}{100,000,000 \times t} = \frac{800,000,000}{100,000,000 \times 1.5} = \frac{8}{1.5} = 5.3 \textit{ volts.}$$

EXAMPLE.—If it is desired to develop an e.m.f. of 10 volts by passing a conductor through a flux, the conductor cutting through the flux in 0.5 sec., how many lines must there be in the flux? SOLUTION.—Substitute in the formula (102):

$$\phi = 100,000,000 \times t \times E = 100,000,000 \times 0.5 \times 10 = 500,000,000 \textit{ lines.}$$

EXAMPLE.—If a conductor cuts through a flux of 550,000,000 lines, 1,200 times a minute, what e.m.f. will be induced in the conductor? SOLUTION.—If the conductor cuts through the flux 1,200 times a minute, it cuts through: 1,200 ÷ 60 = 20 *times a sec.* Therefore, the flux cut in 1 sec. is: 20 × 550,000,000 = 11,000,000,000 lines. Now substitute in the formula (100):

$$(b) \qquad E = \frac{\phi}{100,000,000 \times t} = \frac{11,000,000,000}{100,000,000 \times 1} = 110 \textit{ volts.}$$

445. A Classification of the Methods of Producing Induced E.m.fs.* may be made as follows: (1) *Stationary flux and moving conductor.* (2) *Moving flux and stationary conductor.* (3) *Stationary conductor and variable flux.* (4) *Variable flux and moving conductor.* Each of these four methods is discussed in following articles. Practically all commercial cases where an e.m.f. is induced by a conductor being cut by lines of force will fall under one of the above classifications.

446. Generation of an Induced E.m.f. with a Stationary Flux and a Moving Conductor.—With this case the exciting m.m.f. (Art. 220) and the flux due to it are stationary and the conductor, in which the e.m.f. is induced, cuts through the flux. The flux may be produced either by a permanent magnet, an electromagnet or it may be that enveloping a conductor through which a current is passing.

EXAMPLES.—Several examples of this method of inducing e.m.fs. are illustrated and described in preceding articles. Figs. 208 and 209 illustrate the principle. Fig. 217 shows another example; if it be assumed that there are 200,000,000 lines in the uniform field and that just 1 sec. is required to move the bar at a uniform rate through them, an e.m.f. of 2 volts (Art. 444) will be generated during that second. Since the resistance of the circuit is: ¼ + ¼ = ½ ohm, the current will be: $I = E \div R = 2 \div 0.5 = 4$ *amp.* The power required to move the conductor (Art. 164) would be: $P = E \times I =$

* Karapetoff, THE ELECTRIC CIRCUIT.

2 *amp.* × 4 *volts* = 8 *watts.* This illustrates the principle of the generator and circuit.

447. Direct-current Generators and Motors Depend for Their Operation on This Principle of a Moving Conductor Cutting a Flux.—The *armatures* of the machines are made to rotate in magnetic fields produced by electromagnets. Conductors, carried on and rotating with the armatures, are thereby forced to cut through these fields and thereby e.m.fs. are induced in the conductors. *Rotary converters* and *homopolar generators* also utilize this principle. Generators are discussed in an elementary way in Art. 509 and motors in Art. 625.

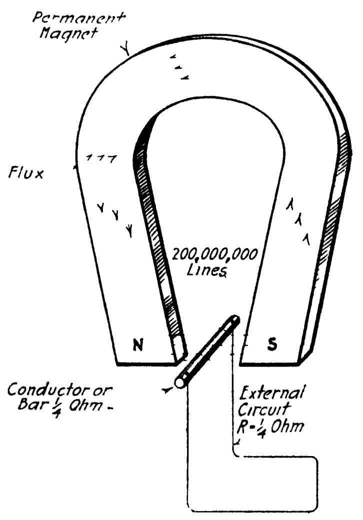


FIG. 217.—Generation of e.m f. with moving conductor and stationary flux.

448. Generation of an Induced E.m.f. with a Stationary Conductor and a Moving Flux (Fig. 218).—With this method, both the exciting m.m.f. and the flux produced by it move. This is

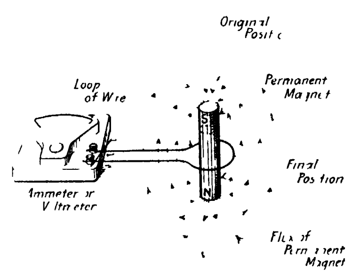


FIG. 218.—Induction of an e m.f. in a loop of wire cut by the flux of a permanent magnet.

merely a variation of the method described in Art. 446. Insofar as the e.m.f. induced is concerned, it makes no difference whether it is the flux or the conductor that moves, so long as the conductor—by some means or other—cuts through the flux. Frequently, this method and that of Art. 446 are treated as being the same method, it merely being necessary for the induction of an e.m.f. that there be a relative movement of the flux and the conductor. Specific examples of e.m.f. induction where the conductor is stationary and the flux moves are:

EXAMPLE.—If the bar magnet shown in Fig. 215 is thrust down into the solenoid—a coil of insulated wire wound on a spool—the lines of force of the

flux of the magnet cut the turns of the coil or conductor on the spool. An e.m.f. is thereby induced (while the magnet is being moved) in the conductor. This e.m.f. forces a current through the closed circuit which includes the ammeter or voltmeter and the coil. This current will cause the ammeter or voltmeter to indicate.

As the magnet is being thrust into the solenoid, the induced e.m.f. and resulting current will be in one direction As the magnet is being withdrawn they will be in the reverse direction—and some force will be required to effect the insertion and the withdrawal of the magnet, all as described in Art. 434. A very strong bar magnet and a solenoid of a large number of turns might be necessary to effect an indication on an ordinary ammeter or voltmeter. But with sufficiently delicate instruments an indication could be obtained readily—by using any bar magnet and solenoid.

448A. The E.m.f. Induced Is Proportional to the Number of Turns in The Solenoid.—If the bar magnet described above were thrust entirely to the bottom of the solenoid, every line of the flux emanating from the N pole of the magnet would cut every turn of the solenoid. Each turn of the solenoid may be considered as and behaves like a separate conductor. However, all of these turns are connected in series, since the conductor constituting the solenoid is wound onto the spool in one continuous length. Therefore, the total e.m.f. induced in the solenoid would be equal to *that induced in each little conductor or turn, by the flux cutting it, multiplied by the number of turns* in the solenoid. This may be expressed in a formula—which is merely another way of denoting the general law of Art. 444, thus:

(103)
$$E = \frac{N \times \phi}{100,000,000 \times t} \qquad \text{(volts)}$$

and

(104)
$$t = \frac{N \times \phi}{E \times 10^8} \qquad \text{(sec.)}$$

or

(105)
$$N = \frac{E \times 10^8 \times t}{\phi} \qquad \text{(sec.)}$$

or

(106)
$$\phi = \frac{E \times t \times 10^8}{N} \qquad \text{(lines)}$$

Wherein E = e.m.f., in volts, induced in the solenoid while the lines of the flux ϕ are cutting the turns of the solenoid. ϕ = number of lines or total flux that cuts the turns. t = the time, in seconds, required to effect the cutting. N = number of turns in the solenoid.

EXAMPLE.—If a flux of 100,000 lines emanates from the N pole of the bar magnet of Fig. 215, what e.m.f. will be induced in the solenoid, which has 400 turns, while the magnet is being thrust to the bottom of the solenoid? It requires 1 sec. to thrust the magnet from above the solenoid to its bottom. It is assumed that all of the flux of the magnet cuts each turn of the solenoid.

SOLUTION.—Substitute in the formula (103):

$$E = \frac{N \times \phi}{100{,}000{,}000 \times t} = \frac{400 \times 100{,}000}{100{,}000{,}000 \times 1} = 0.4 \textit{ volts.}$$

If the above described magnet were drawn from the solenoid in 1 sec., an e.m.f. of 0.4 volts would also be induced, but this e.m.f. would be in the opposite direction. The direction of any such induced e.m.f. can be determined by using the hand rule of Art. 427. The flux developed by a moving solenoid carrying current (Fig. 219) would, provided it had the same number of lines, as the permanent magnet induce an e.m.f. in another

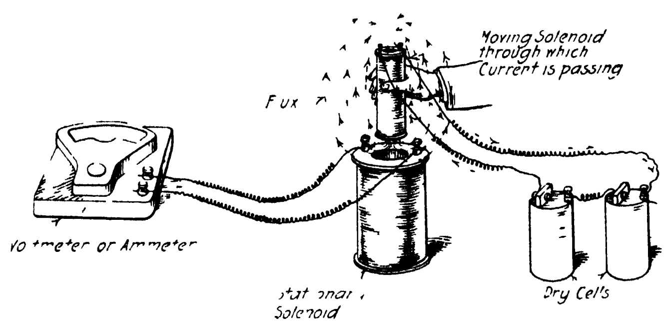


FIG. 219.—Flux of a moving solenoid, through which current is flowing, cutting turns of a stationary solenoid and inducing therein an e.m.f.

stationary solenoid in precisely the same way as does the flux of a permanent magnet:

EXAMPLE.—If a solenoid-carrying current (Fig. 219) be thrust into another stationary solenoid, the flux of the first will cut the turns of the second and an e.m.f. will be induced in the stationary solenoid, precisely as with the bar magnet of Fig. 215. If the moving solenoid had a flux of 100,000 lines, the stationary solenoid 400 turns, and it required 1 sec. to push the first solenoid down into the second, an e.m.f. of 0.4 volts would be induced in the stationary solenoid—the same as in the bar-magnet example above. An e.m.f. of 0.4 volts would also be induced if the first solenoid were removed from the stationary solenoid in 1 sec. The direction of the e.m.f. induced while the moving solenoid was being withdrawn would be opposite to that induced while it was being pushed down.

In either of the above examples, if the bar magnet or the mov-

ing solenoid were turned end for end, so that the S pole of the bar or moving solenoid would first enter the stationary solenoid, the directions of the induced e.m.fs. would be reversed. This follows from the rules of Art. 427.

449. Alternating-current Generators and Synchronous Motors Depend for Their Operation on This Principle of a Moving Flux Cutting a Stationary Conductor, that is, these generators and motors that have *stationary armatures* and *rotating fields.* These machines are discussed in an elementary way in Art. 698.

450. Generators of an Induced E.m.f. with a Stationary Conductor and a Variable Flux.—With this method, the exciting m.m.f. (Art. 220), which produces the flux, and the winding or conductor in which the e.m.f. is induced are stationary relatively to one another. The e.m.f. is induced, as will be explained, by the flux as it varies (that is, as it increases or decreases) and cuts the conductor. The variation of the flux may be effected either: (1) By varying the applied m.m.f. (Art. 220) as in the example given below, in the induction coil (Art. 463) and in the stationary transformer (Art. 821). (2) By varying the reluctance of the magnetic circuit as in the example of Fig. 220 and in the inductor-type alternator.

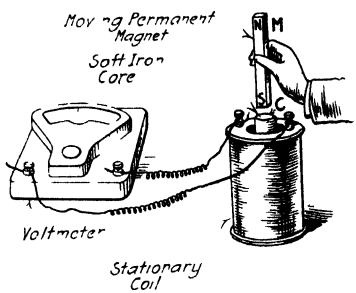


FIG. 220.—Inducing an e.m.f. by changing the reluctance of the magnetic circuit.

EXAMPLE.—An e.m.f. can be induced in circuit A, Fig. 221 (as will be indicated by deflections of the delicate voltmeter V), if circuit B be opened and closed with key K. This e.m.f. in A is induced from B. The voltmeter will deflect only during the instant after K is opened and during the instant after K is closed. There will be no further deflection so long as K remains open or closed, it being assumed that the current from the dry cell is absolutely steady after K has been closed for an instant.

This is because the e.m.f. is induced only during the periods during which there is a change of current—hence a variation of flux—in the inducing circuit B. As will be shown in following articles, the e.m.f. in circuit A is produced because circular lines of force, emanating from and returning to conductor B, cut conductor A while the flux is varying.

451. How Lines of Force of a Variable Flux May Cut a Conductor is explained thus: It was shown in Art. 204 that a magnetic field or flux (circular lines of force) enshrouds, as shown in

Fig. 221,*II*, every conductor through which electricity flows. These circular lines do not surround any conductor which is not carrying a current of electricity. It is the circular lines of such a flux emanating, for example, from *B* (Fig. 221) which, while the flux is varying, cut *A* and induce therein an e.m.f.

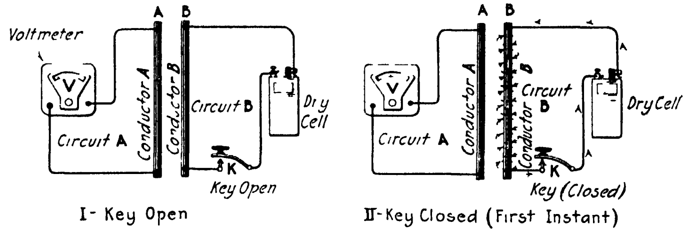


FIG. 221.—Example of the generation of an induced e.m.f. with a stationary conductor and a variable flux.

EXAMPLE.—After *K*, Fig. 221 is closed, it is several instants (by an instant is meant a small fraction of a second) before the current and consequently the flux of circular lines of force surrounding *B* reach their maximum intensities. In the meantime the flux is varying—increasing. In Art. 481 it is explained *why* the current and consequent flux of any conductor can not reach their maximum values instantly. The fact of interest here is that they *do not* attain their maximum values instantly.

In Fig. 221,*II*, the key *K* has just been closed and a few lines of force encircle conductor *B*. Fig. 222,*III*, *IV*, *V* and *VI* show in a rough diagrammatic

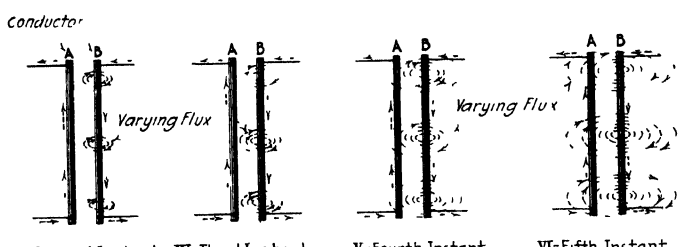


FIG. 222. -Variable flux consisting of circular lines of force cutting conductor A.

way the increasing—varying—numbers of lines of force that might be formed at successive instants as the current in *B* increases. In *VI*, it is assumed that the current and flux has reached its maximum, steady value and will continue at that value until *K* is opened. The maximum number of circular lines of force that the current flowing can generate is, then, shown in *VI*.

As the flux increases—varies— these circular lines of force may be thought of as emanating—expanding outwardly—from the axis of a conductor, in much the same way as concentric smoke rings emanate from the stack of a

locomotive or the ripples emanate from the point where a pebble is dropped into a pond. However, circular lines of force around a current-carrying conductor always form and expand at right angles to the conductor (Fig. 222) and they form very rapidly. It will be noted from Fig. 222 that certain circular lines produced by current in conductor *B* have cut conductor *A*. The lines that cut *A* generate an e m.f. therein in the direction indicated by the arrows (check this direction with the hand rule of Art. 427). In *VI* the flux or number of lines is a maximum. Therefore, there will be no further variation of flux. No additional lines of force will be formed nor will the lines that have been formed move from the positions indicated—until there is a change of current in *B*.

If key *K* be opened, the current in *B* will rapidly but not instantaneously decrease to zero. The flux will vary (decrease) to zero. The circular lines of force will therefore return to the axis of the conductor and vanish. In returning to the conductor some of the lines will cut through *A*. The lines that cut *A* in returning will be those that cut through it, in the reverse direction, when the flux was forming. By again cutting *A*, they again induce an e m.f. in it. But this e.m.f will be in the opposite direction from that induced during the instants while the flux was varying (increasing) after *K* was closed

Hence an e m.f. will be induced in conductor *A* and the voltmeter will be deflected, momentarily, during the instants after *K* is opened or is closed. That is, there will be an e m.f induced while the flux is varying. When there is no change in the current in *B*, there can be no variation in flux; no lines can cut *A* and no e.m f. can be induced in *A*.

452. Induction of an E.m.f. by Varying the Reluctance of the Magnetic Circuit.—This method is, as outlined in Art. 450, merely one of inducing an e.m.f. with a stationary conductor and a variable flux. As the reluctance of the magnetic circuit is varied the flux through it varies accordingly in compliance with the "Ohm's law" of the magnetic circuit given in Art. 231. The m.m.f. is assumed to be constant. As the flux varies, its lines expand or contract—in effect they move—and they then cut the stationary conductor in a manner similar to that described in preceding articles. The following example illustrates the principle:

EXAMPLE.—Consider the apparatus of Fig. 220. The permanent magnet is the source of m.m f. and creates a flux The magnetic circuit of this flux is: (1) Out of the N pole of the magnet. (2) Through the air to the lower end of the soft-iron core *C*. (3) Up through the soft-iron core. (4) Across the air gap from the top end of the core to the S pole of the permanent magnet. (5) Through the permanent magnet to N, completing the magnetic circuit.

Now if *M* be moved closer to *C* the reluctance of this circuit will be decreased. Then the flux will be increased. In increasing the flux expands (varies) occupying more volume, as the additional lines of force form. In expanding some of the lines cut the conductors of the stationary coil and

thereby induce therein an e.m.f. If M is moved away from C the flux is decreased, its volume contracts and an e.m.f. in the reverse direction is induced in the stationary coil.

453. Generation of an Induced E.m.f. with a Variable Flux and a Moving Conductor.—With this method (Fig. 223) a conductor is forced through a pulsating magnetic field. An e.m.f. is then induced in the conductor. *It will be due to two agencies:* (1) The movement of the conductor through the flux induces an e.m.f. (2) The pulsating flux as its lines expand out of and contract into the current-carrying conductors that produce them

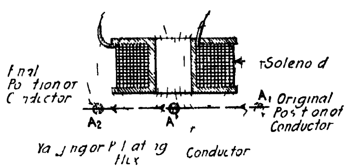


FIG. 223.—Moving conductor in a variable flux.

also effect a cutting of the conductor and induce in it another e.m.f. That is, the aggregate induced e.m.f. is due to a combined "transformer" (Art. 828) and "generator" (Art. 509) action. This action occurs in single-phase motors.

EXAMPLE —Assume that a varying current is flowing through the solenoid of Fig. 223. Then the flux it develops will be variable. That is, it will be continually moving, expanding out of or contracting into the conductors of the solenoid. Now if conductor A be forced through this varying flux from position A_1 to position A_2 it will cut the lines of the flux. Thereby an e.m.f. will be induced in the conductor. Also, the flux lines as *they* move will cut the conductor and induce another e.m f. in it. The total e m f. induced in the conductor will be the sum of these two component e.m.fs.

SECTION 22

MUTUAL INDUCTION

454. Mutual Induction is the electromagnetic induction produced by one circuit in a nearby circuit due to the variable flux of the first circuit cutting the conductor of the second circuit. Figs. 221 and 222 and the text accompanying them describe the principle. "Mutual induction" is merely another name for the phenomenon of the generation of an induced e.m.f. with a stationary conductor and a variable flux (Art. 450). Electrostatic induction is also sometimes referred to as mutual induction but is an entirely different phenomenon from electromagnetic mutual induction.

455. The Principles of Mutual Induction Are Very Important in the study of both direct- and alternating-current circuits. It was stated in Art. 100 that one of the most essential properties of an electric current is its power to generate an induced current in a neighboring circuit by its own variation. It follows, therefore, since there must always be an e.m.f. before there can be current, that an e.m.f. can be generated or induced in a circuit by a change of current in a second circuit not in electrical contact with the first. How such currents are induced is explained in the following articles.

456. Primary and Secondary Coils and Circuits.—These terms are used in discussions of mutual induction phenomena. A *primary* or exciting coil, winding or circuit is the inducer, that is, the one wherefrom the induction emanates. A *secondary* coil, winding or circuit is one under induction, that is, one in which an e.m.f. is induced from a primary coil or circuit.

NOTE.—Often the word *primary* is used as an abbreviation for "primary coil" or "primary circuit." Likewise, the word *secondary* is used as an abbreviation for "secondary coil" or "secondary circuit."

EXAMPLES.—In Fig. 221,*I*, *B* is the primary conductor and *A* the secondary conductor. In Fig. 219 the moving solenoid is the primary coil and the stationary solenoid the secondary coil. See also Fig. 224 showing a primary and a secondary winding.

457. Mutual Induction Between Two Concentric Coils.—As outlined in Art. 454, every case of mutual induction is one where the variable flux of one circuit cuts a conductor of another circuit. An e.m.f. is thereby induced in the second circuit. As will be shown, the phenomenon of mutual induction between two

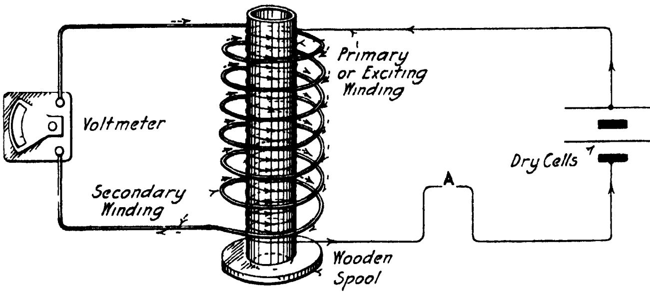


FIG. 224 Diagram of apparatus for showing mutual induction between two concentric coils.

concentric coils (Fig. 225) is due to the same causes as the mutual induction between two parallel conductors, explained in Art. 450. The following example illustrates the mutually inductive action between two concentric coils:

EXAMPLE —Consider the apparatus of Fig. 225 of which Fig. 224 shows a diagram. As shown in Fig 226, there is no electrical connection between the

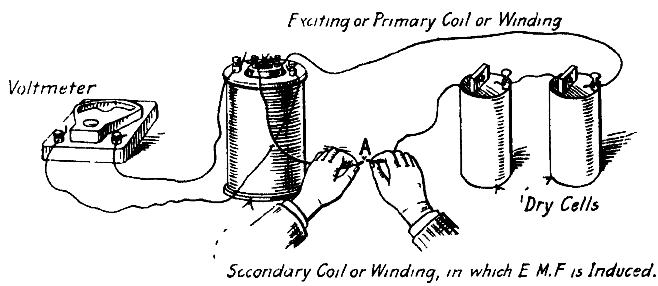


FIG. 225.--Apparatus for showing mutual induction between two concentric coils.

primary and the secondary coils. If the two wire ends be touched together at A, Fig. 225, so as to make electrical connection, a current will be forced, by the e.m.f. of the dry battery, around the primary circuit in the direction indicated by the arrows in Fig. 224.

Furthermore, a momentary current will be induced in the secondary winding, as will be indicated by a momentary deflection of the voltmeter. The

secondary current will be in the opposite direction from the primary current as the arrows indicate. The momentary current continues an instant only and then decays to zero.

If the wires now be disconnected at A, a momentary current will again be induced in the secondary winding. This current too lasts for an instant only and then dies out. When the wires were first touched together the momentary current which flowed in the secondary winding was in a direction opposite to that of the current then being started in the primary winding. When the circuit was opened at A, stopping the current in the primary circuit, momentary current was induced in the secondary in the same direction as that of the primary current which was being stopped.

When the primary circuit was being closed at A and when it was being opened, the induced current in the secondary circuit was in such a direction that it opposed the *change* in current in the primary circuit. The induced current opposed the building up of a current in the primary circuit when the circuit was opened at A. When the *change* in current in the primary circuit ceased, the induced current ceased also.

Now considering the magnetic fields generated by the induced currents in the secondary winding: When current flowed through the primary (Fig. 224) the upper end of the primary coil became a north pole (Art. 48). The induced current in the secondary opposed this production of a north pole at the top end of the coil by trying itself to produce there a south pole (Art. 48). After the field due to the primary current was once established, the induction of an e m f and a current in the secondary ceased. Opening the primary circuit at A tended to kill the field generated by the current from the battery. But the current induced in the secondary circuit by the dying down—varying—of the primary current was in such a direction that it tended to continue the field, that is, to oppose the killing of it, by setting up a field of its own in the same direction.

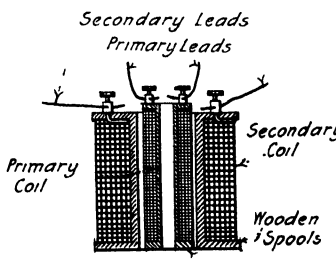


FIG. 226.—Vertical section of two concentric coils.

The above-described phenomena are merely further manifestations of the truth of Lenz's law (Art. 435). How it is that the *change* of current in one coil will induce an e.m.f. in another coil concentric with it is explained in following articles. Explanations of current and electromagnetic reactions above outlined will also be given.

458. Mutual Induction—What Occurs When the Flux of the Primary Coil Cuts the Turns of the Secondary Coil of the two concentric solenoids is diagrammed in Figs. 227 and 228. Fig. 228 shows a diagrammatic section of the two coils of Figs. 224 and 225; the wavy arrows in the top view show the directions of

the e.m.fs. and the currents impelled thereby, as the current is increasing (varying) in the primary. In the longitudinal section, a cross in the sectioned end of a conductor indicates that the current there is going in (Symbols, Fig. 124). A dot indicates that the current is coming out. A discussion of what is happening, just after the primary circuit is closed at A, as the primary current is increasing, is given in the following example:

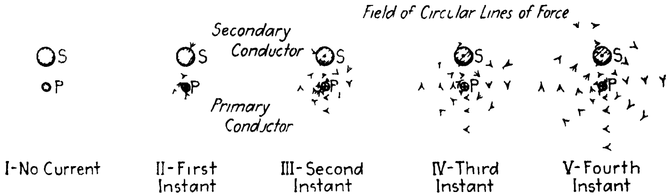


FIG. 227.—Showing how the increasing field around the primary conductor cuts the secondary conductor as the primary current increases.

EXAMPLE.—Since the current flows in the primary winding in the direction shown (Fig. 228) a field will establish around each turn of the primary coil as indicated (hand rule, Art. 427). As the current increases, as outlined Art. 451, the field of circular lines of force will grow in about the same manner as with the straight conductors of Figs. 221 and 222. If any pair of adjacent primary and secondary turns of Fig. 228, as for example S and P, be considered individually, the growth of the field associated with them will be about as diagrammed in Fig. 227.

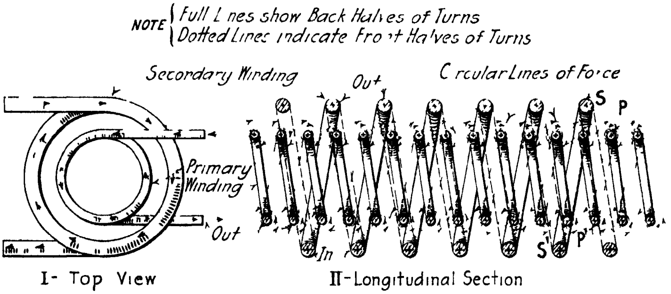


FIG. 228.—Illustrating the formation of circular lines of force around the conductor of the primary coil.

As the circular lines grow or emanate out of P and spread or expand in ever-widening rings, some of these expanding circular lines will cut the secondary conductor S. A larger picture showing how circular lines of force emanate from a conductor is given in Fig. 229. As the force rings (Fig. 227) from P expand upward cutting S, the result is the same as if S were forced down through this flux. In cutting upward through the secondary the

lines induce in it an e.m.f. The direction of this e.m.f. and of the current impelled can be determined, as indicated in Fig. 230, by applying the hand rule of Art. 427. The direction of this e.m.f. and its current is *out* of S. Note that this induced current is in the opposite direction from that of the exciting current in P.

If the hand rule (Art 427) is now applied to the turns on the opposite sides of the coils (for example S' and P' of Fig. 228) it will be found that the exciting current is flowing outward while the induced current is flowing inward Here also, then, the induced e m f in the secondary is in such a

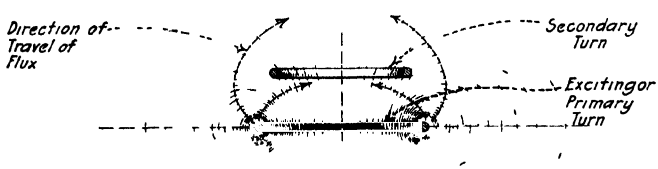


FIG. 229.—Showing how a flux of circular lines of force generated by one conductor cuts another conductor.

direction that it opposes the exciting current in the primary. What occurs with the two turns shown in Fig. 227 happens with all the other turns of the coils. Note that, looking at the ends of the coils (Fig 228,I), the exciting current is in a counterclockwise direction, while the induced current is in a clockwise direction.

Now consider the magnetic polarities produced at the ends of the primary and the secondary coils by the exciting and the induced currents respectively: The exciting or primary current (Fig 231,I) tends, in accordance with the hand rule of Fig. 134 to establish a *north* pole at the upper end of the coil. The induced current tends to establish there a *south* pole. Hence the induced field tends to neutralize or oppose the field due to the exciting current. The induced current tends to oppose any *change* in existing conditions (Lenz's law, Art. 435).

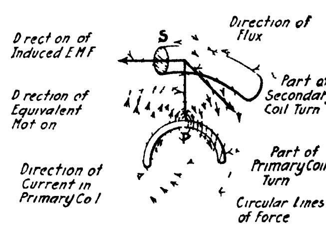


FIG 230—Relative directions of e m.f., flux and motion.

When the exciting current in the primary winding has attained its normal or steady value, the field about its turns also attains a steady value. Then, no further circular lines emanate from the primary turns. Hence, no lines will then cut the secondary turns and no e.m.f. will then be induced in them. Therefore, after the exciting current has reached its steady value the induced current ceases to exist.

459. Now Consider What Occurs When the Primary Circuit Is Opened.—If after the primary current has attained a steady value, the primary circuit is opened, the primary current will

rapidly though not instantaneously decrease to zero. As it does so the following things will happen:

EXAMPLE.—The flux enshrouding the primary turns will gradually contract and disappear into the primary conductor and finally cease to exist, as

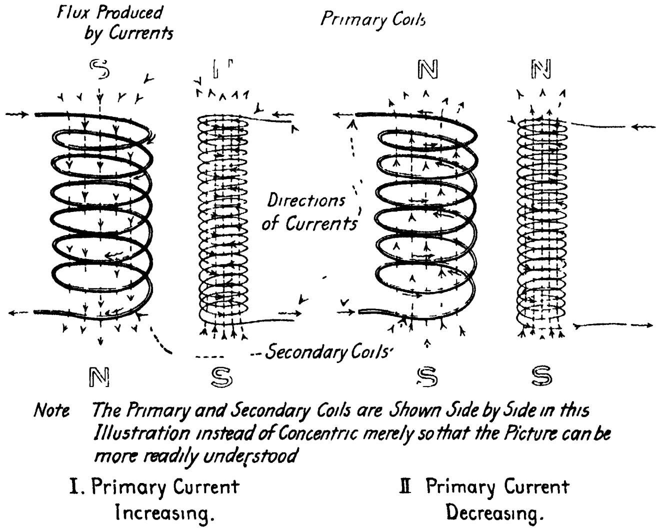


FIG. 231.—Showing polarities of the coils due to the currents therein.

shown in Fig. 232. As they contract, some of these circular lines of force will again cut the secondary turns—but this time in the reverse direction—and again an e.m.f will be induced in the secondary turns. Hence (by the hand

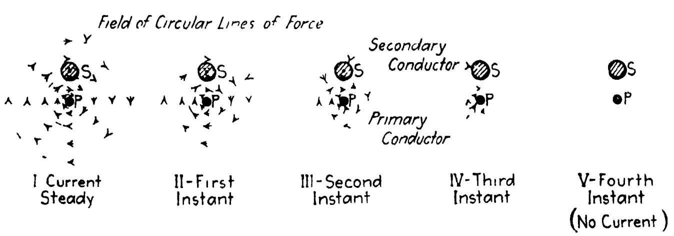


FIG. 232.—Showing how the decreasing field around the primary conductor cuts the secondary conductor as the primary current decreases.

rule of Art. 427), the induced e.m.f. and current in the secondary circuit will now be in the opposite direction to that when the primary current was building up.

Also, the induced current, as shown in Fig. 231,*II* tends to continue the *north* pole already established by the primary current. Again, the induced

current—although it is now in the reverse direction from that first considered (Fig. 231,*I*)—*tends to oppose any change in existing conditions.*

Thus, the principle of mutual induction has been explained and its agreement with the statements of Lenz's law (Art. 435) has been verified.

460. The Mutual Induction Between Two Parallel Conductors is illustrated by Figs. 221 and 222 and the phenomena is described in the article accompanying them. A practical example of mutual induction between parallel wires is given in Art. 451.

461. Circuits Are Mutually Inductive.—Designate one of any pair of mutually inductive circuits as A and the other as B. Then the flux which cuts B, produced by a current of 1 amp. in A, will be the same as the flux which cuts A, produced by a current of 1 amp. in B. Thus the two circuits are mutually inductive.

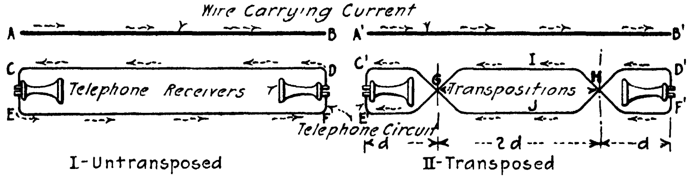


FIG. 233.—Illustrating mutual induction between parallel circuits and showing transposition.

462. Mutual Induction Between Parallel Circuits. Transposition.—A line wire carrying an electric current will induce in wires installed parallel to it (Fig. 233) an e.m.f., as shown in Figs. 221 and 222, when the current in the line wire varies. If the wires in which the e.m.f. is induced constitute a part of a closed circuit, a current will be forced through the circuit. In all cases in practice the e.m.f. thus induced is relatively small. If the inducing wire is far away from the wires in which the e.m.f. is induced, the e.m.f. will be so small as to be imperceptible with even the most delicate instruments.

NOTE.—If the wires are reasonably close together the e.m.f. may be great enough that its effects can be detected and measured and then it may make trouble. The ordinary *telephone receiver* is very delicate—a current of 0.003 to 0.005 amp. will produce an audible click in one. Hence, electric lighting, street railway and power lines, all of which carry relatively heavy currents, frequently induce currents in *telephone circuits* which render the circuits "noisy." Noise in telephone lines may also be due to electrostatic induction as well as to electromagnetic induction. Often the conversation of one telephone circuit may be reproduced in another telephone circuit by virtue of this inductive action. The effect thus produced is then called *cross talk.*

EXAMPLE.—In Fig. 233,*I* the circular lines of force emanating from and returning to the line wire *AB*, as the current in it varies, will cut wires *CD* and *EF* (Art. 451) of the telephone circuit and induce in both an e.m.f. It can be shown (hand rule, Art. 427) that the e.m.f. in both wires will be in the same direction; that is from right to left or from left to right in the picture. However, more lines will cut *CD* than cut *EF*, because *EF* is the more distant. Hence, a greater e.m.f. will be induced in *CD* than in *EF*. This excess of e.m.f. in *CD* will force a current through the telephone circuit in the direction shown by the arrows when the current in *AB* is increasing. When the current in *AB* is decreasing the excess e.m.f. and the current it impels in the telephone circuit will be in a direction the reverse of that indicated. These currents in the telephone circuit may produce *noise* in the receivers.

If now the telephone circuit be transposed as at *II*, the e.m.fs. induced in the telephone circuit, when the current in *A'B'* is increasing, will be in the directions shown by the dotted arrows. But the total em.f. induced in *C'G*, *GJH* and *HD'* will exactly equal the total e.m.f. induced in *E'G*, *GIH* and *HF'*. This is because transposing the telephone wires has the effect of making the average distance between the inducing wire and each side of the telephone circuit the same. Hence, the e.m.f. induced in one telephone line wire will "neutralize" that induced in the other telephone line wire. Then, no current can be impelled and no noise can be produced in the telephone receivers.

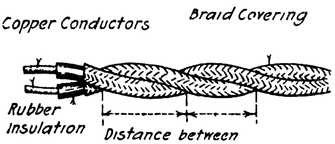


FIG. 234.—Twisted-pair or duplex.

NOTE that the length of the section "2*d*" of the line must be equal to just twice that of sections "*d*" and "*d*" to insure that the e.m.fs. induced in each of the telephone line wires will cancel.

EXAMPLE.—"Twisted-pair" wire (Fig. 234) such as that used for telephone circuits offers a splendid example of transposed wires. Each twist is, in effect, a transposition. Hence, with these conductors, the transpositions are only an inch or two apart and the troubles caused by mutual induction are thereby effectively eliminated.

463. Induction or Ruhmkorff Coils (Fig. 235) are really transformers with an open magnetic circuit, which operate with a pulsating direct current in their primary windings. The induced current in their secondary windings is alternating. Induction coils are used where it is desired to obtain a high secondary voltage of little power-wattage (Art. 159). The induction coil operates by virtue of the principle of mutual induction (Art. 454). The secondary winding consists of many turns of fine wire wound over the primary winding which comprises a few turns of heavy wire. The core is a bundle of iron wires. See "Spark Coils," Art. 468. The principle can be best explained by considering a specific example.

EXAMPLE.—If the switch *S* (Fig. 235) be closed, current will flow in the primary circuit and winding. This establishes a flux through the core within

the primary coil which flux cuts the secondary turns in establishing. Core C then becomes an electromagnet and attracts iron armature A, which breaks or opens the primary circuit at I. As this circuit is broken abruptly the flux returns to its primary turns and vanishes and in so doing cuts all of the secondary turns, inducing in them an e.m.f. This secondary e.m.f. will be much greater than the primary e.m.f. as the number of turns in the secondary winding is greater than the number of turns in the primary winding.

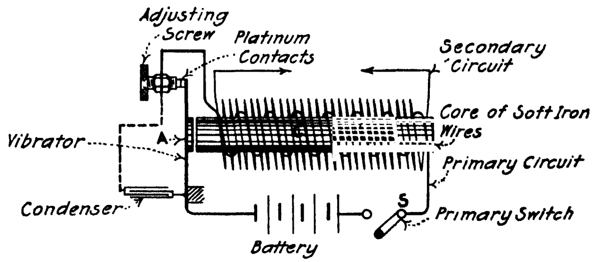


FIG. 235.—Induction or Ruhmkorff coil.

The secondary voltage thus induced is high enough to develop a powerful spark across the air gap or gaps in the secondary circuit.

The vibrator, now that the core is de-magnetized, springs back, due to its resiliency. This closes the primary circuit at the platinum contactors. Now, current again flows in the primary circuit and the cycle of operations just described is repeated as long as the primary switch remains closed. The make-and-break action at the contactors is precisely like that of an electric

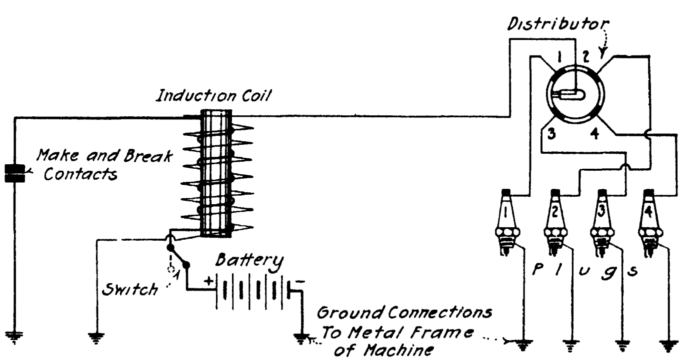


FIG. 236.—Ignition system, using an induction coil, on an automobile.

vibrating bell (Art. 285). The e.m.f. induced as the circuit is opened is much greater than that as the circuit is closed. The condenser decreases the arcing at the make-and-break contact and assures an abrupt breaking of the primary circuit and a consequent rapid contracting of the flux. This (Art. 438) promotes the generation of a high secondary e.m.f. A typical *automobile ignition system* is diagrammed in Fig. 236 which shows how an induction coil is connected into such circuits.

SECTION 23

SELF-INDUCTION

464. Self-induction is that phenomenon, whereby a change in the current in a conductor induces an e.m.f. in the conductor itself. This induced e.m.f. is always in such a direction that it tends to oppose any *change* in the current in the conductor (Lenz's law, Art. 435). The opposing e.m.f. thus produced is called *the counter e.m.f. of self-induction.*

465. Self-induction of a Straight Wire.—Self-induction occurs only when there is a change in current. In Fig. 237 are shown six views of the same conductor cut through. At *I* there is no current. Now current is forced through the conductor and it rapidly, though not instantaneously, attains the maximum, steady

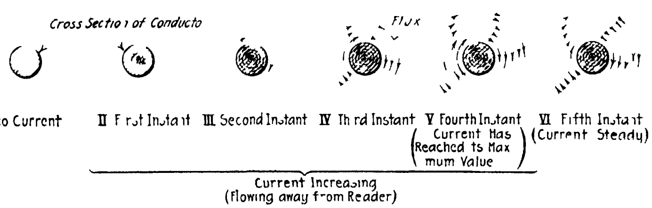


FIG. 237.—Generation of e.m.f. in a conductor by self-induction with increasing current.

rate of flow due to the e.m.f. applied. A flux of circular lines of force is developed around the conductor as the current increases. The circular lines originate at the center of the conductor and expand outwardly as described in Art. 451. At *II, III, IV, V* and *VI* are shown the conditions that might obtain at the ends of 5 successive instants after the switch connecting the source of e.m.f. to the conductor is closed.

As the circular lines of force expand outwardly all of them cut some of the conductor and part of them cut all of the conductor. Thereby a counter e.m.f. is induced in the conductor. At the end of the fourth instant the current has attained its maximum value and no more lines of force are developed. Hence, none cuts the conductor after the end of the fourth instant. The flux will

then remain as diagrammed at *VI* as long as the current is steady. Only a few of the force lines are shown in Fig. 237. Actually there would be many and some would lie at great distances from the conductor.

Now if the circuit of which the conductor of Fig. 237 forms a part is opened, flow of electricity will not cease immediately although it will die out rapidly but not instantaneously. The circular lines of force will then return to the center of the conductor and vanish as diagrammed in Fig. 238. In so doing they will again cut the conductor, again inducing therein an e.m.f. This e.m.f. will be opposite in direction to that induced when the flow was increasing. Every induced e.m.f. is in such a direction as to oppose any *change* in existing conditions (Art. 435).

466. Self-induction in a Coil is produced somewhat as diagrammed in Fig. 239. If an e.m.f. is impressed across the termi-

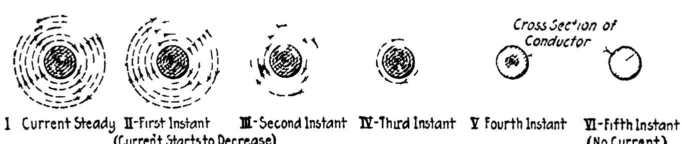


FIG. 238.—Generation of e.m.f. in a conductor by self-induction with decreasing current.

nals of the coil there shown, a current will "build up" in the coil as explained in Art. 458. This current will cause a flux of circular lines of force to emanate from each turn of the coil. These lines will cut the turn which produces them and will also cut all of the other turns of the coil as they expand outwardly. Thus, as the current increases, the expanding lines will induce an e.m.f., in the conductors of the coil, which will be in such a direction (hand rule, Art. 427) that it will oppose the increase of current (Lenz's law, Art. 435). When the switch, connecting the coil terminals to the source of e.m.f., is opened, the current will decrease to zero and the circular lines will then contract into their conductors. In doing this they will again cut—in a reverse direction—the conductors and induce in them an e.m.f., which will be (hand rule, Art 427) in the same direction as that of the now-decreasing current which is producing it. Hence *this* e.m.f. will tend to maintain the current in the coil.

EXAMPLE.—The current, in "building up" in the coil of Fig. 239, reaches its maximum value in the portion of the coil at which it enters before it does in the portions of the coil further away from this first turn. The conditions an instant—a very short interval—after the closing of the circuit are shown in the illustration. The current in the first turn 5_1, develops a flux which cuts 4_1. It induces in 4_1 an e.m.f. opposite in direction to the direction of current flow. The lines emanating from the other turns also cut adjacent turns and also induce in them counter-e.m.f.s.

If the magnetic flux of the entire coil (as in Fig. 133,*I*) be considered, it will be found that the e.m.f. of self-induction tends to set up a flux in opposition to that due to the "rising" or increasing current in the coil. When the current in the coil is decreasing, the e.m.f. then induced tends to set up a flux in the same direction as that due to the now-decreasing current. Note that

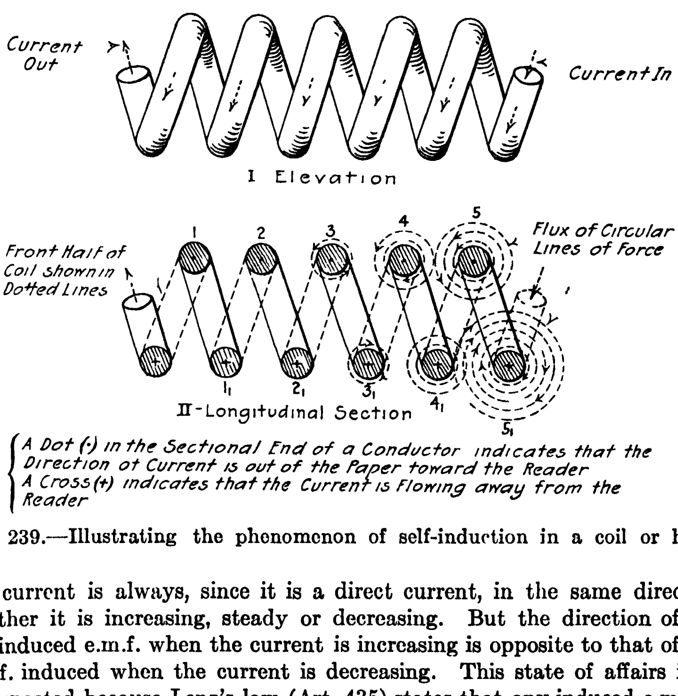


FIG. 239.—Illustrating the phenomenon of self-induction in a coil or helix.

the current is always, since it is a direct current, in the same direction whether it is increasing, steady or decreasing. But the direction of the self-induced e.m.f. when the current is increasing is opposite to that of the e.m.f. induced when the current is decreasing. This state of affairs is to be expected because Lenz's law (Art. 435) states that any induced e.m.f. is always in such a direction as to oppose any change in existing conditions.

467. Elimination of Self-induction.—Self-induction, as explained in Art. 465, is caused by the circular lines of force developed by current in conductors, cutting the conductor. If the formation of these circular lines of force can be prevented, there can be no self-induction. If two conductors lie parallel to one another and the current in each is flowing in an opposite direc-

tion, the field developed about one will oppose the field about the other (Art. 210). Hence, if a circuit is looped back upon itself, as shown in Fig. 240,*I*, the field due to one side, *A*, of the circuit will oppose that due to *B*, the other side. Since the same current flows in *A* as in *B*, the fields around each of these conductors, that the current tends to set up, will be equal and opposite. The result is, that, if the conductors *A* and *B* are very close together, the fields will almost wholly neutralize each other and there will be no appreciable self-induction. The further apart the wires *A* and *B*, the greater the self-induction. Such a circuit is termed a *non-inductive circuit.* Similarly, a *non-inductive coil* can be wound as shown at *II*. The wire with which the coil is wound is looped back upon itself before it is served around the core. Coils so wound, even if on iron cores, have no appreciable self-induction.

EXAMPLES.—Resistance coils used in commercial ammeters and voltmeters and in practically all other electrical measuring instruments are wound non-inductively, as are standard resistance coils. Non-inductive coils are also used to a considerable extent in telephone apparatus where it is desired to obtain resistance without inductance.

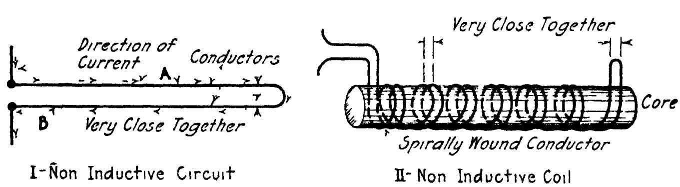


FIG. 240.—Methods of eliminating self induction.

468. Spark Coils (Fig. 241) are merely coils of many turns of insulated copper wire wound on soft iron cores. Due to its self-induction, a spark coil will induce a high e.m.f. in the circuit in which it is connected in series with a battery, at the instant when the circuit is opened. Thus, a spark or arc is produced between the contactors which open the circuit. This spark then ignites the gas in an *electric gas-lighting system* or in the *ignition system of a gas or gasoline engine.* The core of a spark coil is usually a bundle of soft iron wire as this construction decreases the eddy currents (Art. 504) and thereby increases the effectiveness of the coil. For further information concerning spark coils and their applications, see the author's AMERICAN ELECTRICIAN'S HANDBOOK.

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NOTE.—A spark coil which will, when in series with four or five dry cells, develop a spark ample for gas lighting can be constructed by winding about 2 lb. of No. 16 double-cotton-covered wire on a core, 1 in. in diameter and 9 in. long, composed of a bundle of soft iron wire. Induction coils (Art. 463), are now usually applied instead of spark coils forinternal engine ignition.

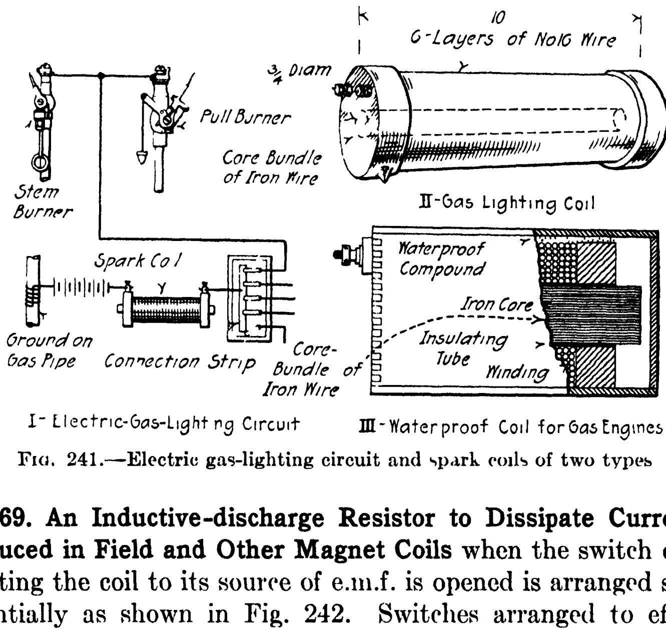

FIG. 241.—Electric gas-lighting circuit and spark coils of two types

469. An Inductive-discharge Resistor to Dissipate Currents Induced in Field and Other Magnet Coils when the switch connecting the coil to its source of e.m.f. is opened is arranged substantially as shown in Fig. 242. Switches arranged to effect

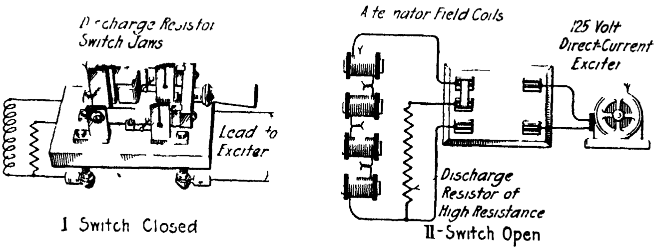

FIG. 242.—Field-discharge switch.

such connections are termed *field-discharge switches* and the resistors used in combination with them are *field-discharge resistors.*

NOTE.—When the field circuit of a generator or that of any powerful magnet (for example, a lifting magnet) is opened, the e.m.f. of self-induction

is high, because the inductance of such a coil is high. If such a circuit is opened instantaneously, the *rate of change of current* (Art. 483) is very high and the induced e.m.f. is correspondingly high—many thousand volts in some cases. Such high e.m.fs. may puncture the insulation between the winding and the frame or core of the coil and thus ground the winding.

However, if as the switch opens, a high resistance is connected across the field coils as at *II*, the current in them dies out rather slowly. The rate of change of current is then low. The e.m.f. of self-induction is correspondingly low so that it can do no damage. This high resistance across the field coil terminals performs somewhat the same function as does the short-circuiting bar across the switch of Fig. 243,*III*. The resistance of a discharge resistor should be determined by the characteristics of the coils that it protects.

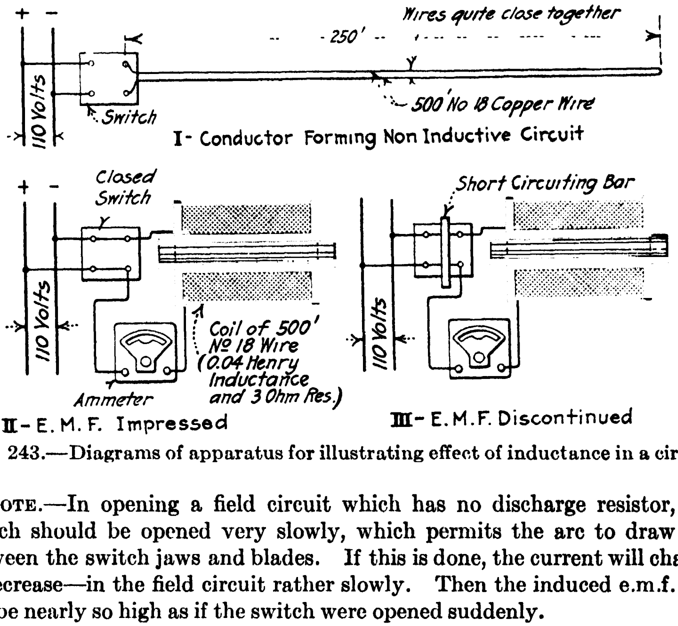

FIG. 243.—Diagrams of apparatus for illustrating effect of inductance in a circuit.

NOTE.—In opening a field circuit which has no discharge resistor, the switch should be opened very slowly, which permits the arc to draw out between the switch jaws and blades. If this is done, the current will change —decrease—in the field circuit rather slowly. Then the induced e.m.f. will not be nearly so high as if the switch were opened suddenly.

470. The Direction of the E.m.f. Induced When a Circuit Is Opened Can Be Shown and its agreement with the rulings of Lenz's law (Art. 435) can be verified with the apparatus diagrammed in Fig. 244, as described in the following example:

EXAMPLE.—When the key is closed, a current flows as at *I*. It divides at *A*, a portion flowing through the galvanometer and a portion through the inductive coil. This causes a deflection of the needle, say, to the right. With one's hand the needle should now be pushed back to the zero position and restrained there with a small block to prevent its turning to the right. If the key is now opened (*II*), a momentary current of self-induction will be

forced around through the galvanometer by the e.m.f. of self-induction developed in the helix. This produces a momentary deflection of the galvanometer needle to the left. This indicates that the current induced in the

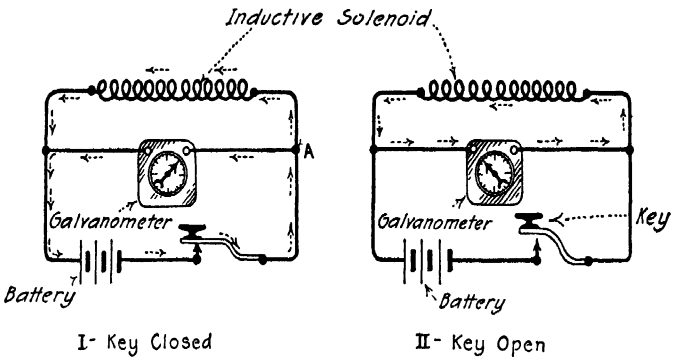

Fig. 244.—Apparatus for demonstrating directions of induced e.m.fs.

helix at the instant the key was opened, was in the same direction as that of the original battery current. The arrows in the diagrams indicate the current directions. A consideration of these will prove the truth of the foregoing statement.

SECTION 24

INDUCTANCE

471. Inductance is the ability of an electric circuit to produce an e.m.f. by electromagnetic induction when the current in the circuit changes or varies. As outlined in Art. 418, an e.m.f. is induced in a conductor when lines of force cut the conductor. When the current in a circuit varies (Fig. 222) the flux of lines of force, due to the current, expands when the current increases and contracts when the current decreases and, in thus moving, the lines will cut any conductor located within their range of action. It is due to this phenomenon, described in detail in Art. 451, that circuits have inductance. The induced e.m.f. is always in such a direction that it opposes the change of current producing it (Lenz's law, Art. 435).

EXAMPLE.—It is due to inductance that an e.m.f. is induced in conductor *A* of Fig. 222 when the current in conductor *B* changes or varies—increases or decreases.

472. The Unit of Inductance Is the "Henry."—It is so called in honor of the American scientist, Joseph Henry, who made important magnetic discoveries. *A circuit has an inductance of* 1 *henry when a current changing at the rate of* 1 *amp. per sec. induces an e.m.f. of* 1 *volt in the circuit.* This induced e.m.f. is always in such a direction as to oppose the force that produces it and to oppose any change in the varying current that is inducing it. These conditions must obtain so that Lenz's law (Art. 435) will be satisfied. As the "henry" is a large unit, the one-thousandth part of it, or the *millihenry,* is most frequently used. The millihenry = $1/1{,}000$ or 10^{-3} henry.

EXAMPLE.—The inductances in henrys of some familiar objects are given in Table 480.

473. The Real Significance of the Unit "the Henry" can best be explained from the consideration of a specific example where the mutual inductance of two adjacent conductors is 1 henry:

EXAMPLE.—Imagine a primary and a secondary conductor arranged as shown in Fig. 245. The number of circular lines of force enshrouding *A*—

or any other conductor-carrying current—is proportional to the current in it. If, then, the current in A is doubled, the number of lines in its flux is doubled. If the current is halved, the number of lines is halved. When there is an increase or decrease in the current in A, there is a corresponding increase (spreading out) or decrease (contracting in) of the number of circular lines in its flux.

Now assume that the proportions of conductors A and B and the distance that they are apart from one another are such (the illustration is not to scale by any means) that when the current in A is changed 1 amp., 100,000,000 lines of force (emanating from A if the current in it is increased or contracting into A if the current in it is decreased) from A will cut B. How it is that these circular lines from A will cut B is explained in Art. 451.

For example, if no current is flowing in A and then a current of 1 amp. is forced through A, there is then a change (increase) in current of 1 amp. Because of the proportions and arrangement of A and B, which is specified above, 100,000,000 lines of force emanating from A will cut B. Conductor A may produce more than 100,000,000 lines when 1 amp. flows in it but only this number cuts B. Likewise if the current in A be changed from 12 amp. to 11 amp., there is a change (decrease, in this case) of 1 amp. in current. Again, 100,000,000 lines —contracting into A—will cut B. *Any* change increase or decrease—of 1 amp. in current in A will cause 100,000,000 lines to cut B.

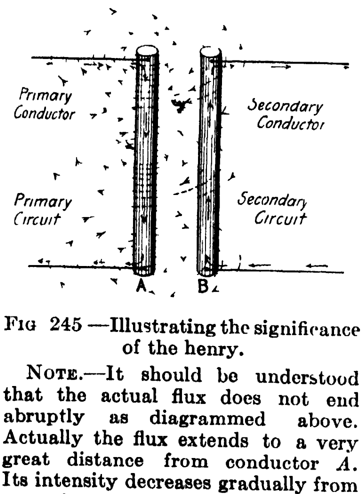

FIG 245 —Illustrating the significance of the henry.

NOTE.—It should be understood that the actual flux does not end abruptly as diagrammed above. Actually the flux extends to a very great distance from conductor A. Its intensity decreases gradually from a maximum at the conductor to zero at a very great distance.

If by using a rheostat or by some other device, the current in A be *gradually and uniformly* changed so that it requires just 1 sec. to effect in A a change of 1 amp., then 100,000,000 lines per sec. will cut B. Therefore (because when lines cut a conductor at the rate of 100,000,000 per sec. an e m f of 1 volt will be induced in the conductor, Arts. 226 and 438), an e.m f of 1 volt will be induced in B.

Hence, it follows from the definition of the henry (Art. 472) that the mutual inductance of conductors A and B is 1 henry—because a change of 1 amp. per sec. in A induces an e.m.f. of 1 volt in B.

474. Inductance Is Proportional to the Number of "Cutting" Lines per Ampere.—If the situation outlined in Art. 418 be considered further it will be noted that the inductance of a circuit or conductor depends wholly on the number of lines developed,

per ampere of current, which cut the conductor or circuit in which the e.m.f. is induced. It is obvious that all of the lines produced by the inducing current, may not cut the conductor in which the e.m.f. is induced. Hence *inductance is numerically proportional to the number of effective or cutting lines per ampere.*

EXAMPLE.—If a current of 1 amp. in a circuit develops 100,000,000 cutting lines, the inductance is 1 henry. If a current of 1 amp. develops 100,000 cutting lines, the inductance is 1 millihenry.

NOTE.—OTHER DEFINITIONS OF INDUCTANCE which are sometimes given, but which when analyzed will be found to have the same meaning as that suggested above are:

1. *Inductance* is equal to the increase in the number of linkages per unit increase in current. When the permeability of the magnetic circuit is constant, the inductance is also constant and is equal to the linkages per unit current (Pender's AMERICAN HANDBOOK FOR ELECTRICAL ENGINEERS).

2. *Inductance* is the total magnetic flux threading the circuit per unit current which flows in the circuit and produces flux.

3. *Inductance* of a circuit is proportional to that value of e.m.f. produced in it by a unit rate of variation of current through it.

4. *Inductance* is the ratio between the total induction through a circuit and the current producing it.

NOTE.—THE SELF-INDUCTANCE OF A CIRCUIT AND ITS PERMEANCE ARE THE SAME THING.—But the unit of inductance, the henry, is a much larger unit than that of permeance, the perm (Art. 238) just as the mile is a larger unit of measure than the inch. Assuming constant permeability, the permeance of a given circuit, in perms, is numerically equal to *the number of lines of flux developed by that circuit when there is a current of* 1 *amp. in it.* Furthermore, the self-inductance of a circuit is numerically equal to *the number of lines of flux developed by that circuit* ÷ 100,000,000, *when there is a current of* 1 *amp. in it* (Art. 473). Hence: *henrys* = *perms* ÷ 100,000,000, and *perms* = *henrys* × 100,000,000. The permeance of a circuit bears no particular relation to the mutual inductance of the circuit with some other circuit unless the permeance of the imaginary magnetic circuit which carries *only that flux which cuts the second circuit* be considered. Self-inductance (not mutual inductance) can then be expressed in either perms or henrys—but the henry is the usual and preferable unit.

475. Inductance Is a Physical Property of Circuits and Conductors.—Inductance is not a concrete thing. It is merely a name or term which signifies a certain arrangement of an electric conductor and a magnetic circuit. The inductance of a circuit, conductor, coil or of any apparatus is a property of that thing just as its resistance is one of its properties. The impressed voltage does not affect the inductance of an object. A conductor has inductance whether current flows in it or not.

The inductance of a circuit or conductor not associated with

iron or steel, never changes. Where there is iron in the magnetic circuit, the inductance may vary a trifle as the current changes (Art. 252) but such variations are so inconsiderable that in practice they usually can be and are disregarded. The inductance of a circuit or conductor depends on the number of lines of cutting flux (Art. 474) that a current of 1 amp. in it will produce.

EXAMPLE.—Table 480 gives the inductances in henrys of some familiar objects.

476. A Certain Conductor May Have Different Inductances.—A given length of a conductor of a certain cross-sectional area and material has a certain definite resistance in ohms. But this conductor may have as many different inductances as there are different shapes and forms into which the conductor can be bent or twisted.

477. Factors That Determine the Inductance of a Circuit or Conductor.—Any expedient that will increase the *cutting lines per ampere* (Art. 474) of a circuit or conductor will increase its inductance. In general, any arrangement that will increase the flux developed by a conductor, with a given current flowing, will increase its inductance. It follows, then, that if iron be associated with a conductor its inductance will, in general, be increased. The reason for this is that iron, because of its great permeance (Art. 239) increases the flux due to a certain current, much above the flux that would exist if the magnetic circuit were comprised wholly of air or any other non-magnetic material.

NOTE.—Why the flux is greater when there is iron in the magnetic circuit is explained in Art. 217.

Furthermore, if a conductor is wound into a coil its flux is increased as described in Art. 214. If iron is so placed as to provide a path for the flux of the coil, thereby increasing the flux, the inductance of the conductor will be still greater. Straight conductors (Art. 495) have relatively little inductance while coils, especially those having iron cores, may have very great inductances as shown in Table 480.

EXAMPLES.—(1) Study the values of Table 480. (2) A coil of, for example, 40 turns wound on an iron core has a much higher inductance than the same coil of 40 turns without an iron core. A coil of 15 turns wound on an iron core has less inductance than a similar coil of 30 turns wound on an iron core.

478. Methods of Determining Self- and Mutual Inductance.—Inductance can be calculated with fair accuracy for certain special cases, examples of which are given in other articles. The self- or the mutual inductance of a conductor or of conductors of non-symmetrical shapes must usually be determined by experiment. Mutual inductance is particularly difficult to predetermine. Nearly all formulas used in practice for computing inductance give approximate results; however, the values obtained by using them are usually accurate enough for ordinary engineering work.

SECTION 25

SELF-INDUCTANCE

479. Self-inductance (Art. 471) is the ability of a circuit to produce an e.m.f. within itself by induction when the current in it changes. The e.m.f. is induced by the process indicated in Art. 471. As the circular lines of force enshrouding the conductor expand out from or contract into the center of the conductor they cut the conductor. Then an e.m.f. is induced in the conductor. This e.m.f. is always in such a direction that it opposes the change of current producing it and therefore is often called the *counter e.m.f. of self-induction.* See Lenz's law, Art. 425. Self-inductance is measured in Henry's (Art. 472).

EXAMPLE.—It is due to the self-inductance of the conductor of Fig. 243 that a counter e.m.f. is induced in the conductor when the current in it changes.

NOTE.—COEFFICIENT OF SELF-INDUCTION is a name, now little used, meaning "inductance." The term "coefficient of self-induction" was formerly applied to a numerical value, in henrys, for inductance.

480. Self-inductances of Some Familiar Objects.*

Object	Resistance in ohms	Inductance in henrys
Coils of electric, vibrating, call-bell	2.5	0 012
Armature of a magneto (telephone) generator:		
Plane of coil in plane of pole pieces	500 0	2 7
Plane of coil perpendicular to plane of pole pieces	500.0	7.3
Bell telephone receiver, with diaphragm	75.0	0.075 to 0.100
Without diaphragm	75.0	0.048 to 0 065
Astatic mirror galvanometer	5,000.0	2.0
Coil of Aryton and Perry spring voltmeter, without iron core. Length of coil, 2.88 in.; external diameter, 3 in.; air core of coil, 0.6 in. in diameter	333.5	1.5
Common Morse telegraph relay:		
Armature against the poles	148.0	10.47
Armature 0.02 in. from the poles	148.0	3.71
Armature in working adjustment	148.0	5.00

* The inductance values tabulated above were determined by test and, for the most part, are taken from Jackson's ALTERNATING CURRENTS AND ALTERNATING-CURRENT MACHINERY, Vol. II, pp. 48 and 49.

Object	Resistance in ohms	Inductance in henrys
Telegraph sounders, armatures 0.004 in. from the poles:		
Bobbin, 1¼ in. by 1 in.	20.0	191.0
Bobbin, 1½ in. by 1¼ in.	20.0	150.0
Single coil of a Morse telegraph sounder: having an iron core 0.31 in. in diameter and 3 in. long; bobbin, 0.94 in. in diameter	32.0	0.094
Complete telegraph sounder with a core like that above but with a bobbin 1.25 in. in diameter	50.0	444.0
Bare Copper Wire, No. 12, B.&.S. or American Wire gage, erected on a pole line 23 ft. from the ground is calculated by Kennelly to measure, per mile	8.5	0.315
No. 6 wire same as above, per mile	2.1	2.95
Secondary winding of an induction coil, capable of giving a 2-in. spark	5,700.0	51.2
Induction coil, 19 in. long, 8 in. diameter:		
Primary winding	0.145	0.013
Secondary winding	30,600.0	2,000.0
Generator field circuits	....	1 to 1,000
Armatures, direct current, between the bushes	....	0.02 to 50.0
Field coils of a 3.5-kw., 110-volt, direct-current generator	44.0	13.6
Armature of above machine	0.215	0.005
Transformer primary and secondary windings, depending on their output and the voltage for which they are designed	....	0.001 to 50.0

481. Self-inductance in a Circuit Prevents the Current from Attaining Its Maximum Value Instantly.—As the current increases, a flux emanates from the conductors of the circuit, cuts them and thereby induces in them a counter e.m.f. as described in Art. 465. Refer to Art. 791 and Figs. 474 and 475 which explain a hydraulic analogy. What occurs and how it occurs can best be understood from the consideration of a specific example:

EXAMPLE.—A piece of No. 18 copper wire 500 ft. long has a resistance of about 3 ohms (Table 157). If this wire be formed into a loop 250 ft. long as shown in Fig. 243,*I*, it will have practically no inductance (Art. 471). If now the switch be closed impressing 110 volts (direct current) across the terminals of the wire, a current of: $I = E \div R = 110 \div 3 = 36.7$ *amp.*

will immediately flow in the circuit. The current will probably have reached its maximum steady value of 36.7 amp. 0.001 sec. afr the switch is closed, because this loop circuit has practically no inductance.teAfter having attained its steady value of 36.7 amp. it will remain at this value until the switch is opened or the circuit otherwise disturbed.

If this same 500 ft. of No. 18 wire is now wound into a coil (Fig. 243,*II*) of certain proportions it will have an inductance of 0.04 henry. The arrangement and dimensions of the coil were so selected that it would have this inductance. Now, if a pressure of 110 volts (direct current) be impressed on this coil, the steady-current value of 36.7 amp. will not be attaiend immediately. Obviously, the steady current through the coil will be the same as that through the looped wire of *I*, because both have the same—500 ohms—resistance. But an appreciable time interval will elapse after the switch is closed before the current attains this steady value of 36.7 amp. If readings of the ammeter shown in *II* could be taken at 0.01-sec. intervals after the closing of the switch they would be about as tabulated in *Column C* of Table 482. Fig. 224 shows a graphic statement of these values.

Immediately after the switch is closed, the 110-volt e.m.f. starts to force a current through the coil but this current, because of the self-inductance of the coil, at once induces in the conductors of the coil a momentary counter e.m.f., as explained in Art. 465. The intensity of this instantaneous counter e.m.f. is proportional to the *rate* at which the current in the coil is *changing* at that instant (Art. 438). The e.m.f. which at any instant actually forces current through the circuit—which will be called here the impelling e.m.f.—is the difference between the counter e.m.f. of self-induction and the impressed e.m.f. The impressed e.m.f. pushes one way, as it were; the counter e.m.f. the other. Current is then forced in the direction of that of the stronger e.m.f.

For example, consider the conditions affecting the circuit of Fig. 243,*II* (Table 482 and Fig. 246), 0.02 sec. after the switch is closed. The current is then (*column C*) 28 amp., this value having been obtained by experiment. Now, if 28 amp. flow in a circuit of 3 ohms resistance, the e.m.f. that is forcing it through, that is the *impelling e.m.f.*, is: $E = I \times R = 28 \times 3 =$ 84 *volts*.

But 110 volts is impressed on the circuit. Hence the counter e.m.f. of self-induction at that instant is: $110 - 84 = 26$ *volts*. The counter e.m.f. in this circuit 0.02 sec. after the closing of the switch is then 25 volts (*column F*). Similar values have been worked out in Table 482 indicating numerically the conditions in this circuit as the current rises to its maximum value. Note that the counter e.m.f. becomes less and less at the end of each hundredth-of-a-second interval. This shows that the current increases more slowly as it approaches its steady value of 36.7 amp.

482. Table Showing How the Current in an Inductive Circuit Increases after an E.m.f. Is Applied to the Circuit.

These values relate to the circuit diagrammed in Fig. 243,*II*. It has 3 ohms resistance, 0.04 henry inductance and the impressed e.m.f. is 110 volts. Fig. 246 shows these values graphically.

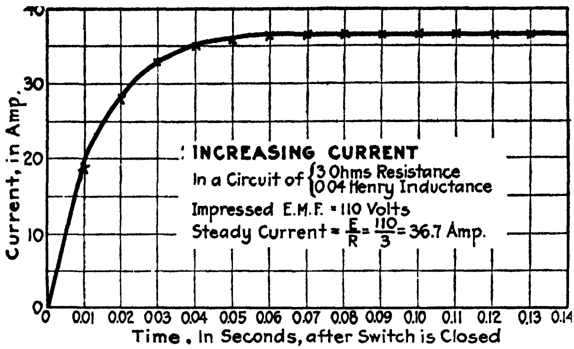

FIG. 246.—Graph showing how current in a closed inductive circuit increases gradually though rapidly when an e.m.f. is applied to the circuit.

A Reading number	*B* Time after closing switch, seconds	*C* Current intensity	*D* Impressed e.m.f., volts	*E* E.m.f. impelling current, volts	*F* Counter e.m.f. of self-induction, volts
1	0.00	0.0	110	0.0	0.0
2	0.01	18.0	110	54.0	56.0
3	0.02	28.0	110	84.0	26.0
4	0.03	33.0	110	99.0	11.0
5	0.04	35.0	110	105.0	5.0
6	0.05	36.0	110	108.0	2.0
7	0.06	36.3	110	108.9	1.1
8	0.07	36.6	110	109.8	0.2
9	0.08	36.7	110	110.0	0.0
10	0.09	36.7	110	110.0	0.0
11	0.10	36.7	110	110.0	0.0
12	0.11	36.7	110	110.0	0.0

483. Self-inductance in a Circuit Prevents the Current in the Circuit from Decreasing Instantly to Zero When the E.m.f. Is Discontinued.—That is, as the current in a circuit decreases, the flux of circular lines of force due to the current contracts into the conductor (Art. 465). The lines of this flux cut the conductor and induce in it an e.m.f. which tends to maintain the current. In a practically non-inductive circuit, like that of Fig. 243,*I*, there is little tendency to prolong the current after the e.m.f. is removed. But, with a circuit like that of *II* and *III*, which has inductance, the tendency is pronounced (Art. 471):

EXAMPLE.—Assume the switch of *I* to be closed so that the e.m.f. of 110 volts is impressed on the circuit, and that the steady current of 36.7 amp. is

flowing. Now, if the switch on this circuit be opened to discontinue the e.m.f. it will "pull" no arc or spark—or at least only a very small one.

The *arc* that bridges between the contacts of a switch when they open a circuit is a manifestation of the e.m.f. of self-induction. As such a switch is opened, the current in the circuit changes—decreases—and, due to the inductance of the circuit, an e.m.f. is produced which tends to maintain the current (Art. 459). Where the inductance is considerable, the induced e.m.f. is great enough to force the current across the air gap which is formed as the switch contacts separate. This induced current is often of sufficient intensity to heat the air in its path to incandescence. Thus that which is called *an arc* is produced.

If, when a steady current is flowing, the switch of Fig. 243,*II* is opened, there will be a relatively large arc, indicating a considerable tendency toward the prolonging of the current in the circuit. When the switch is opened far enough, the resistance of the air gap will be so great that the

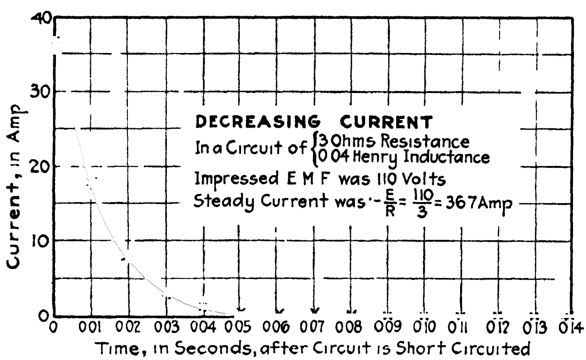

FIG. 247.—Graph showing how current in a closed inductive circuit decreases gradually though rapidly when the e.m.f. is discontinued.

induced e.m.f. can no longer force current across it. Then the arc will disappear and the current in the circuit will cease.

The faster the switch is opened, the greater will be the induced e.m.f. because the induced e.m.f. depends on *rate* of change of current (Art. 472). Obviously, the faster the switch is opened, the more rapid will be the rate of decrease of current. It is apparent then that, where the e.m.f. is discontinued from a circuit by opening a switch, the intensity of the e.m.f. induced thereby, depends to a considerable extent, on the rapidity with which the switch is opened. The rate of decrease of the current also depends on this.

Now assume that the steady current of 36.7 amp. is flowing in the coil of Fig. 243,*III*, and that the e.m.f. is discontinued from it by short-circuiting" the switch with a conducting bar laid across the switch blades as shown. (This would also short-circuit the 110-volt supply main but it is assumed that this main is protected at its source by circuit breakers which open immediately after the short-circuiting bar is placed across the switch blades.)

This leaves the coil of 0.04 henry inductance and 3 ohms resistance in an independent circuit, including the short-circuiting bar and the ammeter.

The current in this independent circuit would, under these conditions "die down" gradually—though in a very short time interval. If ammeter readings could be taken at successive instants after the short-circuiting bar was placed, they would be about as given in column I of Table 484 and shown in the graph of Fig. 247. Now the original source of e.m.f. (110 volts) has been eliminated. Therefore, the current now flowing must be due wholly to the induced e.m.f. For example: At the end of 0.02 sec., the current (*column I*) is 8 amp. The resistance of the coil is 3 ohm. Therefore, by Ohm's law (Art. 134), this induced e.m.f. will be: $E = R \times I = 3 \times 8 = 24$ *volts*. This is shown in *column L* of the table in which are given the induced e.m.fs. at the ends of the other successive intervals.

484. Table Showing How the Current in an Inductive Circuit Decreases When Its E.m.f. Is Discontinued.

These values relate to the circuit diagrammed in Fig. 243,*II*. It has 3 ohms resistance, 0.04 henry inductance and the e.m.f. that was impressed on it was 110 volts. Fig. 247 shows a graphic statement of these values.

G Reading No	*H* Time after shunting out coil switch, seconds	*I* Current intensity, Amperes	*J* Impressed e m f, volts	*K* E m f. impelling current, volts	*L* Counter e m f of self-induction
1	0 00	36 7	110	110 0	0 0
2	0 01	18 0	0	54 0	54 0
3	0 02	8 0	0	24 0	24 0
4	0 03	3 0	0	9 0	9 0
5	0 04	1 5	0	4 5	4 5
6	0 05	0 8	0	2 4	2 1
7	0 06	0 3	0	0 9	0 9
8	0 07	0 1	0	0 3	0 3
9	0 08	0 0	0	0 0	0 0
10	0 09	0 0	0	0 0	0 0
11	0 10	0 0	0	0 0	0 0
12	0 11	0 0	0	0 0	0 0

485. The Rate of Increase of Current in Any Circuit Can Be Computed if the applied e.m.f., the inductance, and the resistance of the circuit are known. However, although the formula involved is not difficult of solution, its manipulation requires a knowledge of a branch of mathematics with which most practical men are not familiar, hence it is not included here. It can be found in almost any electrical engineering text book.

486. Inductance Is Sometimes Called "Electric Inertia" and the Tendency of Currents to Flow in an Inductive Circuit After It Has Been Opened Is Sometimes Called "Electric Momentum."—Inductance does produce effects analogous to those produced by mechanical inertia. For example, if an endeavor is made to change the speed of a flywheel, the change is opposed by the wheel because of its inertia. Likewise, if an effort is made to change the intensity of the current in a circuit, this effort to change is resisted by virtue of the inductance of the circuit (Art. 471). As soon as the speed of the flywheel becomes steady, the inertia effect disappears. When the current in a circuit becomes steady, the inductive effects vanish.

However, it should be remembered that electricity may be thought of as a weightless fluid (Art. 90)—hence it can not have inertia. These so-called inertia effects are due to properties of the circuit through which the electricity flows. Electricity then does not have inductance. It is the conductors which have the inductance and as the conductors are rearranged and bent into different forms their inductances will be changed.

487. The Induced E.m.f. Is Greater When a Circuit Is Opened Than When It Is Closed.—When a circuit is opened the rate of change—decrease—of current is very high. When a circuit is closed the rate of change—increase—of current is not, usually, nearly so high. The facts just recited in connection with the explanation of Art. 438 offer an adequate explanation.

488. The Greater the Self-inductance of a Circuit the Larger the Time Taken for the Current In It to Attain Its Steady Value.—This is obviously true, because the greater the inductance, the greater will be the counter e.m.f. of self-inductance.

EXAMPLES.—Fig. 248 shows graphically the rates of increase of currents in circuits of different inductances. In each of the three cases illustrated, the impressed e.m.f. is 10 volts and the resistance of the circuit is 1 ohm. Note that, in the circuit of 1 henry inductance the current attains its steady value in about 8 sec.; with 10 henry, in about 70 sec. With 20 henry inductance, it requires something more than 180 sec., or over 3 min. If the circuit had an inductance of only 0.1 henry, the current would be steady in a trifle over 1 sec.

489. How Permeance Affects Inductance.—The number of lines of force that a current of 1 amp. in a conductor will develop in the magnetic circuit associated with the conductor will be directly proportional to the permeance (Art. 238) of the magnetic

circuit as described in Art. 240. That is, the number of cutting lines per ampere (Art. 474) will, in general, be directly proportional to the permeability. The greater the permeability of the magnetic circuit, the greater the number of cutting lines per ampere and *vice versa.* Now inductance is proportional to the number of cutting lines per ampere (Art. 474). It follows that the inductance of a conductor will be proportional to the permeance of the magnetic circuit associated with that conductor.

The permeance of air and of all non-magnetic materials is the same and it never varies (Art. 242). It follows that the cutting lines per ampere, and therefore the inductance, of any given conductor associated in a given way with non-magnetic materials

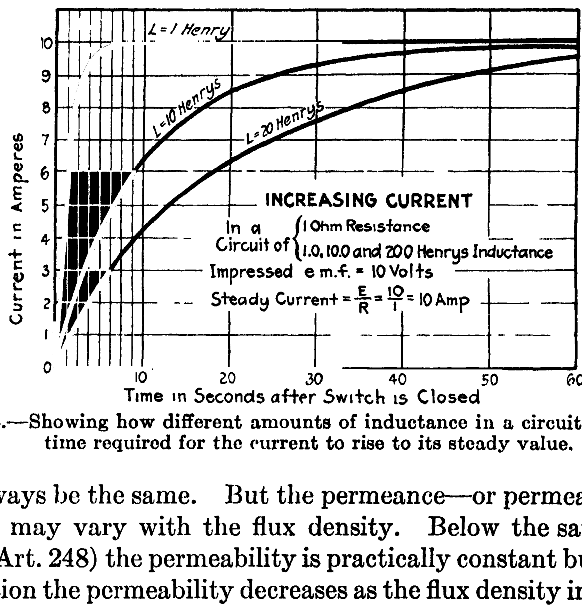

FIG. 248.—Showing how different amounts of inductance in a circuit affect the time required for the current to rise to its steady value.

will always be the same. But the permeance—or permeability—of iron may vary with the flux density. Below the saturation point (Art. 248) the permeability is practically constant but above saturation the permeability decreases as the flux density increases. Hence the cutting lines per ampere developed by any given conductor which is associated in a given way with iron will vary if the iron is worked above the saturation point. In refined computations of the inductances of circuits associated with iron it is necessary to recognize this variation of permeability with flux density. Also, in such cases, the flux density must be known before the permeability can be ascertained. However, in commercial apparatus iron is nearly always worked below the saturation point. Hence, in general, the inductance of any given piece

of apparatus may be and usually is considered constant whether or not iron is associated with it.

490. Computation of the Counter E.m.f. of Self-induction.—If the self-inductance of a circuit in henrys be multiplied by the *rate of change* of current in the circuit, the result will be the counter e.m.f. induced by this change of current. This follows from the definition of the henry in Art. 472 and explained in Art. 473. Obviously, the more rapidly the current intensity changes, the faster the flux will cut the conductor and the greater will be the e.m.f. induced in it (Art. 438).

NOTE.—Rate of change of current can be expressed by *amp. ÷ sec.* Thus, if a current changes from 0 amp. to 4 amp. in 2 sec., the rate of change is: $4 \div 2 = 2$ *amp. per sec.* If a current changes from 20 amp. to 8 amp. in 4 sec., the rate of change is: $(20 - 8) \div 4 = 12 \div 4 = 3$ *amp. per sec.*

Expressed as a formula the above-stated rule becomes:

(107) $$E_I = L \times \frac{I_c}{t} \qquad \text{(volts)}$$

and

(108) $$L = \frac{t \times E_I}{I_c} \qquad \text{(henry)}$$

(109) $$I_c = \frac{t \times E_I}{L} \qquad \text{(amp.)}$$

(110) $$t = \frac{L \times I_c}{E_I} \qquad \text{(sec.)}$$

Wherein E_I = the induced average e.m.f. in volts. I_c = change in current, in amperes, during the time t, in seconds. L = inductance of the circuit, in henrys.

EXAMPLE.—If a coil of wire wound on an iron core has a self-inductance of 10.4 henrys, what average voltage would be induced in the coil by a change from 32 amp. to 6 amp. in 4 sec.? SOLUTION.—Substitute in the formula (107):

$$E_I = L \times \frac{I_c}{t} = 10.4 \times \frac{32 - 6}{4} = \frac{10.4 \times 26}{4} = 67.6 \textit{ volts}.$$

EXAMPLE.—If the average counter e.m.f. of self-induction of a coil is 20 volts when the current in it changes from 24 to 32 amp. in 0.5 sec., what is the inductance of the coil? SOLUTION.—Substitute in formula (108):

$$L = \frac{t \times E_I}{I_c} = \frac{0.5 \times 20}{32 - 24} = \frac{10}{8} = 1.25 \textit{ henry}$$

491. Significance of the Formulas for Computing Inductance.—As outlined in Art. 473, the inductance of a circuit or conductor

in henrys is equal to: *the number of cutting lines developed by it per ampere in it* ÷ 100,000,000. It follows, then, that any formula for the computation of inductance is merely an expression for the cutting flux developed per ampere divided by 100,000,000. Examples of formulas for inductance derived in this way are given in other articles.

492. Self-inductance of Any Coil.—The flux developed by a coil cuts practically every turn of the coil in expanding out of or contracting into the conductor from which it emanates when a current through the coil starts or ceases. It follows, therefore, from this and preceding information that for any coil the following formula is (not exactly but nearly so) true:

$$L = \frac{\phi_a \times N}{100,000,000} \qquad \text{(henry)} \tag{111}$$

Wherein L = inductance of the coil, in henrys. N = number of turns in the coil. ϕ_a = flux or number of lines of force developed through the coil by a current of 1 amp. in it.

EXAMPLE.—If a current of 1 amp. produces in a certain iron-core field coil which has 480 turns, a flux of 263,000 lines, what is the inductance of the coil? SOLUTION.—Substitute in equation (111): $L = \theta_a \times N \div 100,000,000 = 263,000 \times 480 \div 100,000,000 = 126,240,000 \div 100,000,000 = 1.3$ *henrys*.

493. Computation of Self-inductance of a Coil.—The flux of lines of force developed by any coil is from formula (74), Art. 254: $I \times N \times \mu_a \times A \div l$. That is, the flux per ampere, $\phi_a = N \times \mu_a \times A \div l$. Substituting this expression for ϕ_a, for ϕ_a in (111) in the preceding Art. 492 another formula for the inductance of a coil is obtained thus:

$$L = \frac{N^2 \times \mu_a \times A}{100,000,000 \times l} \qquad \text{(henry)} \tag{112}$$

Wherein L = the inductance of the coil in henrys. μ_a = the absolute permeability of the material of the magnetic circuit in perms per in. cube. l = the length of: (1) the coil if the coil has an air core and is not associated with iron; (2) the core or magnetic circuit if it is of iron. A = cross-sectional area of the inside of the coil or of the core in square inches. N = the number of turns in the coil.

NOTE.—The above formula gives results quite accurate for any straight cylindrical coil of one layer which is long in proportion to its diameter but

it may be used in practical work without excessive error for calculating the inductance of almost any coil. Short coils have greater inductances, proportionately, than long coils.

EXAMPLE.—What is the inductance of a coil of 400 turns wound on an iron core which is 24 in. long and which has a sectional area of 46.5 sq. in.? Assume μ_a = 4,800. SOLUTION.—Substitute in formula (112):

$$L = \frac{N^2 \times \mu_a \times A}{100\,000{,}000 \times l} = \frac{400 \times 400 \times 4{,}800 \times 46.5}{100{,}000{,}000 \times 24} = 14.9 \textit{ henry.}$$

494. Computation of the Self-inductance of a Coil with an Air Core.

—From Art. 242, the absolute permeability of air, that is μ_a, is always = 3.192 perms per in. cube. Then substituting this value in formula (112), the inductance of an air core coil is:

$$(113) \qquad L = \frac{N^2 \times 3.192 \times A}{100{,}000{,}000 \times l} \qquad \text{(henry)}$$

Wherein the letters have the same meanings as with formula (112). The results given by this equation are accurate to about the same extent as those of (112) as indicated by the note thereunder.

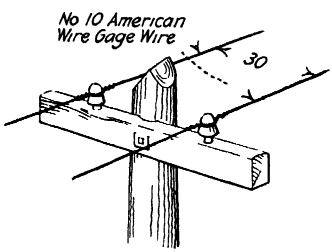

FIG 249.—Transmission line self-inductance problem.

EXAMPLE.—What is the inductance of an air-core coil having an internal diameter of 1 in. (area = 0.79 sq. in) 100 turns and a length of 20 in ?

SOLUTION —Substitute in the formula (113):

$$L = \frac{N^2 \times 3.192 \times A}{100{,}000{,}000 \times l} = \frac{100 \times 100 \times 3.19 \times 0.79}{100{,}000{,}000 \times 20} = 0.000013 \textit{ henrys.}$$

Or, 0.000013 *henry* = 0.013 *millihenry*.

495. Self-inductance of a Straight Conductor.

—Straight conductors have relatively little self-inductance—practically none unless they are quite long. That they must have some inductance is evident from the information of Art. 465. If a straight conductor or wire is bent into a loop with a considerable distance between the sides of the loop or if the conductor is wound into a helix its inductance is greatly increased (Art. 466). The self-inductance of straight conductors can be computed by using certain formulas which are too complicated for inclusion here.

EXAMPLES.—See Table 480 and Art. 496 for specific numerical examples

496. How to Compute the Self-inductance of a Two-wire Transmission Line.—The following approximate formula can be used where the wires are of any non-magnetic material:

$$L = 0.741 \times \log \left(2.568 \frac{D}{d}\right) \qquad \text{(millihenry)} \tag{113a}$$

Wherein L = inductance, in millihenrys, of 1 mile of single conductor or ½ mile of two-wire circuit. D = distance between the centers of the two conductors, in inches. d = diameter of each of the conductors, in inches.

Note that the further apart the conductors, the greater is the self-inductance; the reasons for this are explained in Art. 497. Note also that the greater the diameter of the conductor, the smaller the self-inductance.

EXAMPLE.—What is the self-inductance of 1 mile of a two-wire transmission line (Fig. 249) of No. 10 American Wire Gage (B. & S.) copper—or any non-magnetic—wire, the two sides of the circuit being spaced 30 in. between centers? SOLUTION.—The diameter of a No. 10 wire is almost exactly 0.10 in. Use the above formula:

$$L = 0.741 \times \log \left(2.568 \frac{D}{d}\right) = 0.741 \times \log \left(2.568 \frac{30}{0.1}\right)$$

$$= 0.741 \times \log (2.568 \times 300) = 0.741 \times \log (774.0)$$

Now ascertain the log (logarithm) of 770.4 from any table of common or Briggs logarithms, which may be found in an engineer's handbook. The log of 774. = 2.887. Now use this value in the above equation thus:

$$L = 0.741 \times 2.887 = 2.13 \textit{ millihenrys.}$$

This is the inductance of 1 mile of one wire of the circuit. For both wires (1 mile of circuit, 2 miles of wire) the inductance will be: $2 \times 2.13 =$ 4.26 *millihenrys.*

497. Why Self-inductance is Decreased as the Legs of a Circuit Are Brought Close Together.—Fig. 250,*I* shows a portion of a circuit in which current is flowing. The conductor is bent into a loop. As the current changes in the circuit, the circular lines of force about A will expand or contract and all of them will cut leg A, tending to induce in it an e.m.f. However, only the lines of the flux indicated by C are effective in inducing an e.m.f. in the conductor. The reason is that the lines denoted by D cut both legs, A and B, as they expand out of or contract into A. The direction of the e.m.f. induced by these ineffective

(*D*) lines is the same (from front to back for example) in *A* as in *B* (hand rule, Art. 427).

Hence, the e.m.fs. in *A* and in *B* due to these *D* lines annul one another. The effective "*C*" lines cut only leg *A* and thereby induce therein an effective e.m.f. as the current changes and thus produce self-induction. Leg *B*, although the illustration does not show it, produces a flux of circular lines like that around *A*, except that it is in the opposite direction. It follows, then, that the effective lines of both *A* and *B* contribute toward the self-induction of the looped conductor.

NOTE.—If the legs of the conductor are quite close together as at *II*, the distance *H* between them is small and the number of effective lines is correspondingly small. When the legs are close together (almost touch) only the lines or flux that lie inside of the conductors and between the legs is effective in producing self-induction. If the legs of the looped conductors

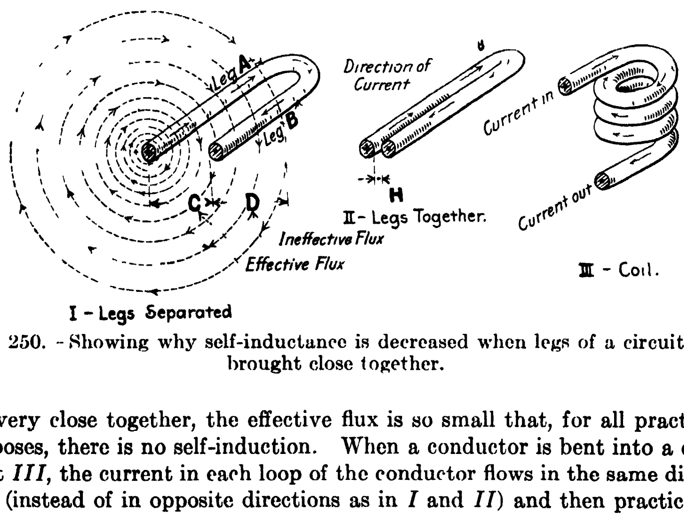

FIG. 250. - Showing why self-inductance is decreased when legs of a circuit are brought close together.

are very close together, the effective flux is so small that, for all practical purposes, there is no self-induction. When a conductor is bent into a coil, as at *III*, the current in each loop of the conductor flows in the same direction (instead of in opposite directions as in *I* and *II*) and then practically all of the flux produced by each loop or turn is effective and cuts every other turn as described in Art. 466. Thus, the self-induction of a coil is large.

498. The Inductance of Stranded Wires is, for all practical purposes, the same as that of solid wires of the same circular-mils area.

499. A Choke Coil (Fig. 251) is merely a coil having considerable self-inductance. Choke coils are sometimes, particularly when used in alternating-current circuits for limiting current, called *reactance coils* (Art. 747). A choke coil should always

be used in combination with a lightning arrester as shown in the illustrations, to tend to prevent the lightning-discharge currents from entering the apparatus which the coil is installed to protect. Choke coils usually have "air cores" so that the flux produced by a current in them will build up very rapidly and thus produce a high counter e.m.f.

EXAMPLE.—In the diagram of Fig. 251, *II*, a choke coil and a lightning arrester* for the protection of the generator are shown. For simplicity, only one wire is shown entering the station. There should, in general, be a lightning arrester and a choke coil on every aerial line entering a station. If no lightning arrester is provided and the line wire is struck by lightning,

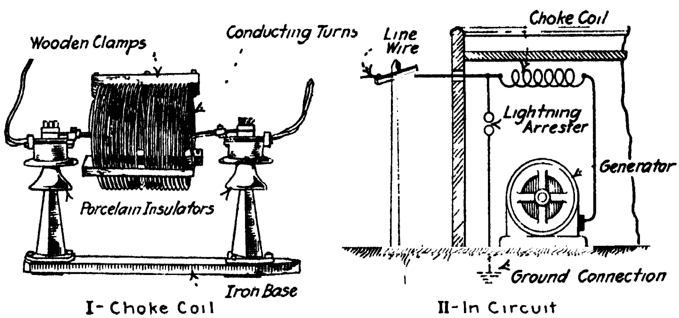

FIG. 251.—Illustrating the choke coil and its application.

the extremely high-voltage lightning-discharge current, in its endeavor to reach ground, may pass into a generator or other apparatus and through it to its metal frame which is usually grounded. Thus a winding may be punctured or possibly "burnt out" and ruined.

If protection be installed as shown, a sudden rush of current into the choke coil will induce in the coil an immense counter e.m.f. This will tend to "choke" or force the lightning-discharge current through the lightning-arrester air gap—instead of through the generator—to ground. Thus the coil tends to protect the station apparatus with which it is associated. For the high-frequency oscillatory current of a lightning discharge, the path through the lightning arrester to ground offers much less opposition than that through the choke coil.

*See the author's CENTRAL STATIONS for more information relating to lightning arresters and lightning protection

SECTION 26

MUTUAL INDUCTANCE

500. Mutual Inductance (Art. 471) is the ability of one circuit to produce an e.m.f. in a nearby circuit by induction when the current in the first circuit changes. However, the second circuit can also induce an e.m.f. in the first when the current in the second circuit changes. The process by which the e.m.f. is induced is outlined in Art. 457. The induced e.m.f. is always in such a direction as to oppose the change of current inducing it (Lenz's law, Art. 435). Mutual inductance is measured in henrys, Art. 472.

EXAMPLE.—As the current in the primary coil of Fig. 228 changes, the e.m.f. induced in the secondary coil is due to the mutual inductance of the two coils. Also, if the current were varied in the heavy-wire (secondary) coil, an e.m.f. would be induced in the fine-wire (primary) coil, by virtue of the mutual inductance of the coils.

NOTE.—"*Coefficient of mutual inductance*" is a term, now little used, having about the same significance as "mutual inductance."

501. Mutual-inductance Calculations are quite complicated except in a few relatively simple cases, one of which is given below. In determining mutual inductance it is first necessary to ascertain the number of lines of force of one circuit which will cut the other circuit when a current of 1 amp. flows in the first circuit. Then this number, representing the cutting flux, is divided by 100,000,000 to get the result into henrys (Art. 473).

502. Computation for the Mutual Inductance of Two Concentric Coils.—Where the two coils are concentric and lie close together their mutual inductance can be readily figured. It is assumed that the flux evolved by each turn of one coil cuts every turn of the other coil. The similarity between the following formula and that for calculating the self-inductance of a coil (Art. 493) is apparent. The derivation of this formula is practically the same as that of the one for computing self-inductance.

$$L_m = \frac{N_1 \times N_2 \times \mu_a \times A}{100{,}000{,}000 \times l} \quad \text{(henry)} \qquad (114)$$

Wherein N_1 = number of turns in one of the concentric coils. N_2 = number of turns in the other concentric coil. The other symbols have the same meanings as given under Art. 493. Where the coils have an air core, the mutual induction (Art. 494) then is:

$$L_m = \frac{N_1 \times N_2 \times 3.192 \times A}{100{,}000{,}000 \times l} \quad \text{(henry)} \tag{115}$$

The above formulas, like practically all of the simple ones for calculating inductance, give approximate results, which however are usually sufficiently accurate for most practical purposes.

EXAMPLE.—What is the approximate mutual inductance of the two concentric coils wound on the wood (non-magnetic or air) core, 1.6 in. in diameter (area = 2.01 sq. in) shown in Fig. 252, *I*? The coils are each 25

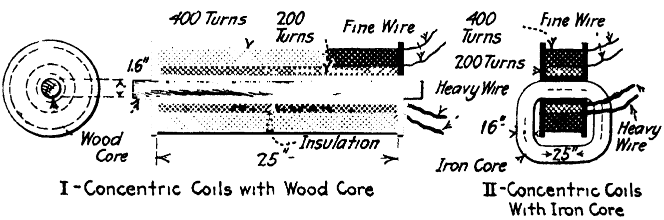

FIG. 252—Illustrating examples in computing mutual inductance of concentric coils.

in. long. One has 400 turns, the other 200 turns. SOLUTION.—Substitute in formula (115):

$$L_m = \frac{N_1 \times N_2 \times 3.192 \times A}{100{,}000{,}000 \times l} = \frac{400 \times 200 \times 3.19 \times 2.01}{100{,}000{,}000 \times 25} = 0.0002 \textit{ henry}.$$

Or, 0.0002 *henry* = 0.2 *millihenry*.

EXAMPLE.—What will be the approximate mutual induction of two concentric coils, each of the same number of turns as specified in the above example, if they are wound on an iron core 1.6 in. in diameter and 25 in. long (Fig. 252)? Assume μ_a = 5,100. SOLUTION.—Use formula (114):

$$L_m = \frac{N_1 \times N_2 \times \mu_a \times A}{100{,}000{,}000 \times l} = \frac{400 \times 200 \times 5100 \times 2.01}{100{,}000{,}000 \times 25} = 0.32 \textit{ henry}.$$

Or, instead, 0.32 henry = 320 millihenry. It is obvious that the inductance in this example is as many times greater than that in the preceding example as the permeability of this iron is greater than the permeability of air That is: $L_m : 0.0002 :: 5100 : 3.19$. Then, $L_m = 0.32$.

SECTION 27

ENERGY STORED IN MAGNETIC FIELD

503. Kinetic Energy Is Stored in Any Magnetic Field.—Energy—foot-pounds—is (Art. 169) capacity for doing work. The power (Art. 159) expended by a current in a circuit is the same whether the circuit has or has not inductance. But the energy spent in *starting* a current in a circuit is greater in an inductive circuit than in a non-inductive circuit as will be shown. The excess of energy is stored in the magnetic field and is all returned to the circuit when the current decreases to zero:

EXAMPLE.—Consider the inductive circuit, which has a resistance of 3 ohms and an inductance of 0.04 henry, for which certain values of increasing currents are tabulated in 482. At the instant when the current is 18 amp. the power loss is: $I^2 \times R = 18 \times 18 \times 3 = 1{,}072$ *watts*. But the total power taken by the circuit at this instant is: $E \times I = 110 \times 18 = 1{,}980$ *watts*. Obviously, there is a difference of: $1{,}980 - 1{,}072 = 908$ *watts* between the instantaneous power taken by the circuit and that actually lost or used. This difference, 908 watts, represents the instantaneous power stored in the magnetic circuit.

If this value—908—in watts be multiplied by the very small time in seconds during which the power is being stored, the result will be the energy (Art. 169), in Joules or watt-seconds, stored during this short interval. By applying the higher mathematics, it is possible to sum up the energy stored during all of the successive instants while the current is increasing and thereby ascertain the total amount of energy thus stored in the magnetic field.

When the current becomes steady, the field becomes steady and no additional energy is imparted to it. Furthermore, no additional energy is required to maintain it. Energy is not required for the maintenance of any magnetic field—except that lost in the conductors whereby the field is excited. A permanent magnet (Arts. 41 and 80) furnishes a striking example of this fact. If the voltage of 110 is discontinued from the circuit described above, the current decreases to zero and then (practically) all of the energy stored in the magnetic field, as described, is returned to the circuit. The process is the reverse of that outlined in the foregoing example.*

*For further information relating to this situation see Karapstoff's THE MAGNETIC CIRCUIT, p. 177.

SECTION 28

EDDY CURRENTS

504. Eddy or Foucault Currents are those currents which are induced in masses of metal whenever the metal is moved in a magnetic field, or when a field or flux moves through the metal. These currents are always in such a direction as to oppose the motion producing them (Lenz's law, Art. 435). Eddy currents are usually of relatively small intensity but may be enormous. They always involve an $I^2 \times R$ loss (Art. 167) which in the aggregate may be considerable.

EXAMPLE.—If a permanent magnet be rotated close to and above a metal plate, as in Fig. 252*A*, *I*, eddy currents will be induced in the plate. Some of the flux of the magnet cuts the metal of the plate and thereby induces an e.m.f. which impels these currents around in the plate. Considerably more force will be required to rotate the magnet than if the plate were not near it, because the induced eddy currents tend to retard the rotation of the magnet.

Now, if the magnet be held stationary and the plate revolved, eddy currents will also be induced in the plate. If the magnet is not restrained, it will tend to rotate, following the plate. The direction of the eddy currents, the plate rotating to the left, is shown in the illustration. One portion of the current follows its own approximately semicircular path in the plate at one side of the magnet. The other, similar, path is at the other side of the magnet. These two currents produce two magnetic poles as shown. The polarities of these are such that they repel the nearest (in the direction of rotation) pole of the permanent magnet. Thus, rotation is opposed and the requirement of Lenz's law satisfied.

If the magnet is not restrained, it will try to assume such a position that, as the plate rotates, its N pole will lie directly over the S pole of the plate and its S pole over the plate's N pole. It can never attain this position, however, because if the magnet moves, the positions of the poles of the plate will shift also. Each of the halves of the plate will always be oppositely polarized, the magnet, as it were, dividing the plate into halves. If the magnet is prevented from turning, and the plate is rotated and a light metal wiper or brush is arranged to bear on the plate under each magnet pole, current will be forced through an external circuit connected to the brushes, as will be indicated by a low-reading ammeter connected in the circuit.

EXAMPLES.—(1) If a slab of copper be abruptly pushed into the field between the poles of a strong electromagnet, the copper acts as if it were being moved in a heavy fluid. The eddy currents induced in it tend to pre-

vent its movement. (2) If a copper penny be suspended on a twisted thread between the poles of a powerful magnet (Fig. 252*A*, *II*), or in any strong electromagnetic field, the penny will spin as the thread untwists *if* the magnet is not energized. But when it is energized motion will cease. (3) The eddy currents circulating in the metal bobbins of the moving elements of permanent-magnet-type direct-current measuring instruments and of D'Arsonval galvanometers tend to stop their motion. The bobbins are suspended in strong magnetic fields and when they move eddy currents develop It is because of this that these instruments are "dead-beat"

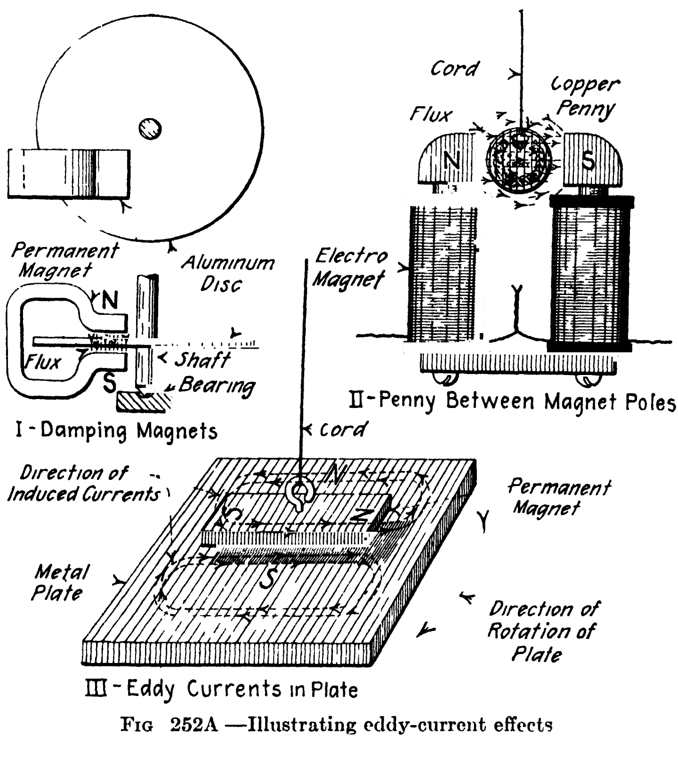

FIG 252A —Illustrating eddy-current effects

EXAMPLE.—The needle of a compass will come to rest much more quickly if it be mounted in a metal case than if it is in a non-conducting case—because of eddy currents.

EXAMPLE.—An important example is that illustrated in Fig. 252*A*, *I*, where is diagrammed the application of damping magnets to a watt-hour-meter movement. These damping magnets, because of the eddy currents due to them, perform two important functions: (1) They prevent an excessively rapid rotation of the moving element of the meter. (2) They insure that the rotational speed of the disc will always be proportional to the load which is being metered. The metal disc—usually of aluminum so that the weight of the rotating element will be a minimum—is mounted on a vertical

steel shaft between the poles of strong permanent magnets. As the disc is forced to rotate by the small electric-motor movement (not shown) which always forms a part of a watt-hour meter, eddy currents are induced in the disc. These, in accordance with Lenz's law, tend to retard the rotation of the disc. The "meter motor" is actuated by the voltage impressed on and by the current taken by the load being metered.

If a watt-hour meter is to register correctly, its moving element must, at any instant, rotate at a speed exactly proportional to the power load being metered at that instant. This is a statement of fact.

Now every meter motor is so designed that the torque which it develops at any instant is proportional to the (power) load being metered at that instant. If the power load is doubled, the meter-motor torque is doubled. If the load is halved the motor torque is halved. It is this motor torque that forces the meter moving element to rotate. If the damping magnets were not provided, a meter motor would—even when metering a very light load—rotate at an exceedingly high speed, because then the only counter torque opposing the rotation of the motor would be that due to the friction and windage of the rotating element and meter movement. In practice this friction and windage is very small, particularly at low speeds. The high speeds which would thus result would be undesirable mechanically. Furthermore, it can be shown that these high speeds would not, necessarily, be directly proportional to the power loads being metered.

It is, as will be shown, imperative, if the speed of the meter motor is to be exactly proportional to the load being metered, that the total counter torque must vary exactly as the speed of rotation varies. As the speed increases, the counter torque must increase directly in proportion. As the speed decreases, the counter torque must decrease correspondingly The counter torque produced by the disc rotating in the damping magnet's field satisfies these requirements.

The flux of the permanent magnets (Fig. 252A) is, obviously, constant. Hence, the intensity of the eddy currents induced, is proportional to the speed of rotation of the disc. Thus the damping or counter-torque effect of the arrangement varies directly as the speed of rotation.

How this phenomenon insures a rotational speed exactly proportional to the power load being metered can be best illustrated by an example: Assume that, with some certain load, an imaginary meter motor develops a torque of 1 oz. at 1 in. radius. Then the counter torque due to the damping magnets would necessarily also be 1 oz. at 4 in. radius. (The negligible friction-and-windage counter torque is here disregarded.) This is evident because any magnet-damped meter motor will always rotate at a speed such that the eddy-current counter torque is equal to the motor torque. Now assume that the power load being metered is doubled. Then the meter motor would, as above suggested, develop a torque twice as great as before or a torque of 2 oz. at 1 in. radius. Since its torque has now been increased, the motor will speed up until it acquires a speed such that an eddy-current counter torque is developed which is equal to the new motor torque. That is, it will speed up until its rotational speed is such that the new counter torque is 2 oz. at 1 in. radius.

Now the counter torque due to the disc rotating between the damping

magnets is, as above stated, directly proportional to the speed so that when the disc has speeded up so as to attain a speed just twice as great as its original speed, its counter torque will then be twice as great as before, or 2 oz. at 1 in. radius, and it will be equal to the now-existing motor-torque. Thus, it is apparent that the speed of the meter motor will always—due to the damping-magnet counter torque—be directly proportional to the load being metered.

EXAMPLE.—The so-called *medical induction coils* or *shocking coils* have cylindrical brass tubes which can be moved in and out between the windings of the core. When the tube is all in and entirely surrounding the core, the secondary e.m.f., that is the "shock power," of the coil is small. The secondary e.m.f. increases as the tube is withdrawn and is a maximum when the tube is entirely withdrawn. The explanation is this: When the tube is "all in" the eddy currents in it are considerable. Consequently the core magnetizes and demagnetizes slowly and the e.m.f. induced in the secondary is low But when the tube is all out, the eddy-current losses are practically

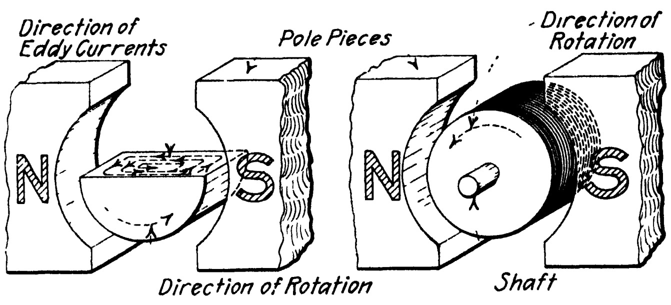

FIG. 252B.—Showing how laminating an armature tends to minimize eddy currents and eddy current losses in it.

eliminated, the core magnetizes and demagnetizes rapidly and the secondary e m.f. is high.

505. Methods of Minimizing Eddy-current Loss.—Since an eddy current tends to flow at right angles to the direction of the flux, the resistance of its path and the intensity of its e.m.f. can be decreased by *laminating* (Fig. 252B, *II*) the metals in which it tends to flow. The laminations should be parallel to the direction of the flux and at right angles to the axis of rotation. Since (formula, 116, Art. 508) the eddy-current loss varies as the square of the thickness of the laminations, it is possible to greatly reduce the loss by this constriction.

In commercial electrical apparatus, all large volumes of metal that are subject to considerable eddy-current loss are laminated. That is, they are built up of thin sheets of metal usually from 0.01

to 0.03 in. thick. Thicker sheets may be used where the tendency for eddy-current loss is not large. The sheets are usually painted to provide insulation between them, but in some cases the oxide on the metal provides sufficient insulation.

EXAMPLES.—(1) Fig. 252*B*, *II*, illustrates the lamination of an armature or rotor. (2) The cores of spark and induction coils (Arts. 468 and 463) are built up of lengths of iron wire to minimize eddy-current loss. An electromagnet composed of an insulated conductor wound on a solid iron core may require 10 or 12 times as long to magnetize as a magnet of exactly the same proportions but having a "laminated" core comprising a bundle of iron wires. The magnet with the solid core will also demagnetize much more slowly than will the other. (3) Fig. 291 shows how the field magnet core of a certain type of generator is laminated to minimize eddy-current losses.

506. Eddy-current Loss.—Whenever there is a current in a conductor there will be a loss: $P = I^2 \times R$ (Art. 167). Hence when eddy currents flow, there is an eddy-current loss which appears as useless heat. Even when these losses have been minimized in so far as is practicable by laminating (Art. 505) they are considerable in generators, motors, transformers and similar apparatus.

507. Eddy Currents in Electrical Machines.—When the rotor of an electrical machine revolves in its field there is a tendency to set up eddy currents, Fig. 252*B*, *I*. There is also a tendency toward setting up of eddy currents in the stationary parts as flux sweeps through, or changes in them. Applying the hand rule of Art. 427, it will be found that the eddy currents tend to flow in the cylinder of the illustration in the directions indicated. Eddy currents flow in a direction at right angles to that of the field. Eddy currents are minimized by laminating (Art. 505).

508. Method of Computing Eddy-current Loss.—The eddy-current loss in any volume of metal that cuts, or is cut by, a flux must obviously depend, among other things, on the specific resistance or resistivity (Art. 126*a*) of the metal and the frequency with which the flux cuts the metal—that is on the rate of cutting. Laminating (Art. 505) increases the resistance; hence, the thinner the laminations the less the eddy-current loss. A formula—the derivation of which can not be given here—for computing eddy-current loss is:

$$P_E = 0.254 \times j \times V(X \times f \times B)^2 \qquad \text{(watts)} \tag{116}$$

Wherein P_E = eddy-current loss in watts. j = a coefficient

varying with the quality and kind of metal in which the eddy currents are induced; the following values are given in Pender's AMERICAN ELECTRICAL ENGINEERS' HANDBOOK: for silicon sheet steel j varies from 0.000043 to 0.000098 with an average of 0.000065—for ordinary electrical sheet steel j varies from 0.00012 to 0.00025 with an average of 0.00022. V = the volume of the metal in which the loss occurs in cubic inches. X = thickness of the sheets in inches. f = frequency in cycles per second. B = the maximum flux density in kilolines (thousands of lines) per square inch.

EXAMPLE.—What eddy-current power loss may be expected in a mass of ordinary laminated electrical steel having a volume of 61 cu. in. if the iron is acted upon by a flux (maximum) of 15,500 lines per sq. in. at a frequency of 60 cycles per sec.? The laminations are 0.016 in. thick. SOLUTION.—Substitute in the formula (116): $P_E = 0.254 \times j \times V(X \times f \times B)^2 = 0.254 \times 0.00022 \times 61(0.016 \times 60 \times 15.5)^2 = 0.0034 \times (14.9)^2 = 0.0034 \times 22.20 =$ 0.075 *watts.*

SECTION 29

PRINCIPLES OF ELECTRIC GENERATORS

509. Electric Generators or Dynamos are machines whereby by utilizing the principle of magnetic induction (Art. 416), mechanical energy may be converted into electrical energy (Art. 179). They are sometimes referred to as "electricity generators." This term is, however, incorrect. Electricity can not be generated. A generator may be thought of as a device by means of which electricity which is already in existence (Art. 94) can be forced to move and thereby transmit energy and do work—light lamps, operate motors and the like. A dynamo does not generate electricity any more than a hydraulic force pump generates water.

510. How a Generator Can Convert Mechanical Energy into Electrical Energy is outlined in Art. 94. Briefly: The mechanical turning of the rotating portion of the generator causes conductors associated with the machine to cut or to be cut by a magnetic flux. This flux is usually produced by electromagnets. The cutting of the flux induces an e.m.f. in the conductors. Now if this e.m.f. be impressed on a closed, conducting circuit it will force through the circuit an electric current—which is a transference of electrons. As outlined in Art. 181, whenever a current is impelled electrical energy is generated.

511. The Principle of the Generator has already been briefly indicated in Art. 418. If the bar shown in Fig. 217 be pushed through the magnetic field between the poles of the magnet an e.m.f. will be induced in the bar. The intensity of the e.m.f. will be proportional to the speed with which the bar is moved—that is, proportional to the rate of cutting. This e.m.f. will force a current (of an intensity inversely proportional to the resistance of the circuit) through the circuit which consists of the bar and the external circuit. The e.m.f. will be in one direction while the bar is being moved up and in the other while the bar is being moved down. The directions of these induced e.m.fs. can be ascertained by applying the hand rule of Art. 427. Power will

be required to move the conductor between the poles (since current is forced through the circuit) and the power, P, thus required (neglecting the weight and the eddy-current loss) will be equal, in watts (Art. 164) to $E \times I$. If the circuit is open $I = 0$. Hence, with an open circuit, $P = 0$, that is, no power is required to move the bar.

EXAMPLE.—If it be assumed that there are 200,000,000 lines in the uniform field of Fig. 217 and that just 1 sec. is required to move the bar at a uniform rate through them (since cutting at the rate of 100,000,000 lines per sec. induces 1 volt, Art. 226), an e.m.f. of 2 volts (Art. 444) will be generated during that second. Assuming the resistance of the entire circuit,

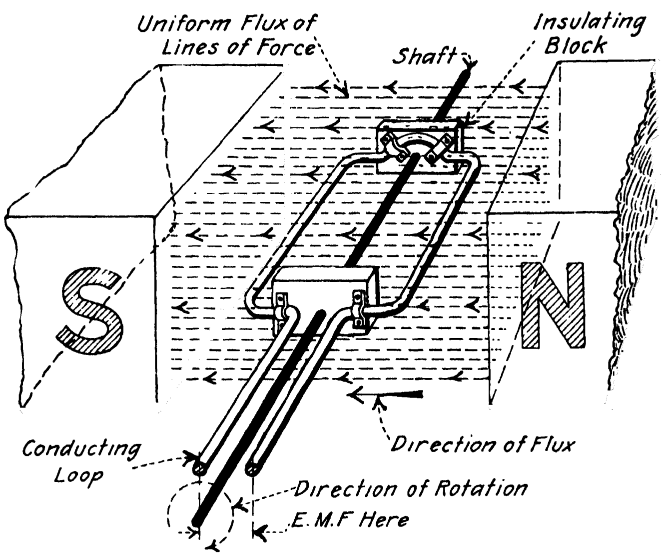

FIG. 253.—Illustrating the principle of the generator.

comprising bar and external circuit, to be $\frac{1}{2}$ ohm, the current will (Art. 134) be: $I = E \div R = 2 \div \frac{1}{2} = 4$ *amp.* The power expenditure—the rate of doing work—will (Art. 164) be: $P = E \times I = 2 \times 4 = 8$ *watts.*

512. In Commercial Generators it is necessary to use strong electromagnets to produce the fields and it is necessary to move conductors through the fields at high speeds to generate the e.m.fs. required. These high speeds, viz., high rates of cutting lines are best attained by rotating conductors formed into loops through magnetic fields. The elements of such an arrangement are shown in Fig. 253.

513. The Hand Rule for Determining the Relative Directions of Motion, E.m.f. and Flux as Applied to a Loop Rotated in a Field is stated graphically in Fig. 254,*I*. This is merely a specific adaptation of the rule given in Art. 427. Note that the *right hand* is always used for a generator as at *I* while the left hand is used for a motor as at *II*.

514. What Occurs When a Conducting Loop Is Rotated in a Magnetic Field will now be discussed in connection with Figs. 255 to 262: Imagine the conducting loop of Fig. 253 arranged in a magnetic field, as shown in Fig. 255, with a *collector ring* electrically connected to each side of the loop and provided with two metallic brushes, each so mounted as to make electrical

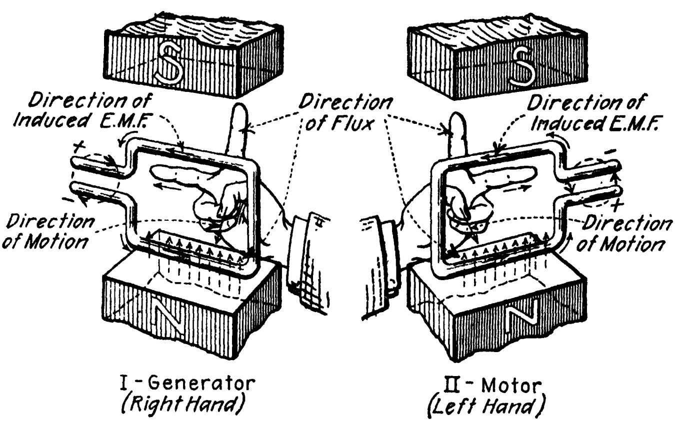

FIG. 254.—Application of hand rules to rotating coils.

contact with one of the collecting rings. The brushes bear down on their rings and always make electrical connection with them, but they do not interfere with the turning of the loop. The external circuit is connected to the two brushes as shown. Furthermore, assume that one-half or side of the loop is painted black, as shown merely so that it can be readily distinguished from the other half which is not painted.

Now if this loop of Fig. 255 be rotated—in either direction—both of its sides, the painted and the unpainted one, will cut lines of force (Fig. 255,*II*) and an e.m.f. will be generated within the loop (Art. 422). If the loop be rotated in the counterclockwise direction indicated, toward the position of Fig. 256, the directions of these e.m.fs. (hand rule, Art. 427) will be *in* (away from

the reader) in the white side of the loop and *out* (toward the reader) in the black side, as shown by the arrows in *I* and by the cross and dot in *II*. Brush *B* (Fig. 256) then will be positive (+), because the direction of the induced e.m.f. at this time is *out* of or away from it, and the brush *W* will be negative (−).

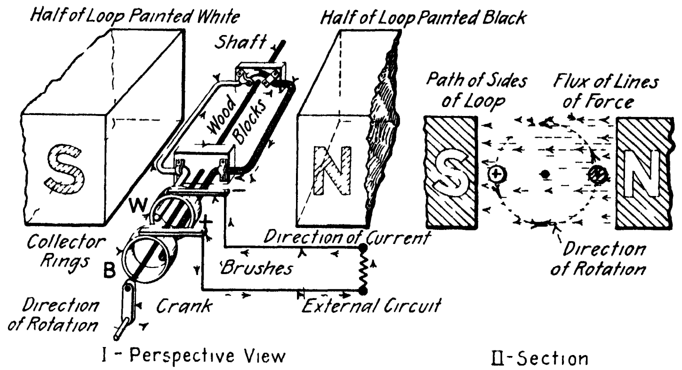

FIG. 255.—Conducting loop being rotated in magnetic field.

Current will be forced in the direction shown through loop and the external circuit by this e.m.f. as outlined in the following example:

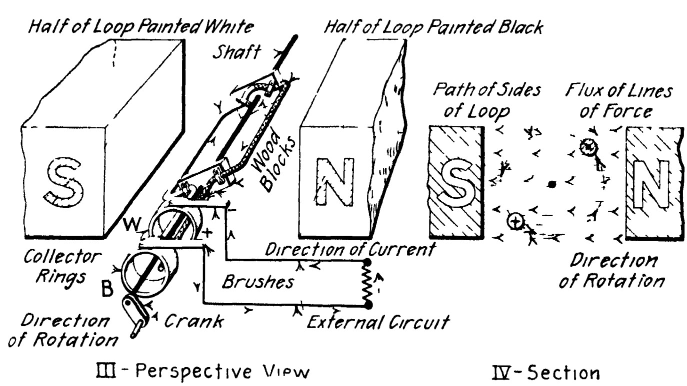

FIG. 256.—Second position of conducting loop in uniform field.

EXAMPLE.—Now assume that the rotation of the above-described loop be continued at a *uniform* speed and in a counterclockwise direction. Its sides will cut lines of force as it moves through the position of Fig. 256 until it reaches the position indicated in Fig. 257. At the instant illustrated in Fig. 257 no lines of force will be cut, because at this instant the sides of the conductor are moving parallel to the lines and can not, therefore, cut them.

Hence, at this instant, the e.m.f. induced will be zero and therefore no current will be forced through the loop or external circuit.

It can be shown (Art. 518) that, as the loop is rotated, at a uniform speed, from the position of Fig. 255 to that of Fig. 257, the e.m.f. induced in it gradually decreases from a maximum to zero. (By "a maximum e.m.f." is

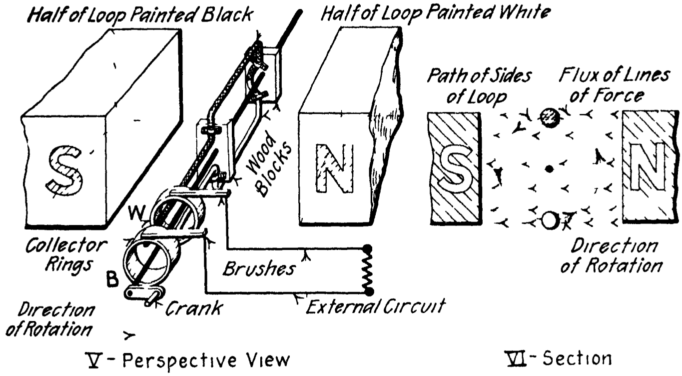

FIG. 257—Third position of conducting loop in uniform field.

meant the greatest possible e m f that can be induced in the loop with the given flux and given speed of rotation; the value of such a maximum e.m f can be determined in any given case by using the formula of Art. 444) The *rate of cutting* gradually decreases as the loop is rotated until, in the

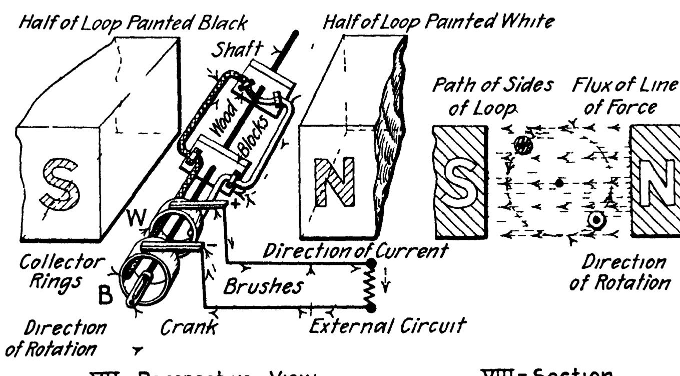

FIG. 258.—Fourth position of conducting loop in uniform field.

position of Fig. 257, no lines are being cut—the rate of cutting is then zero and the induced e.m.f. is therefore zero.

The uniform rotation of the loop is continued. Just as the loop leaves the "neutral" position (Fig. 257) its sides—both of them—will again commence to cut lines. But the e.m.f. now induced in each of the sides of the loop will be in the opposite direction from that induced during the first quarter of

the revolution. Apply the hand rule of Art. 427 and verify the directions of the arrows showing the e.m.f. directions in Fig. 258. The e.m.f. is now *in* in the black side of the loop and *out* in the white side. The brush *W* now becomes positive and *B* becomes negative. Compare this with the reverse condition of Fig. 255.

The current impelled through the loop and external circuit by this induced e.m.f. will reverse in direction as the direction of the e.m.f. reverses As the uniform revolution of the loop is continued through the positions of Figs. 259, 260, 261 and 262, e m.fs. will be induced during the time that the sides of the loop are cutting lines and none will be induced at the instants (Figs. 257 and 261) when they are not cutting. The e m f. induced at the instant of Fig. 259 will again be a maximum because at this instant the loop is cutting directly across lines—cutting at the maximum rate. Study the illustrations and verify the directions of the e.m fs. and the currents indicated by the arrows. Also verify the polarity signs at the brushes.

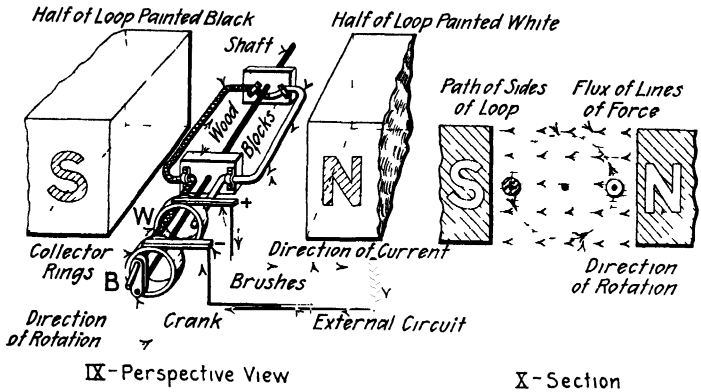

FIG. 259.—Fifth position of conducting loop in uniform field.

Fig. 263 shows in one picture the positions of a rotating loop at five different instants. Note that this loop is being turned in a clockwise direction and that it starts, at *I*, from the neutral position where, for an instant, the sides of the loop are moving in a direction parallel to that of the flux—and the induced e.m.f. at this instant is zero. The *sine curve* (Art. 517) in the lower part of the illustration indicates graphically how the e.m.f. induced varies in strength and direction as the uniform rotation continues. The vertical distance at any point between the horizontal line and the sine curve itself is proportional to the e.m.f. induced at the corresponding instant.

515. Why the E.m.f. Induced in a Loop Rotating at a Uniform Speed in a Uniform Field Is Different at Different Instants may be better understood from a consideration of the example of

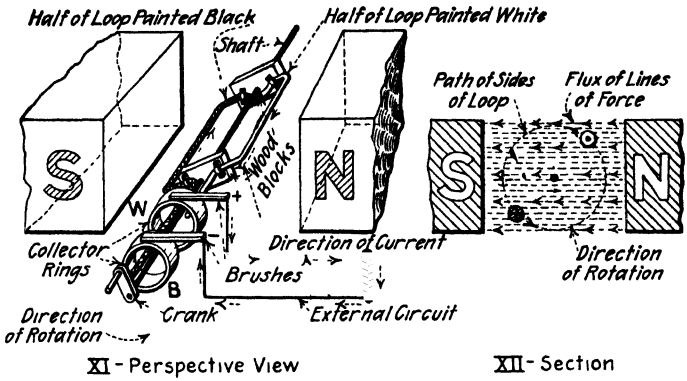

FIG. 260.—Sixth position of conducting loop in uniform field.

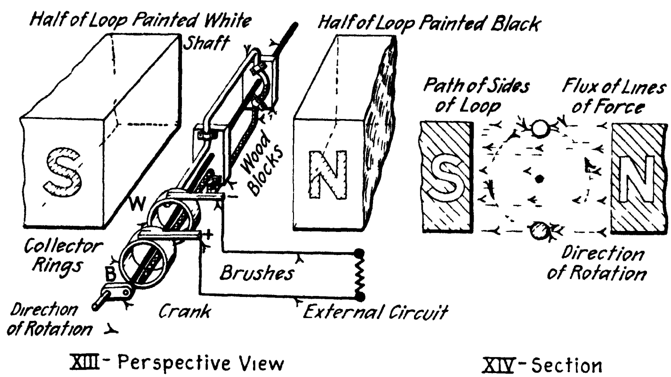

FIG. 261.—Seventh position of conducting loop in uniform field.

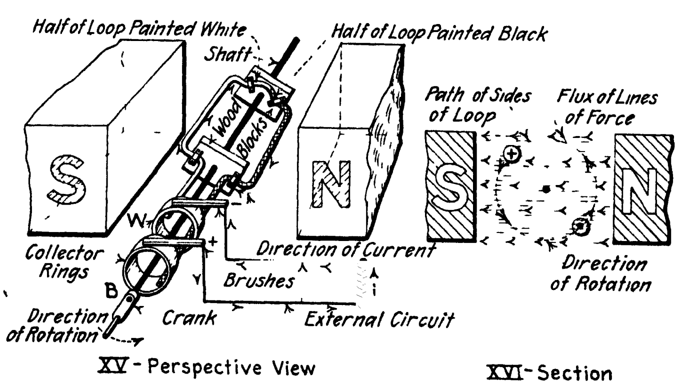

FIG. 262.—Eighth position of conducting loop in uniform field.

Figs. 264 and 264A. Imagine that the loop $A'A$ is revolved at a uniform rate (at a steady speed of 1 complete revolution in 16 sec.) in the uniform magnetic field (Art. 61) shown.

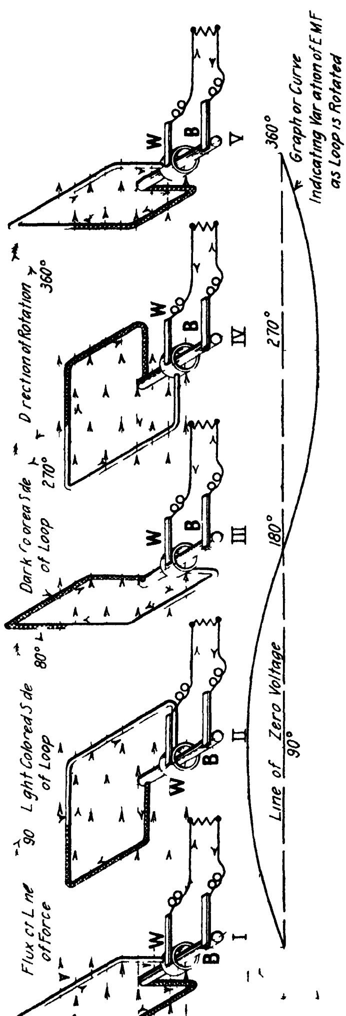

FIG. 263.—Illustrating the relation of the values of a sine curve to the e. m. fs. induced at different instants during the revolution of a conducting loop.

EXAMPLE.—As the loop moves from position $A'A$ to $B'B$, both of its sides S' and S will cut lines. Thereby an e. m. f. will be induced in the loop as described in a preceding article. One complete revolution, that is the circumference of a circle, is always equivalent to 360 degrees. From position $A'A$ to $B'B$ is $\frac{1}{16}$ revolution, hence it is $360 \div 16 = 22\frac{1}{2}$ degrees. Since

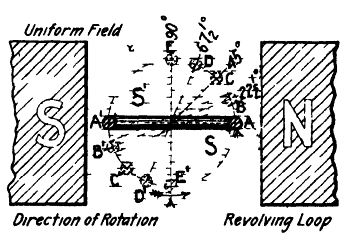

FIG. 264.—Showing how the rate of cutting lines varies as a loop is rotated in a uniform magnetic field.

16 sec. is required for 1 revolution, 1 sec. will be required for $\frac{1}{16}$ revolution or $22\frac{1}{2}$ degrees. In moving from A to B side S cuts 5 of the lines of force shown—in 1 sec. But in moving from B to C, S cuts only 3 lines—in 1 sec. In moving from C to D, S cuts 2 lines—in 1 sec. In moving from D to E, S cuts 1 line—in 1 sec.

Obviously the *rate of cutting*—the number of lines cut per second—decreases as side S approaches position E. The other side of the loop S' is, also, cutting lines at the same rate as is S. And the e. m. f. induced in side S' acts in unison around the loop with that induced in side S, as suggested in Fig. 255. The e. m. f. induced in S, while it is moved from E around to A', will obviously increase as A' is approached. It will be a maximum at the instant of position A' when S will be cutting directly

across lines. Likewise, the e.m.f. induced in the sides of the loop varies as rotation is continued

The example given above explains the situation in general terms and outlines in an illustrative way an important truth. However, the values given should be considered as qualitative rather than quantitative. The actual rate of cutting at position *E* is, at that instant, zero.

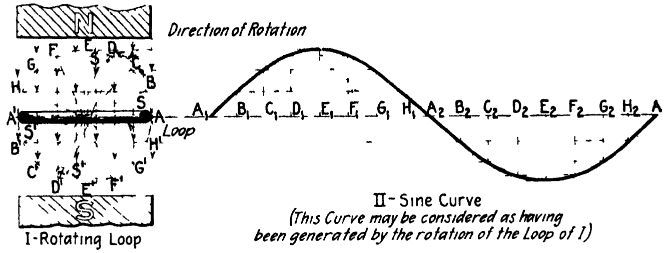

FIG 264A.—The "generation" of a sine curve.

516. The Sine (abbreviated "sin") of an acute angle of a right-angled triangle is *that value by which the length of the hypotenuse must be multiplied to obtain the length of the side opposite the angle.* Elaborate tables of sines of all angles have been computed and may be found in engineering handbooks. There is a condensed table of sines of angles in the author's AMERICAN

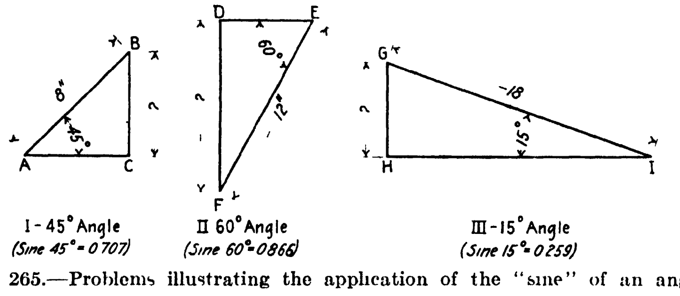

FIG. 265.—Problems illustrating the application of the "sine" of an angle

ELECTRICIANS' HANDBOOK under the heading "Trigonometric Functions."

EXAMPLE.—The angle *A* of the triangle of *I* of Fig 265 is known to be 45 degrees and the length of the hypotenuse *AB* is known to be 8 in. What is the length of the side *BC*, which is opposite angle *A*? SOLUTION.—Referring to any table of sines of trigonometric functions, it will be found that the sine of 45 degrees is 0.707. Hence, it follows from the above-given

definition of "sine" that if the length of the hypotenuse AB be multiplied by this value "0.707," the length of BC will be the result. Thus: $8 \times 0.707 = 5.65$ *in.*, which is the length of the side BC.

EXAMPLE.—What is the length of the side DF of triangle DEF? SOLUTION.—From a table it is ascertained that the sine of 60 degrees is 0 866. Then: $0.866 \times 12 = 10.4$ in., which is the length of DF.

EXAMPLE.—The sine of 15 degrees is 0.259, therefore the length of GH of triangle GHI is: $18 \times 0.259 = 4.66$ *in.*

517. A Sine Curve or Sinusoid (Fig. 266) is a curve whose abscissas (horizontal distances from the origin or starting point) represent the lengths of arcs and whose ordinates (vertical distances from the base line) represent the corresponding sines.

EXAMPLE.—The circumference of the circle of Fig. 266, *I* (having a radius equal to 1) is divided into equal arcs AB, BC, etc. The length of a line, such as Bb, Cc, etc., drawn from the left termination of each section (AB, AC, AD, etc.) to the horizontal diameter IA, is the sine of the opposite or included angle. For example: length Bb is equal to the sine of $22\frac{1}{2}$

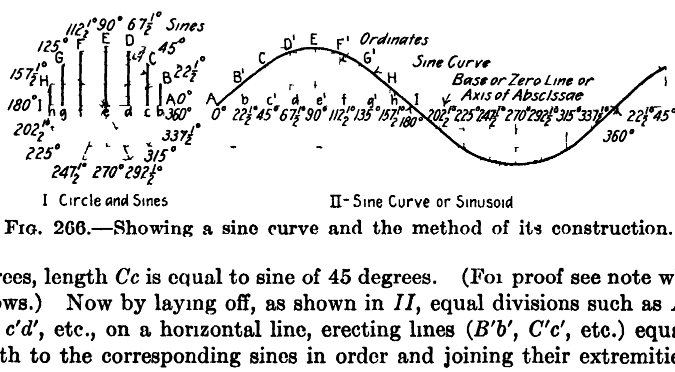

FIG. 266.—Showing a sine curve and the method of its construction.

degrees, length Cc is equal to sine of 45 degrees. (For proof see note which follows.) Now by laying off, as shown in *II*, equal divisions such as $A'b'$, $b'c'$, $c'd'$, etc., on a horizontal line, erecting lines ($B'b'$, $C'c'$, etc.) equal in length to the corresponding sines in order and joining their extremities, a sine curve or sinusoid is formed.

NOTE.—Why the lengths of the lines Bb, Cc, Dd, etc., are equal to the sines of the included angles opposite them may be explained in this way: It follows from the definition of a sine of Art. 516. The hypotenuses eB, eC, eD, etc., of the different triangles in the circle are all the same length, being in each case the radius of the circle. The length of this radius may be taken as being equal to 1. Then if length eB be multiplied by sine of $22\frac{1}{2}$ degrees, the result should be length Bb—from the definition of sine of Art. 516. Hence length of $Bb = \sin 22\frac{1}{2}$ degrees The same sort of proof can be written for the other sines, Cc, Dd, etc.

518. The E.m.f. Induced at Any Instant in a Loop, Rotated at a Uniform Speed in a Uniform Field, Varies as the Sine of the Angle Through Which the Loop Has Been Turned from the neutral plane which lies at right angles to the field. Art. 515

and Fig. 264 indicate *why* the e.m.f. induced in a rotating loop varies at different instants as the loop is passing through different positions in the field—but they do not indicate "how much"

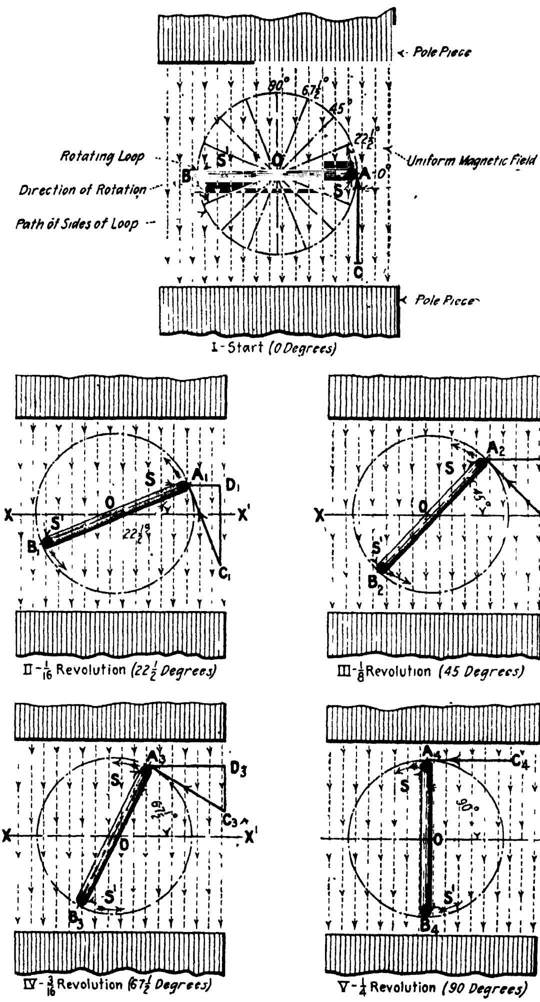

FIG. 267.—Showing why the e.m.f. induced in a loop varies as the sine of the angle through which the loop is turned.

the e.m.f. varies. By carefully considering the diagrams of Fig. 267 in combination with the following example it will be evident that the variation, from instant to instant, in the in-

tensity of the induced e.m.f., as the loop is rotated, will be as stated in the opening paragraph of this article.

EXAMPLE.—The loop BA (Fig. 267, I) is being rotated in a counterclockwise direction in the uniform field shown. It is a fact that a line, AC, drawn at right angles to BA (that is, tangent to the circle outlining the path of the loop) will when drawn to some—any—scale, represent the direction and velocity of the movement of the side S of the loop at the instant shown. The line AC should be drawn to scale proportional in length to the velocity of the side S. The direction of the line shows the direction of S at that instant.

Diagrams I to V inclusive of Fig. 267 show different positions of the loop as it is being rotated in the field. In each of the five positions, the length of line AC (or A_1C_1, A_2C_2, etc.) indicates the actual velocity or speed of the loop at the instant pictured. Since the speed is uniform, AC, A_1C_1, A_2C_2, etc., are all the same length in each of the five diagrams shown. The direction of the line AC, etc., in each of the five diagrams shows the actual direction of the side S of the loop at each of the five instants pictured.

Now at the instant of I, the side S of the loop is, as represented by line AC, in the neutral plane and is moving directly parallel to the direction of the flux. Side S is, therefore, cutting no flux at this instant and the e.m.f. in it is zero. But consider the instant of II: A_1C_1 represents the direction and speed of side S at this instant. Now *any* line, for instance A_1C_1, which represents graphically the movement of a thing, may be resolved into two components or parts at right angles to each other.

Thus, A_1D_1 and D_1C_1 are components of A_1C_1. It is a fact that A_1D_1 actually represents the instantaneous movement of side S in a direction at right angles to the lines of force The direction of A_1D_1 represents the direction of movement at this instant. The length of A_1D_1 represents or is proportional to the speed of S in the horizontal direction at this instant. The actual rate at which S cuts lines is, obviously, proportional to the speed with which it cuts horizontally across the lines that is, at right angles to the lines. This speed is represented at the instant of II by the length A_1D_1.

Now A_1D_1 is proportional to the sine of angle C_1 (Art. 516). But, *angle* C_1 = *angle* O, as can be shown by geometry. And angle O is the angle through which the loop has turned from the neutral plane. Since A_1D_1 represents the speed of S at right angles across lines at the instant shown, it represents the actual rate of cutting lines at this instant. Hence, it is apparent that the rate of cutting lines is proportional to the sine of the angle through which the loop has been turned from the neutral plane. Since O in picture II is 22½ degrees the rate of cutting lines—or the length of A_1D_1—is at this instant, proportional to the sine of 22½ degrees.

Similar reasoning is followed for the three other positions of the loop diagrammed: At the instant of III, the actual *rate of cutting lines* is represented by line A_2D_2 which is proportional to the sine of 45 degrees. At the instant of IV, the actual rate of cutting lines is represented by A_3D_3 which is proportional to the sine of 67½ degrees. And, at the instant of V, the actual rate of cutting lines—now a maximum—is represented by A_4C_4, the actual speed of rotation, because the sine of 90 degrees is 1. It

has, therefore, been shown that *the induced e.m.f. varies as the sine of the angle through which the loop has turned.*

519. The Elementary Alternating-current Generator is, obviously, shown in Figs. 255 to 262. The e.m.f. induced in the loop of this arrangement and plotted in the graphs of Figs. 263 and 268,*I* is, since it alternates regularly in direction, by the definition of Art. 113, an *alternating e.m.f.* If the revolving coil be connected to an external circuit since the resistance of the external circuit of Fig. 255 remains constant, the current

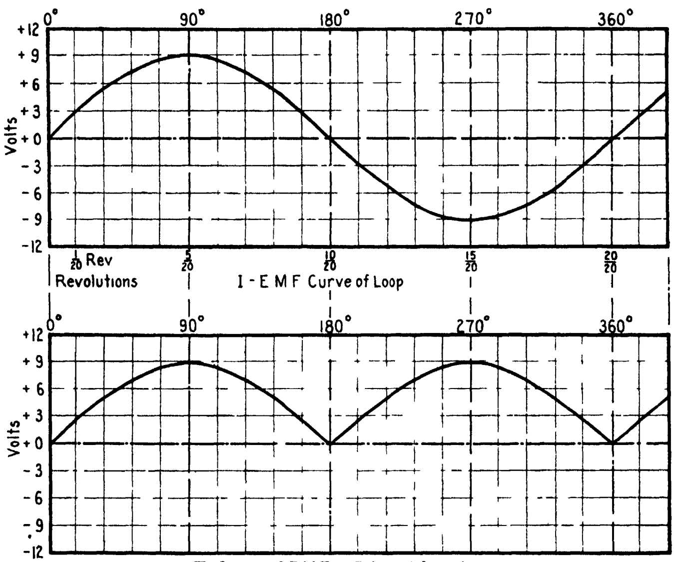

FIG. 268.—Graphs of e.m.fs. in the loop and in the external circuit of an elementary direct-current generator.

in amperes forced through this circuit by the e.m.f. induced in the revolving loop must obviously be an *alternating current.* Now the e.m.f. induced in a coil or loop being rotated in a uniform field varies (Art. 518) as the sine of the angle through which the loop has turned from the neutral plane. It follows, therefore, that the curve of e.m.f. of this loop (Fig. 268) is a sine curve (Art. 517).

520. An Alternating E.m.f. Is Generated in Any Loop Rotated in a Magnetic Field even through the e.m.f. (rectified with a commutator, Art. 529) impressed on the external circuit be a

direct e.m.f. and the current in the external circuit a direct current (Art. 107). Fig. 268,*I* and *II* illustrates this principle. Then, the current in the *armature* (Art. 534) of any direct-current machine is an alternating current—though of course the current in the external circuit is a direct current.

521. Meaning of "Positive" and "Negative" Direction of Rotation.—The counterclockwise direction of rotation is usually called, or assumed to be, the "positive" direction of rotation. *Counterclockwise* means opposite in direction to the direction of rotation of the hands of a clock. Likewise, the *clockwise* direction of rotation is usually designated as the "negative" direction of rotation. The terms as above defined have been arbitrarily selected and, while the definitions given are the generally accepted ones, there is no real reason why they could not be reversed. When used to indicate direction of rotation, then, the term "positive" merely means the opposite of "negative" and neither term has anything to do (except possibly indirectly) with the positive or negative polarities of the circuit.

522. Meaning of "Positive" and "Negative" Directions of Alternating E.m.fs. and Voltages (review Art. 112).—In Fig. 255,*I* the current is flowing away from brush *B* and into brush *W*. But in Fig. 259,*IX*, current flows into brush *B* and out of brush *W*. Obviously, the direction of the current in Fig. 259 is just opposite to that of the current in Fig. 255. To distinguish between these two possible directions of e.m.f. or of current flow in any circuit, one (either one) direction is called the "positive" direction and then the other is then designated as the "negative" direction.

If the loop of Fig. 264 be rotated in a counterclockwise direction from the position AA', the direction of the induced e.m.f. will be *out S and into S'*. Now if the loop from position AA' is rotated in a clockwise direction (Art. 521) the induced e.m.f. direction will again be out of S and into S'. (If the rotation of the loop is started from any position other than one parallel with the flux, AA', Fig. 264,*A*, the e.m.f. will be in one direction if the rotation is commenced counterclockwise and in the other direction if it is commenced clockwise). Verify these statements by applying the hand rule of Art. 427. If the direction of the field is reversed, the direction of the induced e.m.f. will be correspondingly reversed. If in Fig. 264, the direction *into S' and out of S* is taken as the positive direction, then the direc-

tion *out of S' and into S* will be the negative direction. On this basis, considering S alone, the positive direction is *out of S*, toward the reader; the negative direction is *into S*, away from the reader.

NOTE.—It is well to designate the positive direction of e.m.f. in each case, as the direction of the e.m.f. or current in the loop while the loop is being rotated in a *counterclockwise direction* through the first 180 degrees from the neutral axis. Then the direction of e.m.f. during the next 180 degrees will be negative. The reason why it is well to so designate the directions is this: the values of a sine curve that are plotted above the neutral or zero line (as $A'C'D'$ of Fig. 266) are usually most conveniently designated as positive. That is, they usually indicate values of e.m.fs. or currents that are in what is arbitrarily designated a positive direction. Values plotted below the zero line are then regarded as negative. If the positive direction of e.m.f. is taken as that just suggested, this convention will be preserved. This matter of positive and negative directions is, then, merely one of convention and either direction of e.m.f. or current may be regarded as positive as the loop rotates through the first 180 degrees from the neutral plane, provided the directions of e.m.f. and current as the loop rotates through the remaining 180 degrees is regarded as negative.

NOTE.—*Positive* and *negative* directions of currents or e.m.fs. must not be confused with positive or negative polarities of electric circuits. A "positive" or "negative" direction of current or e.m.f. bears no particular relation (except perhaps, indirectly) to the positive and negative polarities of a circuit. However, the polarities of a circuit do change as the directions of the e.m.f. and current change. There are, then, at least three possible usages of the terms "positive" and "negative:" (1) *To indicate polarity of circuits*, Art. 97A. (2) *To indicate direction of rotation*, Art. 521. (3) *To indicate direction of current or e.m.f.* as described in the article just preceding.

523. A Graphic Statement of the Variation of the E.m.f. Values Induced in the Loop of Fig. 255 at the different instants of its revolution can be plotted into a graph like that of Fig. 268, *I*. Assuming that the loop starts from the neutral position of Fig. 257, this graph of Fig. 268, *I* shows how the e.m.f. induced in the loop increases from zero and attains a maximum value in one direction (Fig. 259), which can be designated arbitrarily (Art. 522) as the positive (+) direction, at $5/20$ or $1/4$ revolution.

NOTE.—The maximum e.m.f. in the illustration plotted in Fig. 268, *I* is shown to be 9 volts. This means that the uniform speed at which the loop is being rotated and the number of lines in the flux are such that at the instants when the sides of the loop are passing through the positions of Figs. 259, *IX* and *X*, and 255 the e.m.f. induced is just 9 volts—$4\frac{1}{2}$ volts in each side of the loop.

Fig. 268, *I*, also shows how, as rotation is continued, the e.m.f. decreases to zero at $^{10}/_{20}$ (or $\frac{1}{2}$) revolution (Fig. 261), how it reaches its greatest value in the other direction, which we can call the negative (—) direction (Art. 522), at $^{15}/_{20}$ or $\frac{3}{4}$ revolution (Fig. 255) and how it then again becomes zero at $^{20}/_{20}$ or 1 revolution (Fig. 257). Note that the maximum negative and positive e.m.fs. are equal in value but opposite in direction. Any corresponding negative and positive (Art. 522) locations on the curve are of equal value but of opposite directions. The e.m.f. generated by the loop will continue to vary as shown by the graph (Fig. 268, *I*), as long as the loop is rotated uniformly. This graph or curve is a sine curve (Art. 517).

EXAMPLE.—In Table 525, in the column headed "*E*," are shown numerical values indicating how a loop being rotated in a field and at a speed such that

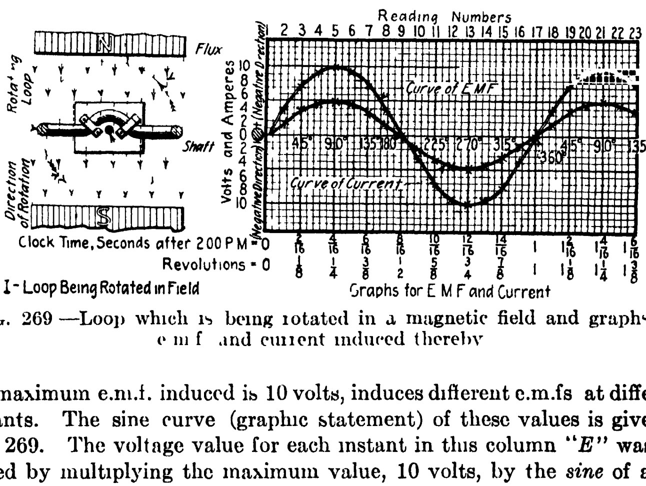

FIG. 269—Loop which is being rotated in a magnetic field and graphs of e m f and current induced thereby

the maximum e.m.f. induced is 10 volts, induces different e.m.fs at different instants. The sine curve (graphic statement) of these values is given in Fig. 269. The voltage value for each instant in this column "*E*" was obtained by multiplying the maximum value, 10 volts, by the *sine* of angle through which the loop had been rotated (from the neutral position shown at Fig. 269, *I*) up to that instant.

524. The E.m.fs. and Currents Developed by Alternating-current Generators Have Sine-wave Forms or wave forms that closely approximate that of a true sine wave (see Arts. 517 and 519). The current in any circuit varies directly as the applied e.m.f. It follows from this that, if the e.m.f. wave has a sine-wave form, any current due to this e.m.f. must also have a sine-wave form. This does not mean that the curve representing current (see Fig. 269) will be the same size as the curve repre-

senting voltage; it only means that both will have the same general shape or form. (See Fig. 269.) The derivations of the formulas for many alternating-current calculations are based on the assumption of perfect sine-wave forms for alternating e.m.fs. and currents. The reason for this is that it is only on the basis of true sine curves that is, true sine-wave forms, that computations can be readily made or equations derived.

525. Table Indicating E.m.fs. Induced at Given Instants (by a loop revolved in a field) and the Instantaneous Currents Impelled thereby.

Resistance of circuit = 2 ohms.

Reading Numbers	Time at which reading is taken: Clock time	Degrees	Revolutions	Direction of e.m.f. and current	E, volts. E.m.f. induced at the given instant	I, amperes. Current forced through the circuit at the given instant, $I = E \div R =$
1	2 : 00 P.M.	0°	0	......	0.00	0 ÷ 2 = 0.0 amp.
2	1/16 sec. after 2	22½	1/16	+	3.83	3.83 ÷ 2 = 1.9 amp.
3	2/16 sec. after 2	45	1/8	+	7.07	7.07 ÷ 2 = 3.5 amp.
4	3/16 sec. after 2	67½	3/16	+	9.24	9.24 ÷ 2 = 4.6 amp.
5	4/16 sec. after 2	90	1/4	+	10.00	10.00 ÷ 2 = 5.0 amp.
6	5/16 sec. after 2	112½	5/16	+	9.24	9.24 ÷ 2 = 4.6 amp.
7	6/16 sec. after 2	135	3/8	+	7.07	7.07 ÷ 2 = 3.5 amp.
8	7/16 sec. after 2	157½	7/16	+	3.83	3.83 ÷ 2 = 1.9 amp.
9	8/16 sec. after 2	180	1/2	+	0.00	0.00 ÷ 2 = 0.0 amp.
10	9/16 sec. after 2	202½	9/16	−	3.83	3.83 ÷ 2 = 1.9 amp.
11	10/16 sec. after 2	225	5/8	−	7.07	7.07 ÷ 2 = 3.5 amp.
12	11/16 sec. after 2	247½	11/16	−	9.24	9.24 ÷ 2 = 4.6 amp.
13	12/16 sec. after 2	270	3/4	−	10.00	10.00 ÷ 2 = 5.0 amp.
14	13/16 sec. after 2	292½	13/16	−	9.24	9.24 ÷ 2 = 4.6 amp.
15	14/16 sec. after 2	315	7/8	−	7.07	7.07 ÷ 2 = 3.5 amp.
16	15/16 sec. after 2	337½	15/16	−	3.83	3.83 ÷ 2 = 1.9 amp.
17	1 sec. after 2	360	1	−	0.00	0.00 ÷ 2 = 0.0 amp.
18	1 1/16 sec. after 2	22½	1 1/16	+	3.83	3.83 ÷ 2 = 1.9 amp.
19	1 2/16 sec. after 2	45	1 1/8	+	7.07	7.07 ÷ 2 = 3.5 amp.
20	1 3/16 sec. after 2	67½	1 3/16	+	9.24	9.24 ÷ 2 = 4.6 amp.
21	1 4/16 sec. after 2	90	1 1/4	+	10.00	10.00 ÷ 2 = 5.0 amp.
22	1 5/16 sec. after 2	112½	1 5/16	+	9.24	9.24 ÷ 2 = 4.6 amp.
23	1 6/16 sec. after 2	135	1 3/8	+	7.07	7.07 ÷ 2 = 3.5 amp.

526. A Sine Curve May Represent the Variation of an Alternating E.m.f. or of an Alternating Current with the Time. (See Art. 719.)—Sine curves may be drawn to any convenient scale.

EXAMPLE.—The heavy line lying radially in the circle to the left of Fig. 270, which may be considered as revolving (counterclockwise) in the direction of the arrow, is called a vector; see also Art. 719. Its length is proportional to the maximum value (Art. 689) of the alternating e.m.f. or the current which it represents. The vertical distance from the arrow-head point, *A*, of the vector to the horizontal zero line, *CD*, is, at any given instant as the vector revolves proportional to the instantaneous value of the e.m.f. or the current at that instant. The lengths of the lines *AB* represent the instantaneous values at the 45-degree instant.

The length of any such line representing an instantaneous value is proportional to the trigonometric sine (Art. 516) of the angle lying between the vector and the horizontal line. It is because of this fact that a curve like that to the right of Fig. 270 is called a sine curve.

527. The Alternating Current (amperes) which is forced through the circuit of Fig. 255 by the alternating e.m.f. (Art. 680) induced in the loop, will also vary as the loop is turned, that is, it varies as the time elapses. The rate of variation will, as with the e.m.f., be proportional to *the sine of the angle* through which the loop has been turned from the neutral position.

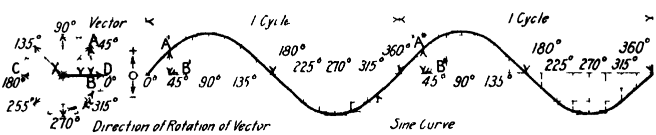

FIG. 270.—Showing a rotating vector and the sine curve "generated" thereby.

The graph or curve indicating this rate of variation will be a sine curve. For example, again starting at the neutral position of Fig. 257, *V*, the current through the circuits at this instant would be zero. As the loop is rotated (Fig. 258,*VII*), current is forced through the circuit in a direction from collector ring and brush *W* toward and into *B*.

The current (amperes) gradually increases in accordance with the sine law from position Fig. 257,*I*, to that of Fig. 259, *IX*. The loop has now been turned through 90 degrees and the direction of motion of its sides is at right angles to the direction of the flux—the e.m.f. and current are now a maximum. As the rotation of the loop is continued (Fig. 260), the current decreases until, it having turned through 180 degrees, the position of Fig. 261,*XIII*, is reached—and the e.m.f. and current are again zero. Rotation continuing (Fig. 262,*XV*), current is again forced through the circuit but it now flows away from *B* and into *W*—it has reversed in direction.

The same sequence of increases and decreases and changes in direction occur periodically and repeatedly so long as rotation is continued. Since the current reverses periodically, that is with the time, in direction, obviously, it is by the definition of Art. 113 an alternating current. Note that the current attains its maximum value when the loop has been turned through 180 degrees from the neutral or starting position.

EXAMPLE.—The last column of Table 525 shows the values of current at different instants as the loop of Fig. 271 is rotated in its field. The maximum e.m.f. induced is 10 volts. The resistance of the external circuit is 2 ohms and the resistance of the loop itself is assumed to be zero. Fig. 269,*II* shows the graph of this current.

528. A Graphic or Sine-curve Portraiture of an Alternating E.m.f. and the Current It Produces is shown in Fig. 269 for the circuit of Fig. 271. The current in any circuit varies as the e.m.f. which is forcing it through the circuit varies. This must be true in order that the requirements of Ohm's law (Art. 134) be satisfied. Since an alternating e.m.f. can be represented by a sine curve (Art. 526), it follows that its alternating current may also be represented by a sine curve. However, the sine curve of a current will be numerically equal to (the same size as) the curve of the e.m.f. that produces it only when the resistance is numerically equal to one. This follows, because $I = E \div R$; now if $R = 1$ then, $I = E \div 1$, that is then, $I = E$. If R is greater than 1 *and the graphs of both e.m.f. and current are plotted to the same numerical scale,* the current curve will lie within the e.m.f. curve as shown in Fig. 269. But if R is less than 1, the current curve will lie without the e.m.f. curve.

EXAMPLE.—Consider the elementary generator and circuit of Fig. 271. It will be assumed that the loop there shown is being rotated at the *uniform* rate of 1 turn (or 1 revolution) per sec. The loop is assumed to have no resistance but the external circuit has 2 ohms resistance. The flux in the field is assumed to be such that the maximum instantaneous e.m.f. induced in the loop is 10 volts. That is, at the instants when the sides of the loop are cutting lines at right angles—the maximum rate—the e.m.f. then induced is 10 volts. The e.m.fs. induced at other instants can be ascertained as suggested in Art. 518 by multiplying this maximum e.m.f. by the sine of the angle between *the neutral position of the loop* and *its position at the given instant.*

The values of the instantaneous e.m.fs. induced at successive instants are shown in column "E" of the Table 525. These values were computed as

described above. Now, consider what occurs as the loop, being started from the neutral position shown, is rotated at this uniform rate of 1 turn per sec.

Assume that rotation is commenced at just 2:00 P.M.—it could be commenced at any other time just as well but 2:00 P.M. will be taken so that there will be a definite starting time. As it is started, at this instant—at 2:00 P.M.—the loop does not cut any lines because, at this instant, its sides are moving parallel to the direction of the flux. Hence, at this instant, the induced e.m.f. is zero as shown in Art. 525. Furthermore, no current is forced through the external circuit at this instant because the e.m.f. is then zero. As the rotation is continued, the sides of the loop begin to shear through lines and thereby e.m.fs. are induced in the loop.

However, the e.m.fs. induced at successive instants increase as the rotation is continued (through the first 90 degrees) because as the sides of the loop move further away from the neutral position their rate of cutting becomes greater—they cut more lines per second.

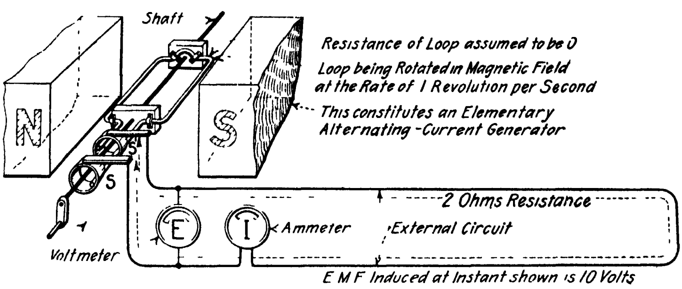

FIG. 271.—Elementary alternating-current generator forcing current through an external circuit. (The sine curves of e.m.f. and current for this arrangement are shown in Fig. 269.)

Now, consider the instant when the loop has been rotated just 22½ degrees away from the neutral position. The e.m.f. induced at this instant is shown as 3.83 volts. Why 3.83 volts? This is the reason: The maximum e.m.f. has been assumed to be 10 volts. If this value of 10 volts is multiplied by the sine of 22½ degrees, which is 0.383, the instantaneous e.m.f. at the 22½-degree position or instant should be the result, as suggested in Art. 518. Thus: 10 *volts* × 0.383 = 3.83 *volts*. Note that this 22½-degree position corresponds in every instance to $\frac{1}{16}$ revolution because 22½ degrees is $\frac{1}{16}$ of 360 degrees and 360 degrees always represents a complete revolution.

Note also that since the loop started at 2:00 P.M. and is being rotated at a uniform rate of 1 turn or revolution per sec., the loop will have reached the 22½-degree position just $\frac{1}{16}$ sec. after 2:00 P.M. It is evident that the position of the loop can be designated by any one of these methods: (1) *by degrees*, (2) *by revolutions* or parts thereof and (3) *by time* or by fractions or multiples of a second. It is so designated for a number of different positions in the table of 524.

What current is being forced through the circuit of Fig. 271 at this 22½-

degree instant? In the preceding article it was ascertained that the e.m.f. developed at this instant is 3.83 volts. The entire circuit on which this e.m.f. is impressed comprises the loop itself and the external circuit. But it has been assumed that the loop has zero resistance. Hence the total resistance of the entire circuit is 2 ohms, the resistance of the external circuit.

By Ohm's law: $I = E \div R$. Then, the current at this $22\frac{1}{2}$-degree instant is: $I = E \div R = 3.83 \div 2 = 1.9$ *amp.* The current curve in Fig. 269 is plotted accordingly.

By proceeding as suggested above, the e.m.f. and current values at any other instants during a revolution of the loop can be ascertained. They have, accordingly, been computed and shown in Art. 525 for successive instants, $22\frac{1}{2}$ degrees—or $\frac{1}{16}$ sec. or $\frac{1}{16}$ revolution—apart. These values have then been plotted in the graph of Fig. 269. This graph shows pictorially the relation of an alternating e.m.f. to the current it forces through a circuit under the conditions specified. Since it requires $\frac{2}{16}$ or $\frac{1}{8}$ sec. for the e.m.f. of this elementary generator (Fig. 271) to complete 1 cycle, it has a *frequency* (Art. 682) of 8 *cycles per sec.*

After the loop has rotated through 180 degrees the direction of the e.m.f. induced in it will be reversed as described in Art. 514. The direction of the current will then be reversed correspondingly. This is shown by the change from the + to the − sign in the table. These signs indicate the *directions* and not the polarities of the e.m.fs. and currents as described in Art. 522.

As long as the rotation of the loop is continued, the e.m.f. and current will continue to vary and reverse regularly. The reversals will occur at each successive $\frac{1}{16}$ sec. The current and e.m.f. will vary in accordance with the *sine law* as indicated in the graph of Fig. 269.

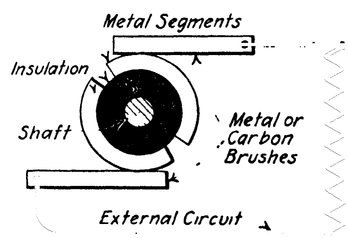

FIG. 272.—Sectional view of two-segment commutator.

529. A Commutator (Fig. 272) may be defined as a device or rectifier for changing in one portion of a circuit the directions of the e.m.f. or current produced in another portion. Ordinarily, a commutator is used for changing an alternating e.m.f. or current to a direct e.m.f. or current. The process involved in this changing may be referred to as *commutation* or *rectification.* Large commutators are shown in Figs. 314 and 315.

EXAMPLE.—The commutator on the loop of the elementary generator of Fig. 273 rectifies the alternating e.m.f. induced in the loop (the graph of which is shown in the lower portion of the figure) in Fig. 268,*I* so that a direct e.m.f. (Fig. 268, *II*) is impressed on the external circuit. This process of rectification is described in another article.

530. To Produce a Direct E.m.f. with a Generator, the Alternating E.m.f. Induced in Its Armature Is Rectified with a Commutator.—A simple commutator is shown in Fig. 273 connected to a loop which may be rotated in a magnetic field. One-half of the loop and its commutator segment are painted black and the other side and segment are white, merely for identification. A consideration of the following example will make it clear as to how a commutator *rectifies* the alternating e.m.f. which is always induced (Art. 520), in a loop which is rotated in a field.

EXAMPLE.—Refer to Fig. 273. The commutator segments are electrically connected to their respective sides of the loop but they are insulated from

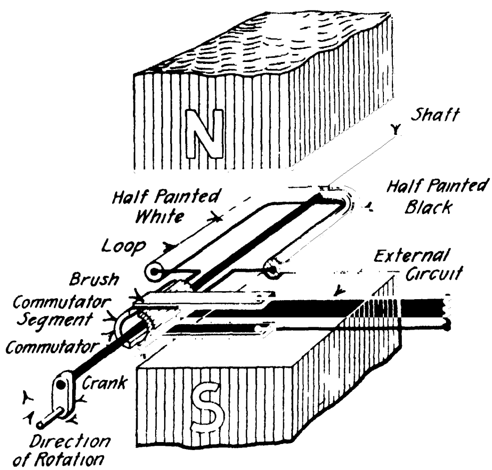

FIG. 273.—Commutator arranged on loop in field.

each other and from the shaft by an air gap. The shaft is insulated from the loop. Assume that the loop is rotated at a uniform rate in a clockwise direction by turning the crank. Fig. 274,*I*, which is a duplicate of 273 except that it is a simplified sectional view, shows conditions at the starting instant. No. e.m.f. is induced in the loop at this instant because its sides are then moving in a direction parallel to that of the flux.

Rotation is continued. After the loop has been rotated through $\frac{1}{20}$ revolution the conditions are as indicated at *II*. At this instant the sides of the loop are cutting lines and an e.m.f. is being induced which forces current through the loop and external circuit. By applying the hand rule of Art. 513 it will be found that the direction of the e.m.f. and current is *in* in the white side of the loop (shown by the cross) and *out* of the black side (as shown by the dot). Hence, the current is forced out through the brush marked + and enters the brush marked −. B_2 is then the positive polarity

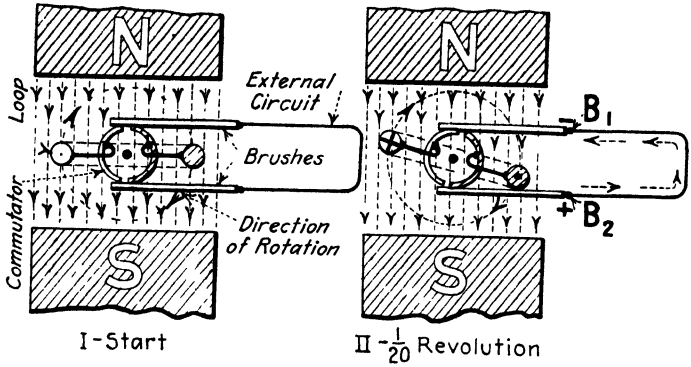

FIG. 274.—Conducting loop with commutator rotating in a field.

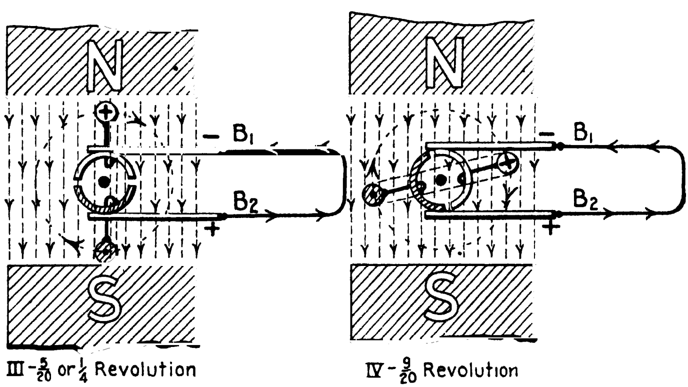

FIG. 275.—First positions as loop is started in rotation.

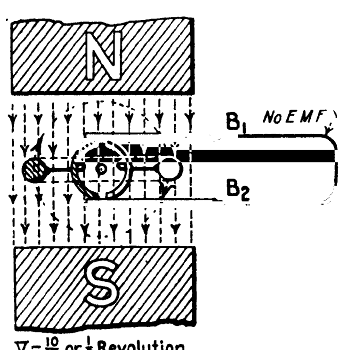

FIG. 276.—Neutral position of loop.

(+) brush because the e.m.f. is directed away from it and because the current flows out of it. B_1 is the negative polarity (−) brush because the direction of the e.m.f. is into it and because the current flows into it.

If this elementary generator is to impress a direct e.m.f. on and force a direct current through its external circuit as long as its rotation is continued, the current must continue to flow out of B_2 and to flow into B_1. It will, by virtue of the commutator, do this as will be shown.

As the rotation of the loop is continued it passes through the positions of Fig. 275,*III* and *IV*. The direction of the e.m.f. and current in the loop remains (hand rule, Art. 427) *out* of the black side and *in* the white side of the loop as indicated by the dot and the cross respectively in the illustration. B_1 retains its − polarity and B_2 its + polarity.

Now, as the rotation is continued, at the instant, pictured in Fig. 276,*V*, when the sides of the loop are moving parallel to the direction of the lines of force, no e.m.f. is induced in either the white or the black side of the loop.

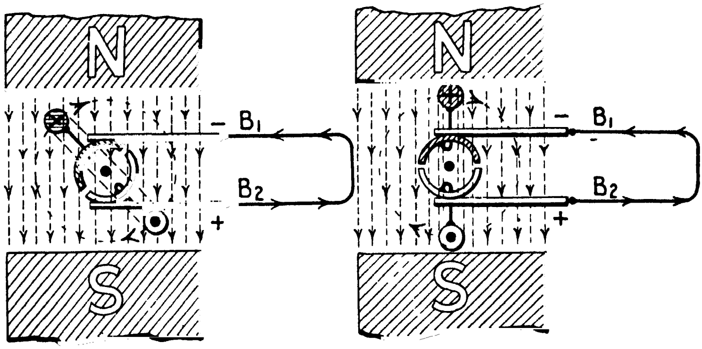

FIG 277.—Positions of conducting loop as its rotation continues.

Hence, at this instant, there can be no current—the brushes are neither positive nor negative. Note that, at this instant, the brushes bridge both commutator bars.

As the turning of the loop is continued the instant depicted in Fig. 277,*VI* is reached. The sides of the loop are again cutting lines and an e.m.f. and current are again induced. But now the current and e.m.f. direction is *in in the black side* and *out of the white side*. Note that the direction of e.m.f. and current has reversed *in the loop*. But at the instant the direction within the loop started to reverse, the white commutator segment slid out from contact with brush B_1 and the black segment slid into contact with it. A similar change occurred with the segments at brush B_2. This change of brush contact from one segment to the other at the proper instant—that is, when the direction of e.m.f. in the loop reverses—maintains the e.m.f. impressed on the external circuit always in the same direction in spite of the fact that the direction of e.m.f. in the loop changes in direction once during each revolution. This illustrates the function and action of the commu-

tator. Fig. 278 shows the situation in one illustration. Compare this with Fig. 263.

As the rotation of the loop is continued it passes successively through the positions shown in Figs. 277,*VII* and 279,*VIII* and *IX*, the direction of

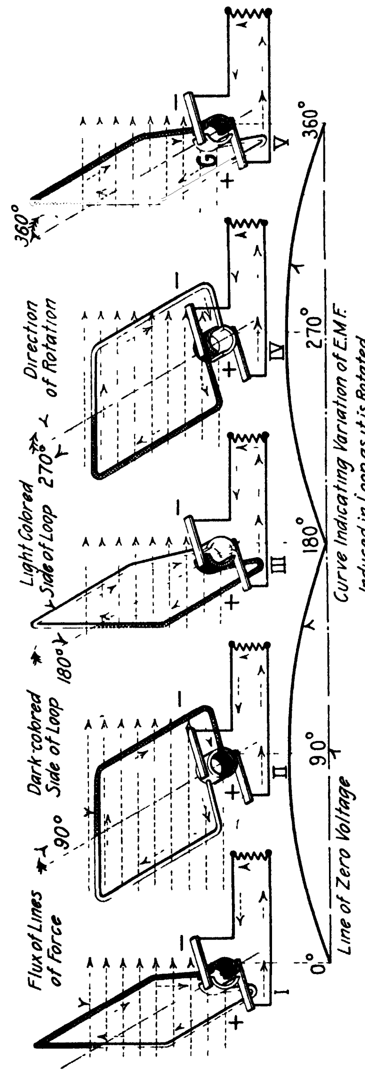

FIG. 278.—Illustrating the rectifying action of a commutator.

e.m.f. and current in the external circuit remaining out of B_2 and into B_1 during the instants when the sides of the loop are cutting lines. As the loop is rotated the e.m.f. induced in it varies at different instants as described in the discussion of the elementary alternating-current generator.

531. A Loop or a Coil of a Few Concentrated Turns When Provided with a Commutator and Rotated in a Field Produces a Pulsating E.m.f.—and current if the external circuit is closed. See Art. 110 for definition of a pulsating e.m.f. or current. Thus the graph of Fig. 268,*II*, for the arrangement of Fig. 273 is that of a pulsating current. All pulsating currents are direct currents but the reverse is not true. To produce a continuous (Art. 108) direct current the graph of which would be about like that of Fig. 298, it is necessary to interconnect several groups of coils to commutator bars in a manner indicated in Art. 565.

532. The Fundamental Difference Between an Alternating-current and a Direct-current Generator is that the alternating-current machine impresses on the external circuit connected to

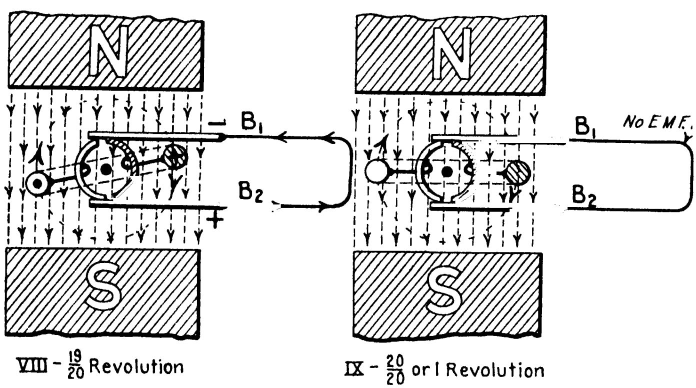

FIG. 279.—Final positions of conducting loop.

it an e.m.f. which regularly varies in magnitude and in direction in accordance with the sine law (Art. 525) as shown in the graph of Fig. 268,*I*. A direct-current generator impresses on its external circuit an e.m.f. (Fig. 249) which is always in the same direction (for a given arrangement of connections) and which remains practically constant in magnitude. These distinctions logically follow from the definitions of a direct current (Art. 107) and of an alternating current (Art. 113). The mechanical differences between these two types of machines are discussed in following articles.

533. A Graphic Comparison of the E.m.fs. of Elementary Alternating and Direct-current Generators is given in Fig. 268, wherein *I* shows the curve of e.m.f. induced in the loop and

impressed on the line by a loop like that of Fig. 255,*I* when it is rotated. It is assumed that at the instant when the sides of this loop are cutting lines at right angles—at the maximum rate—the e.m.f. is then 9 volts. The graph is plotted on this basis. Now if a commutator is added to the loop, as in Fig. 273, the e.m.f. induced in the loop when it is rotated in the field can still be represented by the graph of Fig. 268,*I*. But the e.m.f. impressed on the external circuit would then be represented by the graph of *II*. A comparison of Figs. 263 and 278 will further illustrate this idea.

534. An Armature of an Electric Generator comprises: (1) *the conducting loops in which the e.m.f. is induced,* when the loops cut or are cut by the magnetic flux, and (2) *the structure immediately associated with them.* The armature of an actual ma-

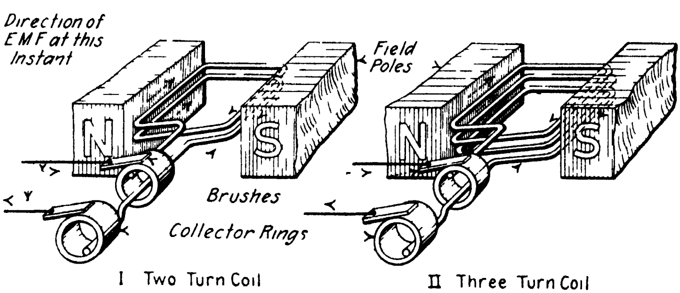

FIG. 280.—Inductor coils of two and three turns.

chine is usually thought of as comprising the armature winding or conducting loops, the iron core or structure on which the loops are wound and the necessary insulation which prevents the turns of the loops from making electrical contact with each other or with the armature iron.

EXAMPLES.—The loops of the elementary generators shown in Figs. 255, 273 and 280 may be considered as primitive revolving armatures. The revolving armature of a magneto generator is shown in Fig. 281. The revolving armature of an actual direct-current generator is shown in Fig. 312. The stationary armature of an alternating-current generator is illustrated in Fig. 394.

535. Every Generator Must Have an Armature and a Field, that is, a magnetic field (Art. 61). This statement is perfectly general and applies to all dynamos or electric generators both alternating and direct current. The distinctive features of the

field structures and armatures of the machines of these two general types are described in the specific articles relating to them.

536. Generators May Have Stationary Armatures and Revolving Fields or the Reverse.—Commercial direct-current generators always have stationary field structures and rotating armatures. Theoretically, direct-current generators could be made which would have stationary armatures and rotating fields. Modern alternating-current generators—except possibly the very smallest ones—always have rotating fields and stationary armatures.

EXAMPLES.—Figs. 285 and 286 delineate a direct-current generator and its rotating armature. Fig. 384 shows a small alternating-current generator having a stationary field structure and a revolving armature while Fig. 389 illustrates a modern alternating-current generator having a stationary armature and a rotating field structure.

537. The Factors That Determine the Voltage Developed by Any Generator are: (1) *The flux* or ϕ, that is, the number of lines of force which are cut by or cut the armature conductors. (2) The *number of cutting conductors* on the armature which cut the flux. (3) *The speed* at which the conductors move through the flux. Obviously these three factors determine the *rate of cutting* which as outlined in Art. 438 always determines the intensity of an induced e.m.f. If the flux, the number of cutting conductors or the speed of the cutting conductors is increased, the e.m.f. induced is increased proportionately. If any one or two or all of these factors is decreased the e.m.f. will be decreased accordingly. The above noted factors are combined into an equation in Art. 542.

538. The Amount of Flux Which Is Cut by the Armature Conductors of a Generator is determined largely by the size and design of the machine. Obviously it is desirable to have the flux in each case as great as is consistent with economical design. To secure the greatest flux compatible with economy the portion of the generator which carries the flux should have low reluctance (Art. 227) that is, high permeance (Art. 238). Consequenty the air gaps which the flux must cross should be as short as possible. The conductors which cut or are cut by the flux are wound on iron cores to insure low reluctance.

539. Increasing the Number of Cutting Conductors can be effected by increasing the number of turns in the armature coil—or by increasing the number of coils as described under "Direct-

current Generators." In alternating-current generators which have stationary armatures (Art. 707) the number of coils or the number of turns per coil may be increased in a somewhat similar way. For example a rotating armature coil may comprise a number of turns as in Fig. 280. The e.m.f. induced in any one turn of the coil will be equal, approximately, to that induced in each of the other turns. These e.m.fs. act in conjunction or in series and the greater the number of turns the greater the e.m.f. If 1 volt is induced in each turn, a coil of 5 turns would induce an e.m.f. of 5 volts. That the e.m.fs. in the different turns do act conjunctively can be verified by applying the hand rule of Art. 427.

While e.m.f. can, theoretically, be increased to any extent desired by increasing the number of turns there are other considerations that tend to limit the number of turns per coil feasible in a practical machine. If a coil of many turns is to occupy the same space as one of a few turns, smaller wire must be used for the many-turn coil. But in every case the wire comprising a coil must be sufficiently large that it will not become excessively hot when the generator operates continuously at its full load and the coil is carrying its full-load current. Furthermore, although the e.m.f. may be increased by increasing the number of turns, the resistance of the coil also increases directly as the number of turns. Great resistance in the armature circuit will involve considerable power loss, heating and voltage drop in the machine, which are obviously objectionable. (The line current in the external circuit served by a generator, or at least a certain definite proportion of it, flows in every armature coil and this current is always relatively large.)

The necessary insulation on the armature or coil conductors further increases the space required by them and since, because of certain design considerations, the space allowed for the conductors is limited (usually it is a slot in the armature core) the number of turns in the average coil does not ordinarily exceed 6 or 8. However, a number of coils can be arranged on the same armature to increase the e.m.f. as described in Art. 537.

EXAMPLE.—The armature of the magneto generator described in Art. 543 is an example of an armature having a winding of many turns.

540. The Speed at Which the Armature Conductors Move, that is the revolutions per minute of the armature, is limited by

considerations of safety and economy. (In modern alternating-current generators the armature conductors are almost invariably stationary and the field structure rotates; in direct-current generators, the armature rotates and the field structure is stationary.) Where a generator is to be direct-connected to an engine or other prime mover, the prime-mover speed determines the generator speed. In any case the probable speed of the prime mover must be considered. Excessively high speeds are not permissible because of the great centrifugal stresses they impose. A peripheral speed of about a mile a minute is probably the upper limit for armatures of ordinary machines but higher speeds can be and have been used. Greater speeds are used in small generators than in large ones.

541. The Inductors (Fig. 282) of an armature are the conductors in which the e.m.f. is induced. Usually, each of the two sides of every turn in an armature coil is an inductor. However, with coils wound in certain ways (Art. 572) only one of the sides of a turn may be an inductor. Thus, as a rule, every turn of an armature coil comprises two inductors. The end turns of armature coils do not ordinarily cut flux but move parallel to the lines of the flux when the coil rotates and are not, therefore, effective in inducing e.m.f. and hence are not inductors.

EXAMPLES.—The armature coil of Fig. 282 has 2 turns and 4 inductors. The loop of Fig. 255 has 1 turn and 2 inductors. The coil of Fig. 280, *I* also has 2 turns and 4 inductors while that of *II* has 3 turns and 6 inductors.

542. The E.m.f. Induced in Any Coil Which Is Rotated in a Magnetic Field may be computed on the following basis: If the number of lines cut *per second* by a single inductor of the coil be divided by 10^8 (Art. 444), the result will be the average e.m.f., in volts, induced in that inductor during that second. Then the total average e.m.f. induced in the coil will be the *average e.m.f. per inductor* multiplied by the number of inductors in series. Proceeding on the basis of the principle just outlined, the following formula (117) for the e.m.f. induced in a coil which is rotated in a magnetic field may be derived:

Let the symbol ϕ_T stand for the total number of lines cut by *each* inductor or side of each turn of the coil during one revolution of the coil. Then the total number of lines cut by all of the inductors of the coil, per revolution, will be equal to the number of inductors in the coil multiplied by the total number of lines

that are cut by each inductor per revolution. Let C = the number of inductors in the coil. Then, $\phi_T \times C$ = the total number of lines cut by all of the inductors of the coil per revolution.

Now, if T = the revolutions per minute of the coil, it follows that $T \div 60$ = the number of its revolutions per second, since there are 60 sec. in a minute. If the number of lines cut per revolution be multiplied by the revolutions per second, the result will be the total number of lines cut per second. Hence, $(\phi_T \times C) \times (T \div 60)$ = *total number of lines cut per second* by all of the turns of the coil. As stated above, if this total number of *lines cut per second* be divided by 10^8 (100,000,000), the result

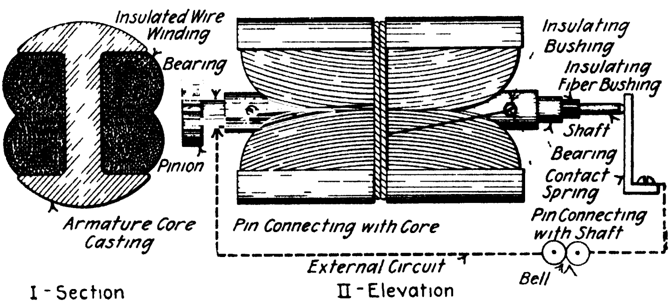

FIG. 281.—Armature of magneto generator

will be the average e.m.f. in volts, the quantity sought. Expressing the above operations in an equation:

$$E_A = \frac{\phi_T \times C \times T}{100{,}000{,}000 \times 60} \quad \text{(volts)} \tag{117}$$

Wherein ϕ_T = flux or total number of lines of force cut by each inductor during one (1) revolution. T = number of revolutions per minute = r.p.m. C = number of inductors or sides of the coil or loop *which are in series and form one circuit between brushes* or collector rings of opposite polarity. E_a = *average* e.m.f. in volts induced in the rotating coil; see note following.

NOTE.—The above formula will not give results which are strictly accurate for alternating-current armatures where the inductors are widely distributed over the surface of the armature (Arts 539 and 563) Where the inductors are concentrated, that is where they are grouped quite closely together as in Fig 281, the result given will be quite accurate for alternating-current machines. The result given will be strictly accurate for all direct-current generators. Note also that the term *average e.m f.* has a specific meaning (Art 690) when applied to an alternating e m f.

EXAMPLE.—What average e.m.f. would be induced in the loop of Fig. 255 if it be rotated at the rate of 1,800 r.p.m. and the flux is 400 kilolines, that is 400,000 lines? SOLUTION.—Each inductor or side of the loop cuts the flux twice per revolution, hence $\phi_T = 2 \times 400{,}000 = 800{,}000$. The number of inductors in series between collector rings is two, that is the coil has two sides. Substituting in the formula (117):

$$E_A = \frac{\phi_T \times C \times T}{100{,}000{,}000 \times 60} = \frac{800{,}000 \times 2 \times 1{,}800}{100{,}000{,}000 \times 60} = 0.48 \ volt.$$

This would be an alternating e.m.f.

EXAMPLE.—See example under Art. 543, the magneto generator, for another illustration of the application of this equation.

EXAMPLE.—What average e.m.f. would be induced in the loop of the elementary direct-current generator (Fig. 273) if it were rotated at a speed of 1,200 r.p.m., assuming that the flux is 800,000 lines? SOLUTION.—Each inductor or side of the loop cuts the flux twice per revolution, therefore $\phi_T = 2 \times 800{,}000 = 1{,}600{,}000$ lines. The number of inductors in series between brushes is two. Now substituting in the formula (117):

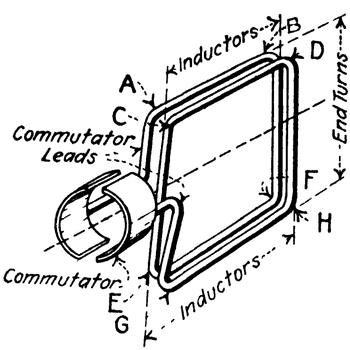

FIG. 282.—Illustrating inductors and end turns.

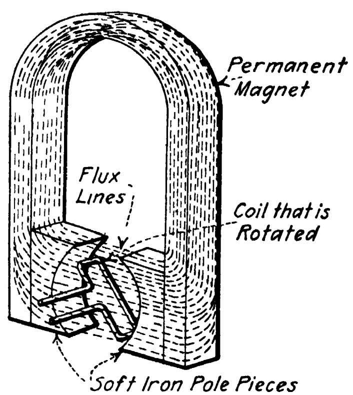

FIG. 283.—Elements of a magneto generator.

$$E_A \times \frac{\phi_T \times C \times T}{100{,}000{,}000 \times 60} = \frac{1{,}600{,}000 \times 2 \times 1{,}200}{100{,}000{,}000 \times 60} = 0.64 \ volt.$$

This is a direct e.m.f.

543. The Magneto Generator (Figs. 281, 283 and 284) such as is used in local-battery telephone instruments and in magneto testing sets offers an example of a small generator illustrating the essential principles. Its construction also illustrates the fact that by connecting a number of inductors or coil turns in series, a relatively high total e.m.f. may be induced. The armature winding consists of a single length of wire, wound into a

coil of many turns, around the armature-core casting; hence all of the inductors are in series. The flux is produced by several steel, permanent magnets. Usually these magnetos are provided with collector rings or their equivalent and hence impress alternating e.m.fs. on the circuit connected to them. However, they can be—and are for certain special purposes—equipped with a commutator and then they impress direct, pulsating e.m.fs. on the external circuit. The effective e.m.f. (Art. 691) of the average magneto generator is about 60 to 75 volts.

EXAMPLE.—When a certain magneto generator is turned at a uniform speed of 100 r.p.m., the alternating e.m.f. it produces at its terminals is 70 volts, as indicated by a voltmeter there connected. The armature is wound

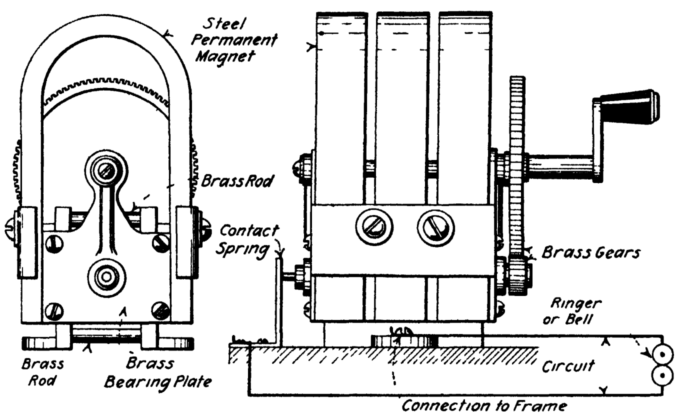

FIG. 284.—Magneto generator assembled

with 3,000 turns of wire. What is the flux produced by the permanent magnets. SOLUTION.—Each turn of the armature winding obviously has two sides or inductors, hence the total number of inductors is: $2 \times 3000 =$ 6,000 *inductors*. A voltmeter connected to an alternating-current circuit always indicates the effective e.m.f. (Art. 695) of the circuit. But the symbol E_a in the formula of 117 stands for the average e.m.f. (Art. 690) or E_{av}. Now (Art. 697): $E_a = 0.901 \times E_e = 0.901 \times 70 = 63.5$ *volts = average e.m.f.* Now substitute in the formula:

$$\phi_T = \frac{10^8 \times 60 \times E_A}{C \times T} = \frac{100{,}000{,}000 \times 60 \times 63.5}{6{,}000 \times 100} = 635{,}000 \ lines.$$

This means that 635,000 lines of force span the gap wherein the armature is rotated. (In the above example it is assumed that a magneto generator produces an e.m.f. of true sine-wave form (Art. 517). In practice this is not strictly true as the wave form is usually rather peaked. The actual e.m.f. induced will be somewhat greater than that given by the above formula.)

544. Constant-potential and Constant-current Generators. All commercial generators may be divided into the two classes or groups just specified.

A CONSTANT-POTENTIAL OR CONSTANT-E.M.F. GENERATOR is one which will, under normal conditions, impress a *practically* constant e.m.f. or voltage on the external circuit connected to it. (The term "constant difference of potential" is a better one than "constant potential" because it better describes the machine.) Nearly all modern, commercial generators, both alternating-current and direct-current, except the relatively few used directly for series street lighting and other series circuits, are constant e.m.f. generators. In practice, the term "constant e.m.f." is applied when the e.m.f. referred to never varies more than, say, 10 per cent. from a constant value. Constant e.m.f. generators are designed to have their armatures rotated at practically constant speeds.

The current in the external circuit of a constant e m.f. generator—and in the generator—will, in accordance with Ohm's law (Art. 134), vary inversely as the resistance of the external circuit. The circuits connected to a constant-potential machine are parallel circuits (Art. 191)—the receiving devices such as lamps and motors are connected in parallel to the circuits.

NOTE.—By adjusting the field rheostat (Art. 562) of a constant e.m.f. generator, the voltage which it impresses on the external circuit may be varied within a considerable range. Also, by adjusting the field rheostat, the impressed voltage may be held practically constant, even if the prime-mover speed decreases or increases somewhat.

EXAMPLE.—Constant e.m.f. generators are used for all indoor incandescent lighting, railway traction, multiple arc lighting and electrical power transmission in North America. The transmission circuits in all of these cases are parallel or multiple circuits.

A CONSTANT-CURRENT GENERATOR (see Art. 109 for definition of "constant current") is one which maintains a constant current in the external series (Art. 187) circuit connected to it, the terminal e.m.f. of the machine varying as the resistance of the external circuit changes. Practically all constant-current generators are direct-current and they are seldom used now. About their only application in North America is for series street lighting. They were formerly used to some extent in this country and are now used in certain cases in Europe for the transmission of electrical energy. See author's AMERICAN ELECTRICIANS' HANDBOOK for method of varying impressed e.m.f. so as to maintain current constant.

EXAMPLE.—In series street lighting circuits the current is maintained at about 10 amp. The lighting devices are connected in series so this constant current flows through them in tandem. The e.m.f. required to force a current through a lighting device having a resistance of 5 ohms would be ($E = I \times R = 10 \times 5$) 50 volts. Then if 100 such lighting devices were connected on one circuit the generator would have to impress: 100 × 50 *volts* = 5,000 *volts* on the circuit to circulate this 10-amp. current. A small additional e.m.f. would also be required to overcome the *IR* drop in the line wires.

The constant-current series system of distribution is economical of copper

where the circuits are very long as in street lighting. No. 6 copper is usually employed for these circuits. Each unit on the series circuit must receive its proper proportion of energy, since the current is maintained constant. Its disadvantages are its high potential and the fact that motors and similar devices to operate on constant current can not be designed readily. Lamps and other devices for use on constant-current circuits are provided with *automatic cut-outs* which automatically provide a short-circuit path around the device if it accidentally opens or develops other troubles.

545. Direct-current Generators.—It has hereinbefore been shown (Art. 520) that an alternating e.m.f. is induced in the

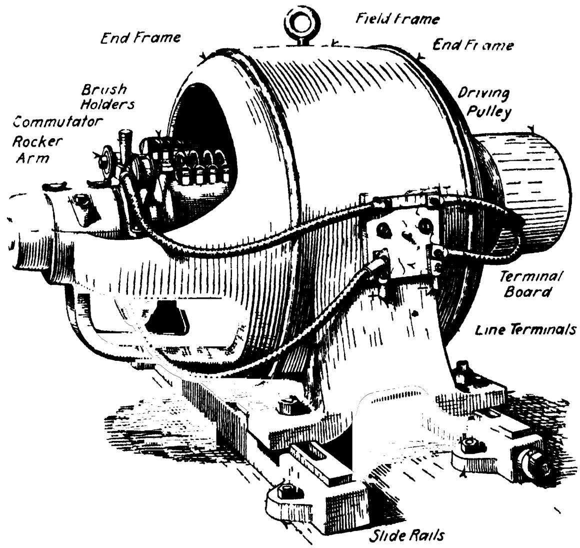

FIG. 285.—Assembly view of a small direct-current compound-wound generator (This is the same Allis-Chalmers machine, a dissembled view of which is shown in Fig. 286.)

armatures of all generators but that, when a suitable commutator is provided, this alternating e.m.f. may be so rectified that an e.m.f. which is always in the same direction—a direct e.m.f.—may be impressed on the external circuit. Generators designed to thus produce direct e.m.fs. are direct-current generators. The fundamental principles of these machines have already been briefly indicated. In following articles their principal characteristics will be considered.

546. The Necessary Components of a Direct-current Generator are:—(1) *A field structure,* (2) *an armature,* (3) *a commutator* and (4) *brushes.* Each of these essential parts is shown

in an assembled machine in Fig. 285, and in a disassembled one in Fig. 286. The functions, construction and general arrange-

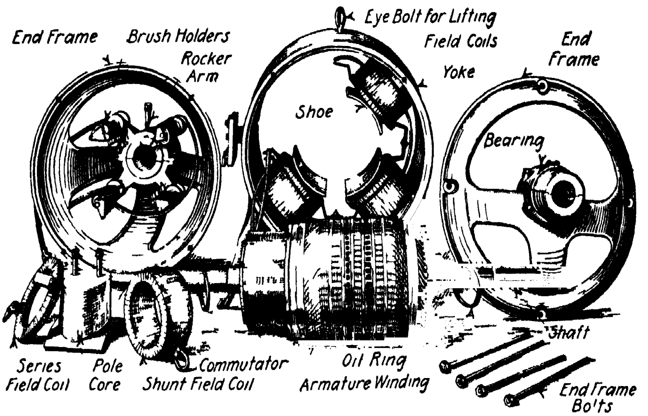

FIG. 286.—Dissembled view of the small direct-current generator (Allis-Chalmers Electric Company).

ment of these different components are treated in articles which follow.

SECTION 30

DIRECT-CURRENT GENERATOR FIELD STRUCTURES

547. A Magnetic Field Is Necessary in Every Generator as indicated in Art. 535 so that the inductors can cut or be cut by this field and thereby have induced in them an e.m.f.

548. Methods of Producing the Field. Field Magnets.—The magnetic field of a generator is produced by *field magnets* which may be either permanent magnets or electromagnets. However, very powerful fields are essential in most commercial generators and these can be obtained only with electromagnets. Another disadvantage of permanent magnets is that with them there is no method of conveniently controlling or varying their strengths. Sometimes the fields of very small generators, such as magnetos (Art. 543) used for telephone signaling and internal-combustion engine ignition, are produced by permanent magnets. In all generators used for electric lighting, energy transmission and industrial applications the fields are produced by electromagnets. The field magnets of every generator are modified horseshoe-shaped magnets. It will be found that in every case each magnet unit consists of a yoke and two legs the ends of which form the poles. The legs are so disposed that the armature revolves between them.

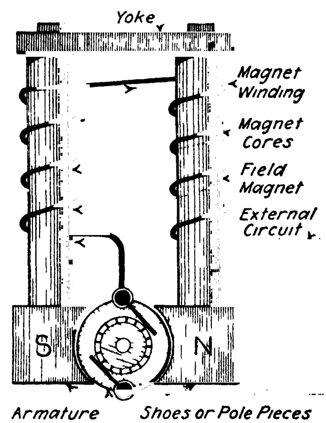

FIG 287.—Diagrammatic drawing of a series generator.

EXAMPLE.—Fig 287 shows (diagrammatically) a generator. Note that the field magnet is essentially of horseshoe form. Note also the same condition in the magneto generator of Fig. 284. Considering the multipolar generator of Fig. 288, each pair of magnet coils and the portion of the magnetic circuit associated with them in reality comprise a horseshoe magnet.

549. The Field Structure (Fig. 288) is that portion of a generator comprising and immediately associated with its field magnets. The *field coils* or magnet coils (Fig. 289) are the insulated copper wire or strap coils through which electrons are

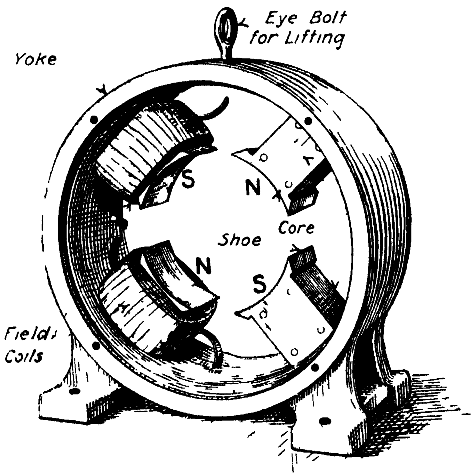

FIG. 288.—Field structure of a multipolar (four-pole) direct-current generator.

forced to produce the magnetic field. Frequently, particularly in the large machines, these coils are (Fig. 289, *I*) provided with air spaces between layers or turns to facilitate ventilation.

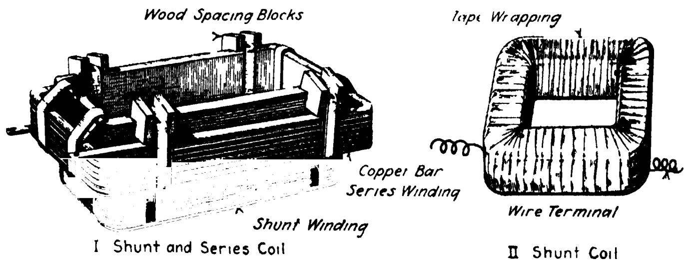

FIG. 289.—Field coils. (Note how the series coil of *I* is composed of bar copper with spaces between its turns to assure effective ventilation and cooling Also note the space between the series and the shunt coil for the same purpose)

The *magnet cores* (Fig. 290) are of soft iron or steel to minimize hysteresis losses and are frequently, particularly in the larger machines, laminated to minimize eddy-current losses. Fig. 291 shows the magnet core of a large machine thus laminated.

The ends of the cores which are nearest the armature are flared out into *pole shoes*, sometimes called pole pieces or, merely, "shoes." The shoes are occasionally separate blocks of metal as in Fig. 287 but more often they comprise enlarged portions of the material of the core. Shoes are provided for three reasons: (1) They decrease the distance between the end of the core and

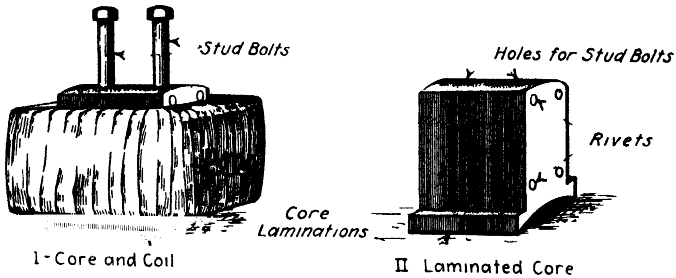

FIG 290.—Laminated magnet-core for a direct-current generator

the armature. (2) They spread the flux along the armature and produce the most effective distribution thereof. (3) They serve to hold the magnet coils in place.

550. Direct-current Generators May Be Classified or grouped in accordance with: (1) *The number of poles*, Art. 551. (2) *The method of field excitation*, Art. 553. (3) *The type of field winding*, Art. 557. All three classifications are applicable to the same generator. That is, to comprehensively describe a direct-current generator one should indicate: (*a*) *How many poles it has.* (*b*) *How its fields are excited* or magnetized. (*c*) *The type of winding used for its fields.* These different classifications and their subclassifications will be described.

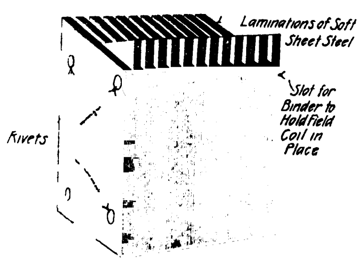

FIG. 291.—Showing the laminated core for a field magnet for a large Fort Wayne direct-current generator.

551. In Classifying Direct-current Generators on the Basis of Number of Poles, they may be grouped into: (1) *bipolar* and (2) *multipolar* machines. Bipolar machines are those which have only two magnet cores or "poles." Multipolar machines have more than two magnet cores or "poles." The bipolar design though formerly used exclusively is now applied for only

the smallest machines. See Art. 594 for an outline of the advantages of the multipolar design.

EXAMPLES.—Fig. 287 shows the general characteristics of the bipolar design while Figs 285 and 288 show multipolar machines.

552. The Arrangement of Poles is always such that they alternate in polarity around the frame. This is true of the poles of all generators, both alternating-current and direct-current.

EXAMPLE.—In Fig. 288, the shoes alternate in polarity thus, *NSNS* around the frame. This same principle is also shown in following illustrations.

553. There Are Two Methods of Field Excitation used for direct-current generators. A machine may be either (1) *separately excited* (Fig. 292) or (2) *self-excited.*

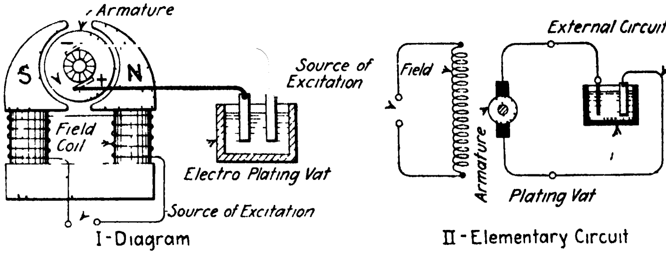

FIG. 292.—Diagrams of a separately-excited, direct- or continuous-current generator serving an electro-plating vat.

554. Separately-Excited Generators are sometimes, though infrequently, used. The current which flows in the field magnet coils is impelled by some source other than the generator itself. Such machines are used in special cases (for testing in electrical machine factories) where very close regulation of the field strength is desirable and for electrolytic or electroplating work where it is of importance that the polarity of the machine be not reversed. Another generator or a storage battery (Fig. 292) may be used for impelling the excitation current. The field magnets may be wound for any desirable voltage which may be available, inasmuch as they have no electrical connection with the armature of the separately-excited machine. Practically all alternating-current generators are separately excited as indicated in Art. 555.

555. Self-excited Generators.—Practically all direct-current generators are self-excited. That is, the current which excites or magnetizes their field magnets is impelled by the armature

of the generator itself. The field current of a self-excited generator is determined almost entirely by the brush voltage of the machine.

EXAMPLE.—Fig. 293 shows typical diagrams of self-excited generators. Note that n each case the field windings are so connected that the e.m.f. induced in the armature of the generator forces the current, which magnetizes the field magnets, to flow around the field coils.

556. The Excitation of Self-excited Generator Field Magnets.—It is not readily apparent why a machine should be self-exciting because the current which magnetizes the fields must be impelled by the e.m.f. induced in the armature inductors when they cut flux. And it seems reasonable that there should be no flux to cut until a current is circulating in the field coils. The explanation is that practically all commercial iron and steel has some *residual magnetism*—magnetism retained in the material—

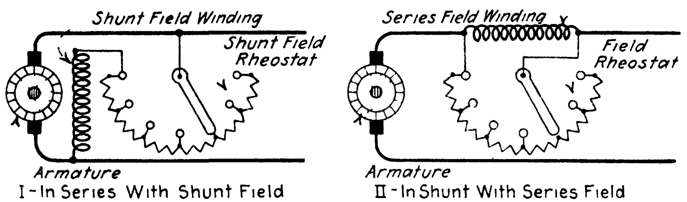

FIG. 293 – Rheostats for field strength control

which causes the generator to build up. Hence, there is practically always, when a machine is not in operation, a weak though appreciable flux emanating from the poles. Then, when the armature is caused to rotate, its inductors (Art. 541) cut this flux and a low voltage is induced in the armature. This voltage impels a current through the field coils which increases the flux. This, in turn, increases the current in the field coils which still further increases the flux. The increases continue until the saturation point (Art. 248) and the normal voltage of the machine are attained. All self-exciting machines "build up" in this same manner.

NOTE.—"Building up" may require 20 to 30 sec. Occasionally machines appear to lose all of their residual magnetism and will not build up. Then, while the machine is being started, the fields must be weakly excited by using some source of low e.m f. (for example, several dry cells in series or a low-voltage, direct-current lighting circuit) to force current through the field coils and produce an initial magnetization. Usually after this treatment

the machine will build up satisfactorily. Sometimes the earth's field, Art. 51, can be made to induce the initial magnetization. See the author's Electrical Machinery for detailed directions as to how to make machines build up.

557. The Three Types of Field Windings.—The field-magnet windings of direct-current, self-excited generators may be arranged or connected so as to produce the required number of ampere turns to develop the necessary flux, in accordance with one of three different methods. Generators may, therefore, be classed as regards their method of field winding arrangement into: (1) *Series-wound generators,* Fig. 294 and Art. 558. (2) *Shunt-wound generators,* Fig. 295 and Art. 559. (3) *Compound-wound generators,* Fig. 296 and Art. 560. Each of these types has distinctive characteristics and is inherently fitted for certain services as will be shown.

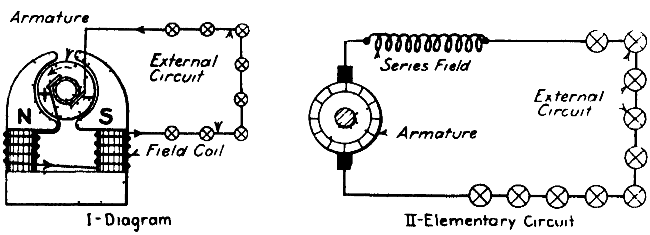

Fig. 294—Diagrams of a series or series-wound generator

558. Series Generators, Fig. 294, have their armatures, field coils and external circuits, all in series. The same current flows in the external circuit, the field coils and the armature—obviously this must be true since these components are in series. See Art. 187, series circuit. The applications and characteristics of series generators are discussed in Art. 618. Since the current in the series field coil is relatively large, a comparatively small number of turns produces the required ampere-turns for magnetization.

559. Shunt-wound Generators (Fig. 295) have their field windings connected in *shunt* or parallel with the armature. Hence the current in the field coils is only a portion of that in the armature. Since the armature e.m.f. is impressed across the shunt field winding, it follows that the current in the field winding will, in accordance with Ohm's law, be determined by its resistance. Usually shunt-wound machines are so proportioned

that from 5 per cent. (small machines) to 1 per cent. (large machines) of the total armature current circulates in the field coils. Inasmuch as the shuntfield current is relatively small a much larger number of field turns is required than in series machine coils to produce the necessary ampere-turns for magnetization.

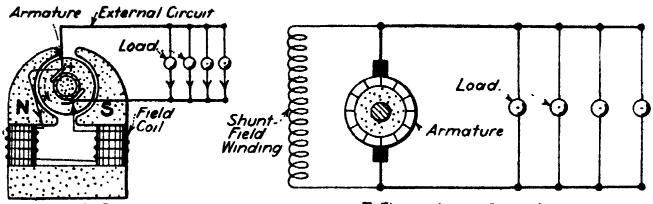

FIG. 295.—Diagrams of a shunt-wound generator.

NOTE.—The current circulating through the shunt field winding equals: (*the e.m.f. across brushes of opposite polarity*) ÷ (*the resistance of the field winding* + *the resistance of the field rheostat*).

The current circulating through the armature of a shunt generator equals: *the current in the field winding* + *the current in the external circuit.*

560. Compound-wound Generators (Fig. 296) are hybrids, partaking of the characteristics of series and of shunt machines.

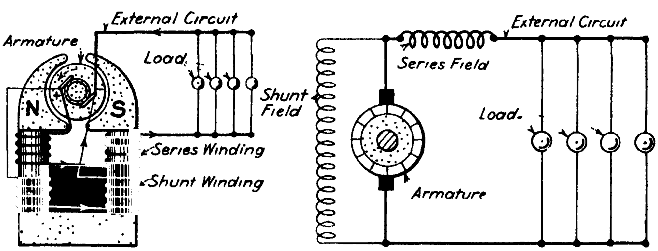

FIG. 296.—Diagrams of a compound-wound generator.

The main-line current (or a certain definite proportion of it when a series shunt is provided) flows in the series coil. Hence the series-coil current varies with the load. The shunt coil is connected directly across the armature and the current forced through the coil by the armature e.m.f. is always the same (practically) and is inversely proportional to the shunt-field circuit resistance.

561. Long-shunt and Short-shunt Connections for compound-wound generators are illustrated in Fig. 297. *A short-shunt field connection* is one wherein the shunt-field winding is connected directly across the brushes or armature. *A long-shunt field connection* is one where the shunt-field winding is connected across the armature and the series-field winding. Any compound-wound generator can be readily changed from a short-shunt to a long-shunt machine or *vice versa* by merely altering the field connection accordingly. The effect of each of these connections on the performance of the generator is discussed in the author's AMERICAN ELECTRICIANS' HANDBOOK and in his ELECTRICAL MACHINERY.

562. The Methods of Controlling Field Strength are indicated diagrammatically in Fig. 293. The rotational speeds (r.p.m.) of generators are constant (that is, they do not vary more than

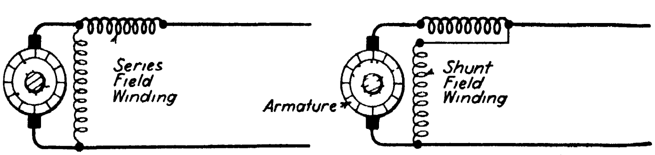

FIG. 297.—Long and short shunt connections of compound-wound generators.

a few per cent.), hence the only feasible way of controlling or varying the rate of cutting flux, the e.m.f. that the machine produces, is by varying the flux or the field strength. Where a generator has a shunt-field winding, the field strength may be varied by altering the shunt-field current by incorporating a rheostat in series with it. Increasing or decreasing the resistance in the field circuit by moving the rheostat handle decreases or increases the field current and consequently the flux.

With a generator having a series winding, field control may be effected by arranging a rheostat in shunt, Fig. 293, *II*, with the winding. With the rheostat circuit open, line current flows in the series coil and the flux is a maximum. But if current is permitted to flow in the rheostat circuit, the current in the series coil is decreased accordingly. By varying the resistance in circuit in the field rheostat, the flux is correspondingly varied.

SECTION 31

PRINCIPLES OF DIRECT-CURRENT GENERATOR ARMATURES

563. The Function of a Direct-current Armature is, for generators for most commercial purposes, to produce an unvarying, continuous e.m.f. The definition of an armature is given in Art. 534. In Art. 530 the method whereby the alternating e.m.f. induced in a rotating loop may be rectified into a direct e.m.f. in the externa circuit is expla ned. However, the e.m.f. thus produced by the loop, as shown in Fig. 268, *II*, is a pulsating e.m.f. because it is always in the same direction. Obviously, this pulsating e.m.f. would not be applicable for a majority of

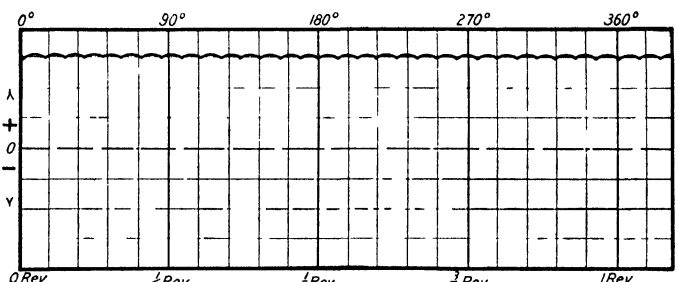

FIG. 298.—Graph of a practically constant e m f.

applications because it consists merely of a series of impulses. How armatures may be arranged so as to produce the "smooth" continuous e.m.fs., such as that shown graphically in Fig. 298, will be explained.

564. Production of a Constant or Unvarying E.m.f. by Increasing the Number of Coils.—Imagine the primitive direct-current generator of Fig. 273 modified by the addition of another coil and pair of commutator segments as suggested in Fig. 299 and compare the two arrangements. With the single-coil arrangement it is obvious that there are certain instants (Figs. 276 and 279) at which no e.m.f. is being induced. Such instants occur twice during each revolution. Also, for a considerable

proportion of each revolution the e.m.f. induced is small as the graph of Fig. 268 shows. But when the two-coil "armature" of Fig. 299 is rotated in the field there is no instant when the e.m.f. impressed across the brushes is zero. The brushes are so disposed that they always connect with the coil in which the greater e.m.f. is being induced.

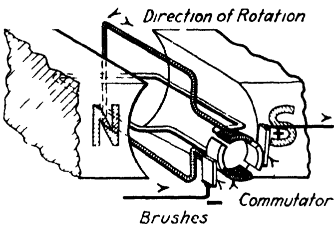

FIG. 299.—Showing an elementary direct-current generator with a two-coil armature.

EXAMPLE.—Consider the elementary generator of Fig. 299. (One coil has been shown dark-colored and the other white merely for distinction, otherwise they are the same.) The light-colored coil is cutting flux at a maximum rate at the instant depicted and the dark coil is not cutting flux. At this instant it is the white coil which is impelling the current in the external circuit. As the coil is rotated, the e.m.f. induced in the white coil will decrease and that in the dark coil will increase. The e.m.fs. in both coils will be equal when the coils have been turned ¼ revolution or 90 degrees from the starting position which is shown in Fig. 299. When the e m.fs. thus become equal, the brushes bridge the commutator segments connected to the coils. An instant later, the brushes contact only with the segments connected with the dark coil. As rotation of the coil is continued the cycle of occurrences just described will be repeated.

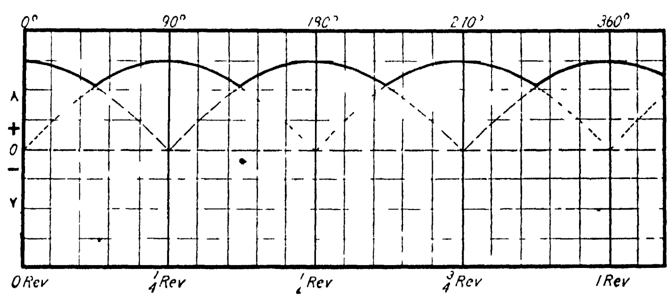

FIG. 300.—Graph of e.m.f. of two-coil generator of Fig. 299

If the e.m.fs. induced by this two-coil armature be plotted into a graph, it will be of the form indicated in Fig. 300. The heavy line indicates the e.m.f. impressed on the brushes. The current circulated in the external circuit will vary in accordance with a similar curve. The dotted line shows the e.m.f. induced in the white coil and the dashed line that in the dark coil—or *vice versa.* Every ¼ revolution or 90 degrees the e.m.f. is a

maximum. Furthermore, every $\frac{1}{4}$ revolution—midway between the maximum instants—the e.m.fs. are equal and less than the maximum. Note that by the addition of 1 coil the brush e.m.f. has been prevented from decreasing to zero at any instant. It is apparent that by adding more coils the variation of brush e.m.f. during a revolution can be still further decreased.

565. To Generate a Constant, Direct E.m.f., many inductors are properly arranged on an iron core so as to form an armature (Art. 534). The conductors are suitably connected together, in ways which will be described, and to commutator bars as shown in Fig. 301. Then to generate an e.m.f. this armature is rotated at uniform speed in a magnetic field. Some of the inductors—not the same ones constantly but different ones at different instants of a revolution—are always cutting flux at the maximum rate, that is, cutting at right angles across the flux. Thereby a direct, constant e.m.f. is impressed on the brushes. The graph of such an e.m.f. is given in Fig. 298. Obviously, the current impelled in an external circuit of constant resistance by this e.m.f. will be constant (practically) also and in every case will be inversely proportional (See Ohm's Law equation) to the resistance of the *entire circuit* (Art. 136).

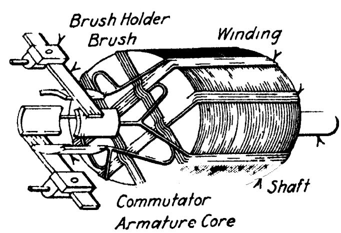

Fig. 301—Showing a four-coil armature. (This is a closed circuit, drum winding.)

566. Open-coil and Closed-coil Armature Windings are diagrammed in Fig. 302. In *open-coil windings* the coils as a whole do not form a closed circuit but each coil is in circuit only when the commutator bars to which it is attached contact with the brushes. In *closed-coil windings* the coils as a whole, together with the commutator segments to which they are connected, form a closed circuit upon themselves and each coil always comprises part of the circuit. Practically all modern armatures are of the closed-coil type. Open-coil armatures have been used to some extent in series arc-lighting generators.

567. Effect of Open- and Closed-coil Windings on Brush E.m.f.—With an open-coil winding, Fig. 302,*I*, the e.m.f. of each coil decreases to zero every $\frac{1}{2}$ revolution, or 180 degrees as shown by the dotted and the dashed curves of Fig. 303. The dotted curve shows the variation with the time of the e.m.f.

of the white coil and the dashed curve that of the dark-colored coil. With a closed-coil winding (Fig. 302,*II*), since the coils are connected in series, the e.m.f. impressed on the brushes is at any instant the sum of the e.m.fs. induced in the coils at that

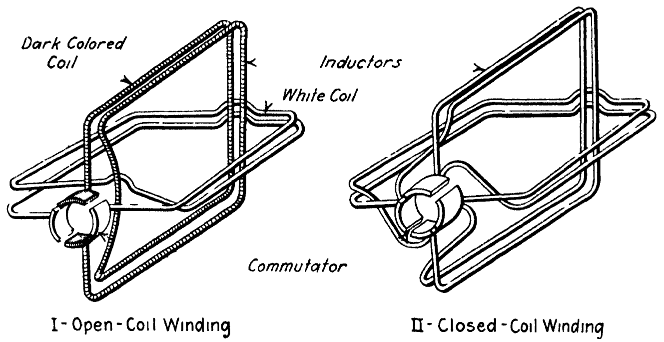

FIG. 302.—Diagrammatic open-coil armature winding and closed-coil armature winding.

instant. Thus the full-line curve of Fig. 303 may be taken as the e.m.f. curve of the closed-coil winding of Fig. 302,*II*. This full-line curve was obtained by adding together the values of the dotted and the dashed curves. Note, then, that a winding

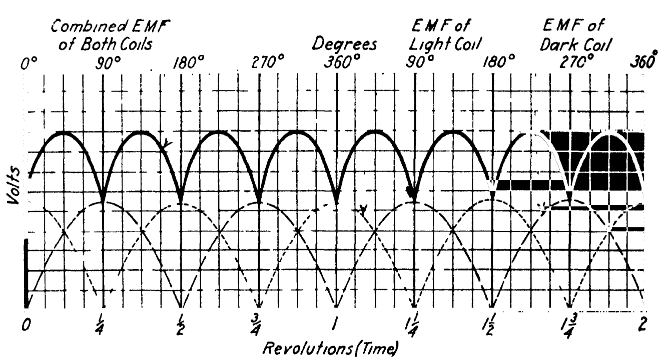

FIG. 303.—Showing e.m.fs. impressed on brushes by open- and closed-coil windings illustrated above.

"connected closed-coil" has the property of producing a higher e.m.f. than the same winding "connected open-coil."

568. The Functions of the Armature Core are two: (1) It constitutes a strong mechanical support for the inductors. (2)

It decreases the reluctance of the magnetic circuit and induces the flux to follow paths where it will be most effective..

EXAMPLES.—If the inductors of Fig. 299 were wound on a cylindrical iron core instead of on an "air core" arranged for rotation between the N and the S pole, the reluctance of the magnetic circuit would be very greatly decreased and a correspondingly fewer number of ampere-turns would produce the flux necessary for the induction of the required voltage. In a multipolar machine, if there were no armature core, the flux would tend to bridge directly between the poles, as at Fig. 304,*I*, where the inductors would not cut across it. With an iron core, incorporated as shown at *II*, practically all of the flux traverses the coil and crosses the air gap where the inductors will cut through it.

569. Construction and Material of Armature Core.—So that the core will decrease the reluctance of the magnetic circuit as

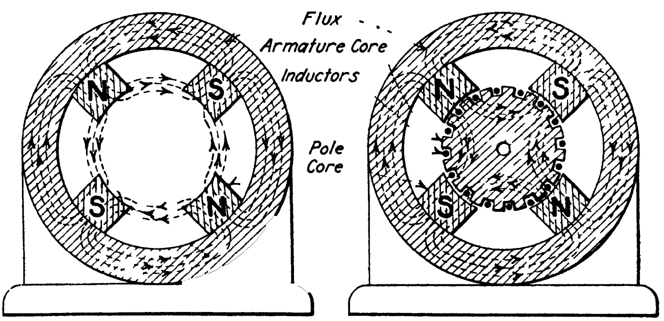

FIG. 304.—Multipolar-generator magnetic circuit showing effect of armature core on flux distribution.

much as possible, it should be of highly permeable material. Hence, in practice, soft iron or electrical steel is used. Since the core is, when the generator is in operation, being magnetized by induction in one direction (*N*) and then demagnetized and magnetized in the other direction (*S*), as portions of its surface pass under the poles of opposite polarity in succession, it is subject to hysteresis (Art. 303). High-grade iron should therefore be used to maintain the hysteresis loss (Art. 305) at a minimum. Eddy currents (Art. 504) are also induced in the iron when it is being rotated in the magnetic field. Hence, cores should not be solid but should be built up or laminated. The cores are composed of sheet-iron laminations called armature-core punchings. To maintain the core temperature at a minimum, ventilating

spaces or ducts (Fig. 305) are provided at certain locations between laminations. Such ducts are always provided in the cores of large generators and frequently in those of small ones. The arms of the armature spider, when it is rotated, act like

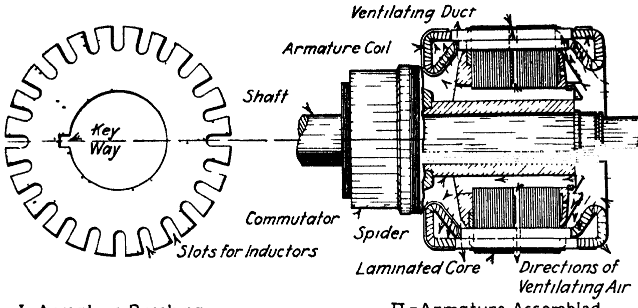

FIG. 305.—Illustrating armature punching in armature construction.

blades of a ventilating fan and force cooling air currents through the ducts and around the conductors.

570. Armature-core Punchings are illustrated in Figs. 305 and 306. These punchings comprise the laminations of which the core is assembled. For generators of the smaller capacities each punching is a suitably perforated, toothed disc as shown in Fig. 305,*I*. Cores for the larger machines are assembled from segmental punchings as shown in Fig. 306. Slots of suitable size are provided in the punchings for the accommodation of the armature winding or inductors.

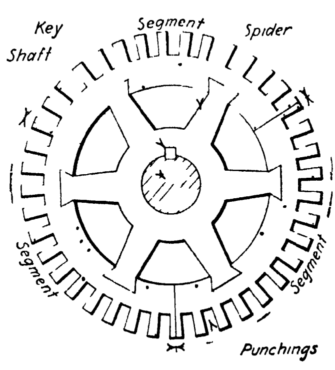

FIG. 306.—Armature core built up of segments.

571. Drum-wound and Ring-wound Armatures are illustrated diagrammatically in Fig. 307. These comprise the two classes into which armatures may be divided as regards disposition of the winding. *A drum-wound armature* is one the inductors of which lie wholly on or in slots in its cylindrical surface; the inductors do not pass through the interior of the rings. *A ring-wound armature* is one the coils of which are wound around the rim of a ring-shaped iron core.

NOTE that armatures may be classified into open-coil and closed-coil as regards the interconnection of the coils, as indicated in Art 567, and also into drum-wound and ring-wound as above as regards the disposition of the winding on the armature core.

In effect, a ring winding constitutes a continuous helix wound around the armature core, as shown in Fig. 308. Ring windings are seldom if ever used for the armatures of modern generators.

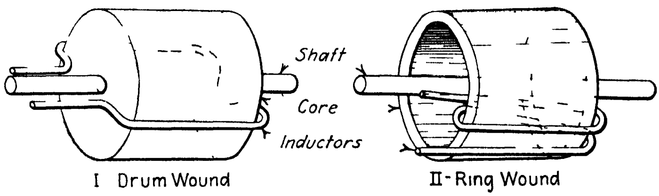

FIG 307 —Diagrammatic illustrations of drum-wound and ring-wound armatures

572. The Principle of Operation of Both Drum- and Ring-wound Armatures Is the Same but since the ring-wound type is now seldom used most of the treatment herein will relate specifically to drum-wound armatures. Note that, while with drum-wound armatures both sides of each loop or coil cut flux, with

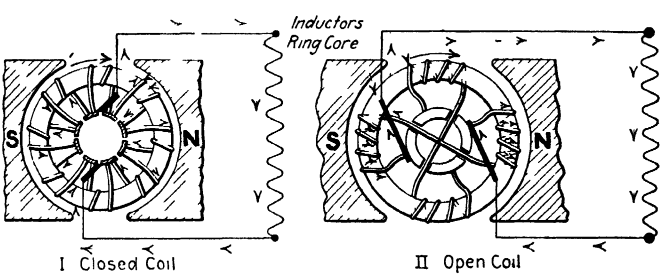

FIG 308 —Diagrams of ring-wound armatures, closed-coil and open-coil.

ring-wound armatures only one side cuts flux; the other side of each coil lies within the ring where there is practically no flux.

573. The Disadvantages of the Ring Winding are: (1) It is necessary in making a ring winding to thread the winding conductor through the space within the hollow-cylinder core which necessitates bending the conductor back and forth. Thus the coils must be wound on the core by hand. (2) Because of this bending it is very difficult to form large conductors into a ring

winding. (3) The insulation on the conductors is likely to be injured by the bending process hence insulation must, in such cases, be placed on the coils while they are being wound. (4) It is difficult to secure ring-wound coils securely in their positions. (5) Only half, approximately, of the length of wire wound on the armature is effective for cutting flux, thus making the armature resistance greater than for an equivalent drum-wound armature.

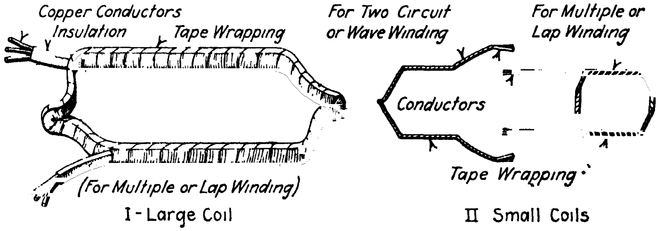

FIG. 309—Form-wound armature coils for large and small generators

574. The Advantages of Drum-windings are: (1) The coils may be *form-wound* (Fig. 309), that is, since all of the coils for a given armature are of the same shape and size they may be wound on forms. In some cases these form-wound coils may be made with automatic machinery. The resulting economy in cost is obvious. (2) Conductors of any reasonable size may be made

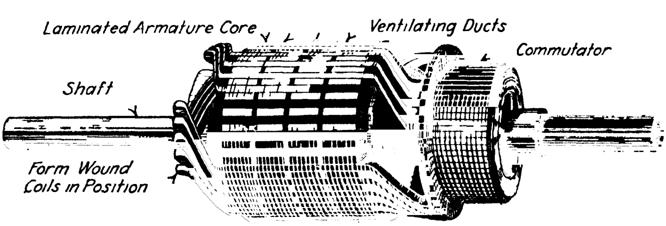

FIG 310.—A drum-wound armature with a portion of its form-wound coils in position

up into form-wound coils. (3) The form-wound coils can be readily placed and firmly secured in the armature-core slots prepared for their reception.

575. The Process of Making a Form-wound Coil Is as Follows. —The conductors which are wires or bars covered with a thin insulation, such as a winding of cotton, are first wound on a form into a coil of the contour and size required. They are

then wrapped with insulating tapes, heated to eliminate moisture and impregnated with some waterproof insulating compound.

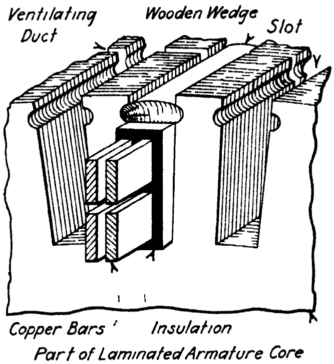

Fig. 311.—Portion of a drum-wound armature for a large generator showing inductors in position in slot.

576. In Winding an Armature with Form-wound Coils, the coils are all placed in their proper positions in the slots, Fig. 310, and are held in place therein with wedges of fiber, wood or

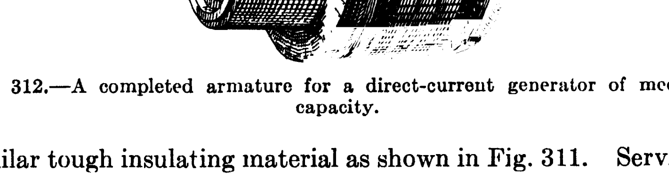

Fig. 312.—A completed armature for a direct-current generator of medium capacity.

similar tough insulating material as shown in Fig. 311. Servings of steel wire wound into *banding* (Fig. 312) are usually also placed around the armatures to further secure the winding in place.

SECTION 32

ARMATURE REACTION, COMMUTATORS, AND COMMUTATION

577. Commutators in Practical Generators always have a number, sometimes a great number, of bars. It is almost obvious from a consideration of Figs. 299 and 313 that as the number of coils in an armature is increased the number of commutator segments must be proportionately increased.

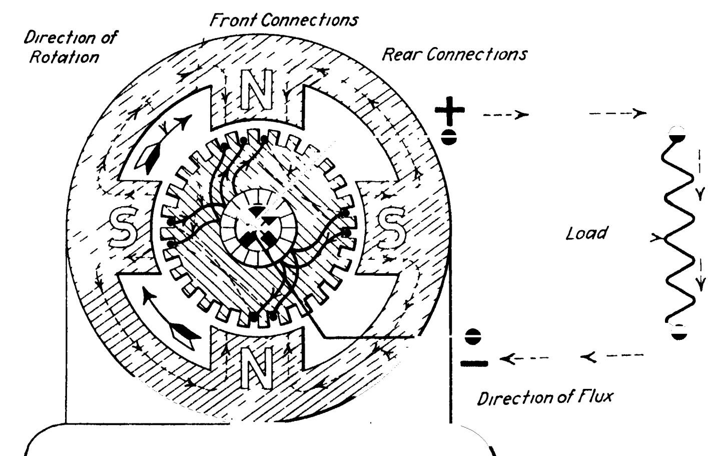

FIG. 313.—Diagram showing part of a direct-current armature winding in position on an armature. Verify the e.m.f. directions shown by applying the right-hand rule.

578. Commutator Construction.—The function of a commutator and its elementary construction are described in Art. 529. Actual commutators, though simple in principle, are very difficult to construct properly. The commutator is the weakest member both mechanically and electrically of any direct-current generator. The great majority of difficulties encountered with direct-current machines are "*commutation troubles.*" The forged copper segments (Fig. 314, *I*) are assembled into a cylinder on a sleeve or spider (Fig. 315) each copper segment being insulated

from its neighbor by a thin segment of mica. The segments are also insulated from the iron or steel member which carries them with mica sleeves and rings. The *wedge rings* (Fig. 315)

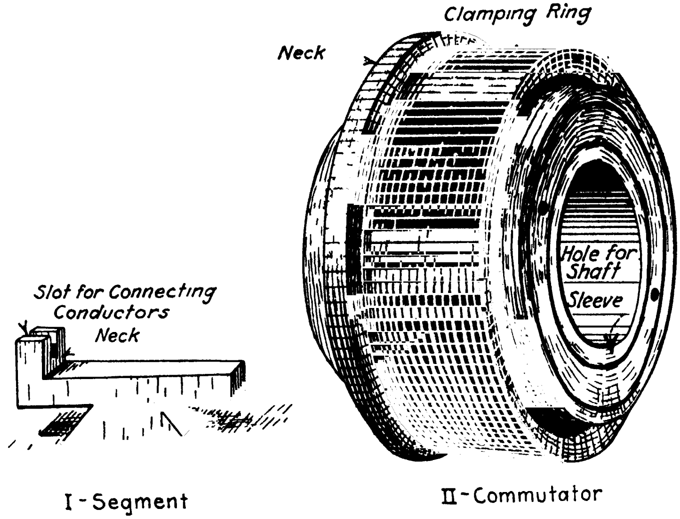

FIG 314.—Commutator segment and assembled commutator

when tightened by the turning of a *clamping ring* or the drawing up of clamping bolts bind the segments securely in position.

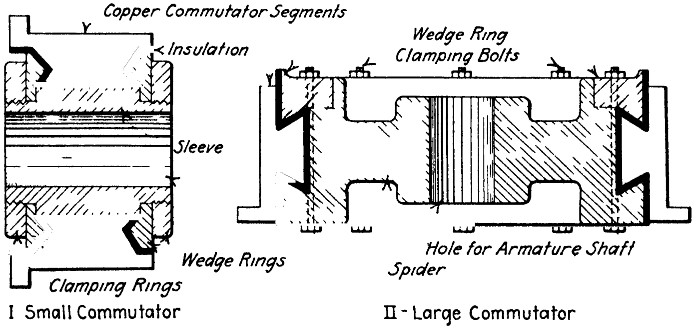

FIG. 315.—Sectional views illustrating typical construction of a small and a large commutator.

The commutator having been thus assembled it is mounted in a lathe and its surface machined into a perfect cylinder.

579. Brushes for most generators are blocks of carbon or graphite. Brushes made of packs of copper gauze or copper wire

are sometimes used for certain low-voltage, high-current electrolytic generators. Graphite is a suitable material because it is in a measure self-lubricating and has a relatively high resistance which, if the brushes are suitably proportioned, tends to minimize sparking between the brushes and the commutator (Art. 588).

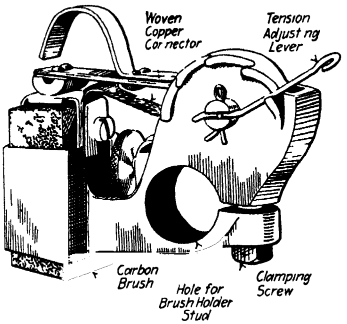

FIG 316 —Brush and brush holder of modern generator

Brush holders (Fig. 316) are used to hold the brushes in correct position in relation to the commutator. Adjustable tension springs are provided whereby the *brush pressure* against the commutator may be regulated. The brush holders are mounted on brush-holder studs which, in turn, are mounted on a *rocker frame*. The rocker frame is so arranged that it may be rotated through a small arc and permits rotating of the brush-holder studs, brush holders and brushes as a unit to different positions (within the limited range) around the commutator. Where the brushes must carry a large current, a number are arranged side by side on one stud. It is seldom,

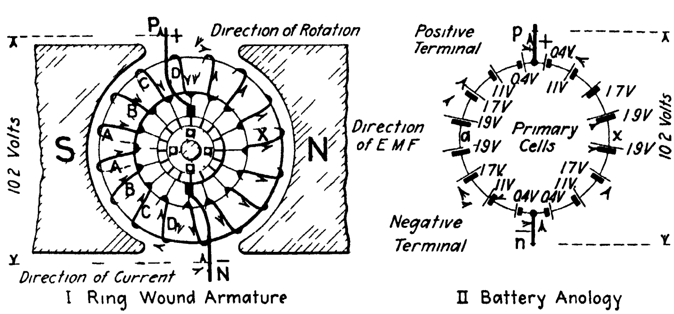

FIG 317 —Illustrating how the e m fs induced in the different coils of a bi-polar generator armature combine to produce the brush voltage.

except for the very smallest generator, that any one brush-holder stud carries less than two brushes. The grade of graphite and the proportions for the brushes for a given machine is in design a matter of considerable importance.

580. How the Voltages Induced in the Different Coils of an Armature Combine to Produce the Brush Voltage is indicated in

Fig. 317. While this illustration shows the armature of a ring-wound machine, the principle involved is the same in the armatures of all machines. In every case it will be found that an armature winding consists essentially of a number of turns or loops of flux-cutting conductors connecting between the brushes of the machine. (The conductors of a wave winding (Art. 598) do not form closed loops but the electrical effects are practically the same as if the loops were closed.) Also, it will be found that the e.m.fs. induced in the different coils or loops combine in every case in about the manner to be described:

EXAMPLE.—Assume that the armature of Fig. 317,*I* is rotated at a uniform speed in the clockwise direction shown. The outside parts of the coils will cut flux and induce e.m.fs. in the directions indicated by the dotted arrows. (Check the e.m.f. directions by applying the hand rule of Art. 513.) It is assumed that the field is uniform. Then the e.m.f. induced in any inductor at a given instant will be proportional to the sine of the angle through which that inductor has been rotated from the neutral position (Art. 518).

Assume that the e.m.f. induced in the coils A at the instant shown is 1.9 *volts*. Then the e.m.fs. induced at this instant in the other coils will be about: B = 1.7 *volts*. C = 1.1 *volts*. D = 0.4 *volts*. The e.m.f. impressed on the brushes will be the sum of the e.m.fs. induced in the separate coils, that is: $1.9 + 1.9 + 1.7 + 1.7 + 1.1 + 1.1 + 0.4 + 0.4 = 10.2$ *volts*. The opposite group of coils around the right-hand half of the armature will obviously impress 10.2 volts also on the brushes. With the conditions given, as the rotation of the armature is continued the pressure of about 10.2 volts will be constantly impressed on the brushes and external circuit.

Primary cells, 16 in number, (Fig. 317,*II*) of the different voltages shown (if it were feasible to obtain cells of these different voltages) might be so connected as to impress 10.2 volts on an external circuit. This diagram may assist the reader to understand what is occurring in the armature.

Since the two groups of coils (that to the left and that to the right of the brushes) are in parallel and each has the same resistance, half of the current that the generator impels in any external circuit connected to it will flow in each of the groups. Half will take the route *NAP* and the other route *NXP*. If the current in the external circuit is 12 amp., 6 amp. will flow in *NAP* and 6 amp. in *NXP*. A similar situation would obtain if the battery at *II* impelled current in an external circuit. Half would pass in the route *nap* and half in the route *nxp*.

In Fig. 328 are illustrated the conditions described above as they occur in the armature of a multipolar generator.

NOTE.—In the diagram of Fig. 317,*I* there is only one turn of the winding connected between adjacent commutator segments. However, in practical generators there are several turns of a coil in series between adjacent segments.

581. The Neutral Plane (Fig 318,*II* and Fig. 319) through an armature is that plane at which the e m f. induced in the inductors is zero, because at this plane the inductors move parallel to the flux and hence do not cut it The *normal neutral plane* (Fig 319) is the plane which is the neutral one when there is no current in the armature inductors, this plane lies midway between the adjacent poles of opposite polarity

582. The Commutating Plane is that imaginary plane, passing longitudinally through the armature and brushes, at which commutation occurs The commutating plane may not coincide with the neutral plane, because, as described in Art 589, it may be necessary (in the case of a generator as will be hereinafter

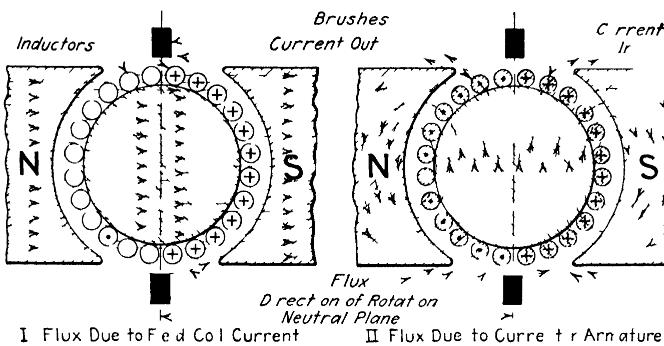

FIG 318 — Showing the two fields which cause armature reaction and field distortion (generator)

shown) to shift the brushes ahead (in the direction of rotation) of the neutral plane in order to insure *sparkless* commutation

583. Armature Reaction is the reactive magnetic influence produced by the current in the armature of a generator or motor, which is under load, on the magnetic circuit of the machine Armature reaction is the cause of the phenomon called *field distortion* How it is that armature reaction and field distortion occur in generators will be explained

EXPLANATION —Consider a portion of the magnetic circuit of a bipolar, direct-current generator, having a drum-wound armature with 12 coils, as shown in Fig 318 The illustration delineates a cross sectional view of the armature and parts of the pole pieces With the fields excited and the armature circuit open at the brushes so that no current can flow in it, the direction of the flux will be straight across from pole to pole as at *I* and the field will be practically uniform The conditions just stated will obtain whether the armature is in rotation or not

Now assume that the armature of *I* is rotated in the counterclockwise direction shown. Its inductors will cut flux and e.m.fs. will be induced in them in the directions indicated (hand rule, Art. 427). To the right of the neutral plane (Art. 581), the e.m.f. in each inductor will be *in* as suggested by the crosses (Fig. 124) in the ends of the inductors. To the left of the neutral plane the e.m.fs. will be *out* as shown by the dots. Since the e.m.f. induced in the inductors at the *neutral plane* is zero (0), the brushes should be set at this plane as shown. Note that the field of *I* is now (while an e.m.f. is being induced in the armature but while there is no current in the armature) uniform—there is no field distortion.

Now assume that the armature circuit be closed through an external circuit. Then, the e.m.f. of the armature will impel a current through the external circuit and armature inductors. This current of itself will produce a field or flux in the direction shown at *II*. This illustration of *II* shows only the flux produced by the armature current; it does not show the flux produced by the field-coil current. The condition shown at *II* could be

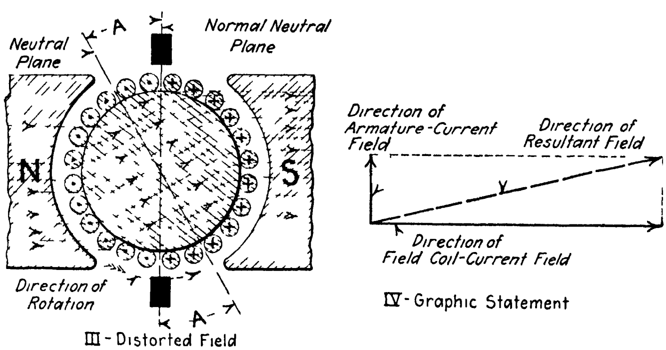

FIG. 319.—Field distortion caused by armature reaction (generator).

reproduced in an actual machine by opening the field circuit, thereby "killing" the main field, and impressing some external source of e.m.f. across the brushes which would create a current, in the proper direction, in the armature, which should be held stationary. It is apparent then that, when a generator is in operation, there are two fields which tend to react on its armature *One*, due to the field-coil current, has a direction between the two poles as at Fig. 318, *I*. *The other*, due to current in the armature inductors, has a direction, almost at right angles to the first as shown at *II*. In an operating generator—one impelling a current—the effects of these two fields are simultaneous and superimposed. They combine to produce a *distorted field* as shown in Fig. 319. The flux across the air gap in the pole pieces and armature core is now no longer uniform but becomes dense toward the toes of the pole shoes in the direction of rotation. Furthermore, the neutral plane is no longer coincident with the normal neutral plane but is shifted through some angle as *A*, Fig. 319,*III*, in the direction of rotation. The field-coil-current field is usually much stronger than the armature-current field as shown graphically in *IV*.

Observe that with *no* current in the armature circuit, the neutral plane coincides with normal neutral plane, as at *I*. But, *with* current in the armature inductors, the neutral plane will shift, in the direction of rotation as at *III*. The greater the armature current, the greater will be the effect of *armature magnetization* (*II*) and the greater will be the shifting of the neutral plane. This means: *The greater the load on a generator the greater is the armature reaction.*

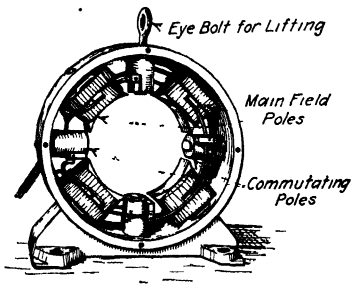

FIG. 320.—Field structure of commutating-pole machine.

584. The Effects of Armature Reaction May be Minimized by providing a magnetic circuit—air gap and pole pieces—of suitable design. Pole pieces of special forms and construction are sometimes used for the purpose. A description of the methods involved would require more space than is here available. *Commutating poles* (sometimes called *interpoles* or *auxiliary poles*, Figs. 320, 321 and 322) provide the most effective means of eliminating the effects of armature reaction and are used in the best, modern direct-current machines. See following note.

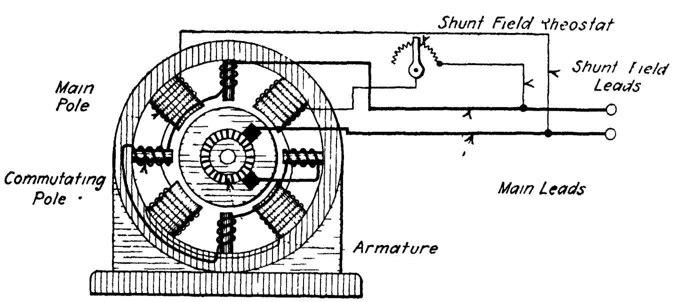

FIG. 321.—Arrangement of main-pole and "interpole" windings in a commutating-pole generator.

NOTE.—*Commutating poles* are small auxiliary poles mounted between the main poles. They are so connected that the armature or line current—or a certain definite proportion of it—circulates in their windings. These windings are so proportioned that they produce a magnetic field in the air gap and in the armature core which is always of somewhat greater strength than that which the armature current (Fig. 323, *V*) tends to produce. As the line current, and hence tendency toward armature reaction increases,

the strength of the commutating poles increases also. Furthermore, the commutating-pole coils are so wound that their field opposes the armature-current field (see Fig. 322). Thereby the armature-current field is neutral-

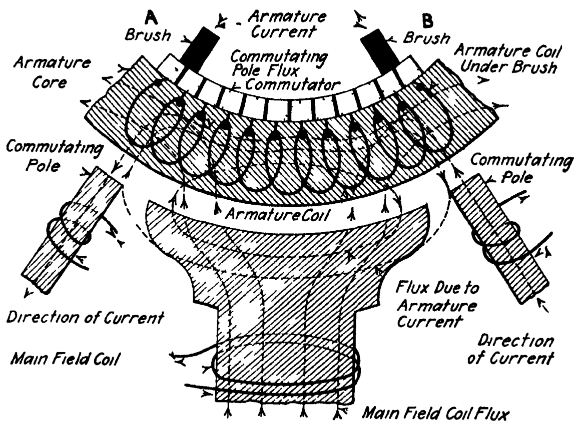

FIG 322 —Illustrating the principle of the commutating pole

ized Since it is the field due to the armature current which causes armature reaction (Art 583), this neutralization of it eliminates the possibility of armature reaction or field distortion

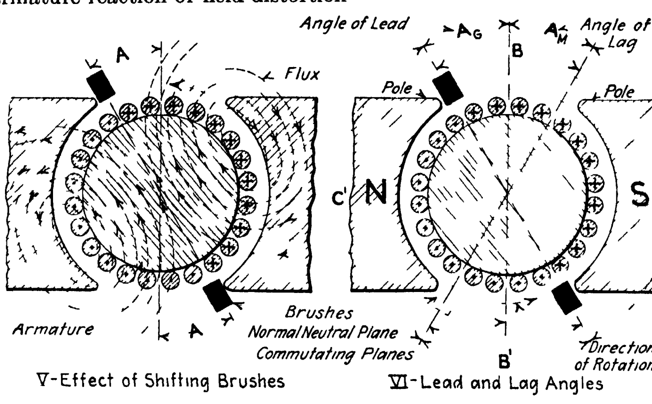

FIG. 323 —(V) Effect of shifting brushes and (VI) lead and lag angles generator.

There is another effect of armature reaction that should be understood: When the field becomes distorted, due to armature current, as in Fig. 319,*III*, the brushes should be shifted to some position (shown in Fig. 323,*V*) so that they will lie in the neutral plane. The magnitude of the angle *A* (Fig. 323),

through which the brushes should be shifted, will depend on the amount of armature reaction. But when the brush position is thus changed, the direction of the current in the inductors included within angle A will be reversed as shown in V. This will alter the direction of the magnetization of the core due to the armature current, from that of II, Fig. 318, to that of V, Fig. 323. The result is a further distortion and weakening of the resultant field (combination of the field-current and the armature-current (fields) of the machine.

585. The Angle of Lead and Angle of Lag, as these terms apply to the brush positions in direct-current machines, are illustrated in Fig. 323,*VI*. *The angle of lead* is the angle (A_G, Fig. 323,*VI*) through which the brushes of a generator must be shifted ahead (in the direction of rotation) of the normal neutral plane so as to lie in the commutating plane. *The angle of lag* is the angle (A_M) through which the brushes of a motor (Art. 640) must be shifted behind (against the direction of rotation) the normal neutral plane so as to lie in the commutating plane.

NOTE.—The terms "angle of lead" and "angle of lag" as above defined have no connection with the same terms as used in alternating-circuit terminology.

586. Cross Ampere-turns and Back Ampere-turns.—The conductors included in the double angle B (Fig. 323,*VI*), which is the sum of the lag angle and the lead angle, are sometimes called the "back turns" and their magnetizing effect is called *demagnetizing ampere-turns.* These terms are applied because the turns within angle B may be considered as being in series with those of angle B', forming a complete helix around the armature core. Their effect is opposed to and tends to weaken that of the field coils of the machine. The turns not included in the lag and lead angles may similarly be called the "cross turns" and their effect *cross-magnetizing ampere-turns.* They tend to set up a field at right angles to the main field.

587. The Process of Ideal Commutation is illustrated in Figs. 324, 325 and 326. The armature conductors of any direct-current machine, in effect, constitute loops or coils between the commutator bars. Therefore, the process of commutation may be examined by considering the diagram, Fig. 324, which shows part of a direct-current-generator armature, commutator and field stripped of the non-essentials. See Art. 529, which defines commutation.

EXPLANATION.—The armature (Figs. 324 to 326) is being rotated at a uniform speed in the clockwise direction shown. Each coil of the armature is marked in a different way to facilitate ready identification. The brush is on the neutral plane and it is located inside of the commutator to simplify the diagrams. What occurs when coil *C*—the white one—passes through the commutating plane will be considered. The current directions are as indicated by the small arrows.

At the instant of diagram I, the brush bears on segment 3. The currents in the left-hand half and that in the right-hand half of the armature meet at this segment (Art. 580), combine and flow through the brush out into the external circuit.

At the instant of II the armature has been rotated far enough so that segment 2 just contacts with the brush. Segment 3 is also in contact with it. Now, a small part of the current from *B* flows directly into segment 2, through the brush and into the external circuit. Only a small part flows from *B* directly into 2 because the area of the brush contacting with 2 is small. Most of the current from *B* flows via coil *C*, through 3 and the brush

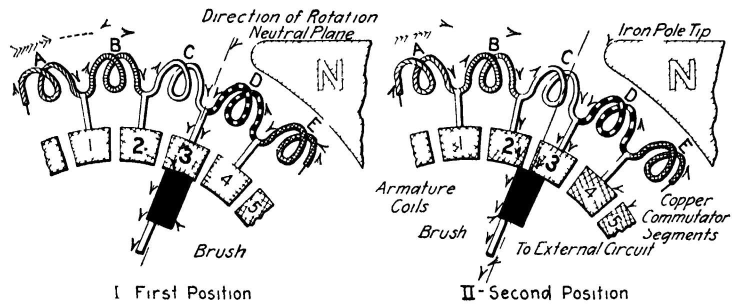

FIG. 324.—Final conditions in process of commutation.

into the external circuit, because the area of brush in contact with 3 is relatively great. (The contact resistance, Art. 156, between the brush and a segment on which it bears will be inversely proportional to the area in contact.) Also, as in *I*, one-half of the total current through the brush flows from *D*. It unites with the portion from *C* and flows through 3 and out through the brush. As the armature is rotated further, the contact resistance between the brush and 2 decreases while that between the brush and 3 increases. The currents through these two paths are, respectively, increased and decreased correspondingly.

At the instant of III (Fig. 325), the brush area contacting with 2 is the same as that bearing on 3. Hence, half of the current—that from *B*—flows directly out through 2 to the brush where it unites with the other half which flows from *D* through 3 to the brush. The coil *C* being in the neutral plane is cutting no flux, hence the e.m.f. and current in it is zero.

At the instant of IV, the armature has been so rotated that the brush is barely in contact with 3. The brush area in contact with 2 is much greater than that in contact with 3. Now the half of the total current from *B*

flows directly into 2 and to the brush. The other half of the total current flowing from D divides, the smaller portion of it flowing directly to 3 and into the brush, while the greater portion flows from D through C and 2 to the brush.

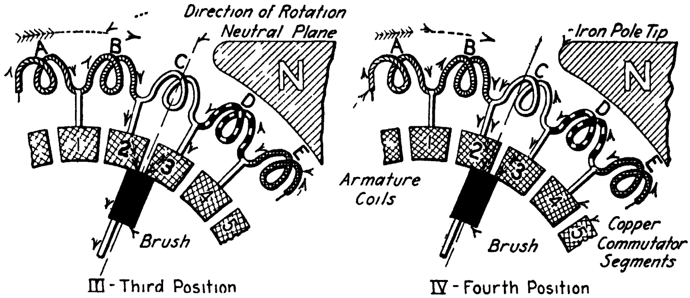

Fig. 325.—Initial conditions in the process of commutation.

At the instant of V (Fig. 326) the brush contacts only with segment 2. Half of the current through 2 and through the external circuit flows from C and the other half flows from B.

The important fact to note from the above explanation is that the current direction in a coil reverses as it undergoes com-

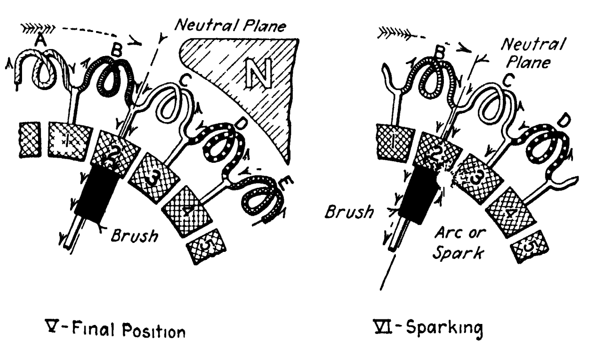

Fig. 326.—Intermediate conditions in the process of commutation.

mutation. For example, in the coil C of Fig. 324, I, prior to commutation, the current direction in C is toward the right. At Fig. 326, V, subsequent to commutation, the current direction is toward the left. The ideal process as above described may not obtain exactly as described in the case of an actual generator for reasons outlined in the following article.

588. Sparking During Commutation is due to the self-inductance (Art. 479) of the armature coils. If armature coils had no inductance, with the brushes set on the neutral plane, there should be no sparking. However, armature coils, since they are wound on iron cores, do have considerable inductance. Also, since armatures revolve so rapidly that frequently the current in a coil must reverse in a hundredth of a second and since they may carry very large currents, the counter e.m.f. of self-induction (Art. 464) may be considerable. How this produces arcing or sparking at the commutator will be described:

EXPLANATION.—In an ordinary generator the current in the coil under commutation must reverse during the short interval between (Fig. 324) the instant before the brush first contacts with one commutator segment of the coil (Fig. 324,*I*) and (2) the instant at which the brush ceases contacting with the other segment (Fig. 326,*V*). At the instant of Fig. 325,*IV*, the current should be reversed in direction in the coil *C*, as shown in the diagram, because this coil has been rotated to the right of the neutral plane.

Actually in an ordinary generator the current will not be wholly reversed at this instant—the instant after the coil has been rotated beyond the neutral plane. The self-inductance of the coil tends to maintain the current in it in its original direction. The consequence is that at the instant pictured at *IV*, there will be (in actual generators) a considerable current circulating through the short-circuit path constituted by the coil and the brush, in a direction from left to right through the coil *C* (in *IV*) and from right to left through the segment 3, the brush and segment 2. The current through the brush tip may be so great as to greatly heat or fuse the edge of the segment 3 and the adjacent edge of the brush.

This short-circuit current still flows at the instant when the armature has been turned so that the brush has ceased contacting with 3 (as shown in *VI*) and is wholly contacting with 2. The result is that when the brush slides from 3 to 2, this current is interrupted with an air gap between 3 and the brush and an arc or spark is formed as shown at *VI*. Obviously, such excessive heating and arcing may, if permitted to continue, prove very injurious to both commutator and brushes.

589. Preventation of Sparking.—Obviously, any means whereby the counter e.m.f. of self-induction in the short-circuited coil (Fig. 326,*VI*) is neutralized will eliminate sparking at the commutator. The common method of accomplishing this is to move the brush a trifle ahead (in the direction of rotation in the case of a generator) of the neutral plane (Fig. 327) so that the coils as they reach the commutating plane can cut some flux. Then, commutation will not occur with the coils as they successively lie in the neutral plane. But it will occur with the coils as they pass through the commutating plane—a location ahead of the

neutral plane—where there will be induced in them an e.m.f. opposite in direction to the counter e.m.f. of self-induction of the coil. The e.m.f. induced in the coil is opposed to and will just neutralize the counter e.m.f.—if the brush has been shifted ahead the right distance. The brush position to insure the least sparking is ascertained by trial. Sparking is automatically eliminated in commutating-pole generators, Art. 584 without its being necessary to shift the brushes.

590. Change of Brush Position with Change of Load.—Armature reaction and field distortion vary with change in load (Art. 583). Therefore, the neutral plane shifts as the load on the generator—the current in its armature—changes. The neutral plane shifts ahead as the load increases. It follows that the brushes of a generator (commutating-pole generators, Art. 584, excepted) should be shifted ahead as the load on the machine increases so that the sparking will be a minimum. The brushes should always be in such a position that the coils, at the instants when they successively pass through the commutating plane, will cut enough flux to neutralize their counter e.m.fs. of self-induction.

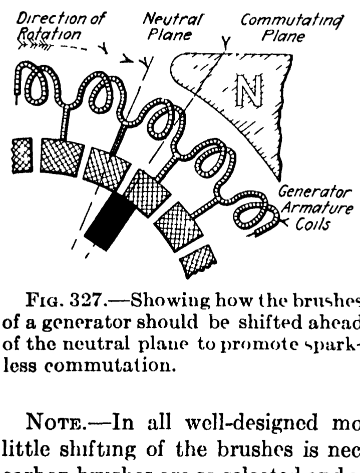

FIG. 327.—Showing how the brushes of a generator should be shifted ahead of the neutral plane to promote sparkless commutation.

NOTE.—In all well-designed modern non-commutating-pole generators little shifting of the brushes is necessary with change of load because the carbon brushes are so selected and proportioned as to have proper resistance to maintain the short-circuit currents in them at a minimum, thus minimizing sparking.

591. In Commutating-pole Generators It Is Not Necessary To Shift the Brushes with Changes in Load to Minimize Sparking at the Commutator.—As suggested in the preceding article shifting of the brushes is necessary in ordinary generators because of changing armature reaction. But with commutating-pole machines the armature reaction effects are not present. Hence, with these machines brush-shifting is not necessary or desirable. The best brush position for a commutating-pole machine is determined at the factory where it is manufactured and usually the brushes are there permanently set at the proper position.

SECTION 33

MULTIPOLAR DIRECT-CURRENT GENERATORS

592. Multipolar Generators.—The term is defined in Art. 551. Multipolar machines have certain inherent advantages. The multipolar construction is used therefore for all but the machines of the smaller capacities, which can be constructed more economically in the bipolar type.

593. The Magnetic Circuit and Generation of e.m.f. in Multipolar Machines is illustrated typically in Figs. 328 and 329 which indicate a four-pole generator. In Fig. 328,*I* is suggested how the e.m.fs. are induced in the different armature coils which

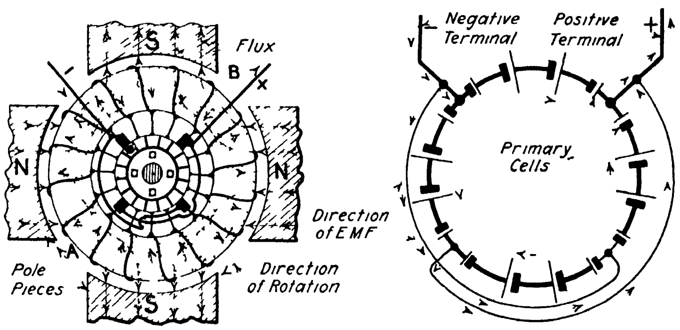

FIG. 328.—Illustrating how the e m fs induced in the different coils of a multipolar generator armature combine to produce the brush voltage.

cut the flux emanating from the different pole pieces. The e.m.f. directions shown by the dotted arrows may be verified by applying the hand rule of Arts. 427 and 513. The e.m.fs. combine to constitute the brush e.m.f. in much the same way as described in Art. 580 in connection with the bipolar machine armature. The battery analogy of *II* may serve to better illustrate how the e.m.fs. of the different coils combine. The e.m.fs. directed from the armature coils to the commutator combine at two locations and those directed from the commutator toward the coils diverge at two other locations. Hence—with a four-pole machine—four

brushes must be used—two of them positive (+) and two negative (−). Except with machines having certain seldom-used types of armature windings (Art. 601), generators always have the same number of sets of brushes as they have poles. The positive and the negative sets of brushes are suitably interconnected and connected to the external circuit as illustrated.

594. The Economics of the Multipolar Design.—Multipolar machines are, in general, more economical in constructive material, largely because of the effective distribution of flux which they provide, than are those of the bipolar type. Because of their greater areas of exposed surface per unit output, they radiate the heat due to losses more effectively. They weigh less per unit

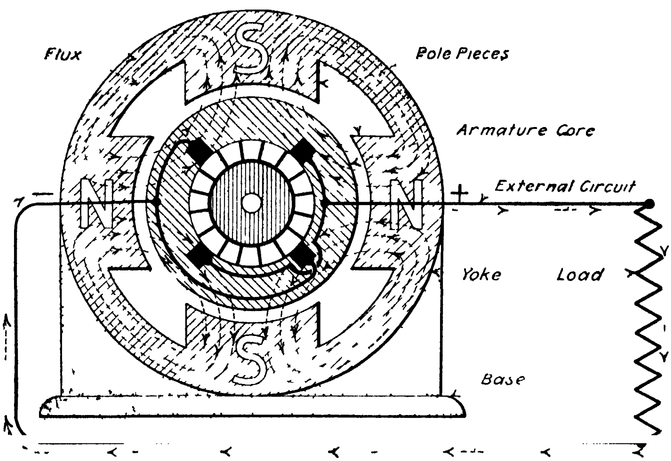

Fig. 329.—Magnetic circuit of a four-pole generator.

output. However, if an endeavor is made to construct a machine, of a given output, with too many poles, the increased cost of manufacture will obviously overbalance the saving in material. It is for this reason that the smaller machines are usually made bipolar.

595. Smaller Armature Conductors Usually Feasible with Generators of the Multipolar Type (see following Note).—With a bipolar generator (Fig. 330,*I*), each armature conductor (closed-coil winding) must carry one-half the line current. Obviously, in machines of considerable power the line current will be very great, which would necessitate excessively large armature-conductor currents in large bipolar machines. In a

four-pole machine (Fig. 330,*II*) the line current passes through the armature via four paths. Hence, in a four-pole machine, the armature-conductor current is but one-half that in a two-pole machine or one-quarter of the total current. There are always (except as detailed in the following NOTE) as many armature paths as there are poles. It is evident that the greater the number of poles a machine has, the smaller, relatively, may its armature conductors be. The smaller the conductors—within reasonable limits—the more readily are they wound and handled. Also, the greater the number of poles, the smaller the current through each set of brushes which facilitates sparkless commutation.

NOTE.—Machines with wave-wound armatures (Art. 601) may have only two paths through the armature, regardless of the number of poles. Hence

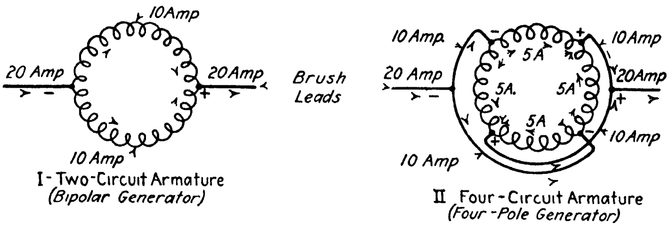

FIG. 330.—Showing division of current in a two-circuit and a four-circuit armature.

the information of the above article may not apply to wave-wound armature machines.

596. Lower Peripheral Speeds Are Possible with Multipolar Machines.—When any coil of a bipolar generator has been rotated through 360 electrical degrees, it has also passed through 360 actual degrees. But when any coil of a four-pole-generator (Fig. 328,*I*) has been rotated through 360 electrical degrees, it has passed only from *A* to *B*, through 180 actual degrees. And when a coil of a six-pole generator has been rotated through 360 electrical degrees (Angle *D*, Fig. 331) it has passed through only 120 actual degrees. It follows that, with the same flux per pole and a given angular velocity or rotational speed, a multipolar generator will induce a higher e.m.f. than will a bipolar machine. Furthermore, with the same flux per pole, the same e.m.f. may be induced with lower angular velocities in multipolar machines than in bipolar. It would not be feasible to build machines of ordi-

narily high voltage and of large capacity in the bipolar type. Then armatures would necessarily have to be large and could not be safely rotated at speeds great enough to induce, in a bipolar field, the e.m.fs. required. Instead of increasing the speeds, in the large machines the number of poles is increased, which is equivalent to increasing the flux.

SECTION 34

DIRECT-CURRENT ARMATURE WINDINGS

597. Armature Windings of modern machines are in almost every case of the *drum* type (Art. 571) hence only these will be discussed. It is obvious (see Figs. 307 and 313) that the e.m.f. in one side of a drum-winding coil must be from *front to rear* at instants when the e.m.f. in the other side of the coil is *from rear to front,* so that the two e.m.fs. will act in unison and combine instead of opposing and tending to neutralize one another. It follows then that the coils on a drum-armature core must be so arranged that two inductors of any coil will never be passing under like poles at the same instant. The coil sides should therefore be separated by an angle (A, Fig. 331) about equal to the angle B between pole center lines. Angle A is called the *angular pitch* or *spread* of the coils or the *winding pitch.* Angle B is the *angular pole pitch.* The subject of armature windings is an extensive and rather involved one, hence only the essential fundamentals can be discussed herein.

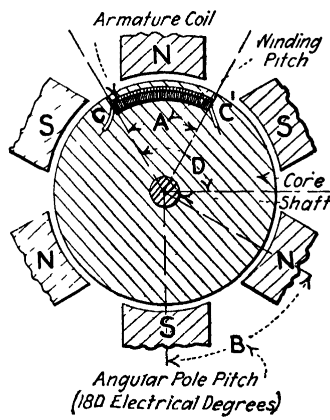

FIG. 331.—Showing "spread" or "winding pitch" of a drum-armature coil.

598. Types of Windings, Lap and Wave Windings.—There are many possible types of windings but it is seldom that any but the *lap winding* (*parallel or multiple drum winding*), or the *wave winding* (two-circuit series drum winding) are used.

THE LAP WINDING is ordinarily used for all direct-current and for some alternating-current armatures. It has the advantages of adaptability and simplicity but is not readily adapted for the inducing of high e.m.fs. With it there are always as many studs of brushes—hence current paths through the armature—as there are poles. A lap winding can be distinguished from the fact that the rear end connections "lap" over one another as shown by

the black-line coil in the winding development of Fig. 332. Coils for lap windings are shown in Fig. 309. With a lap winding there are always as many conducting paths as there are poles.

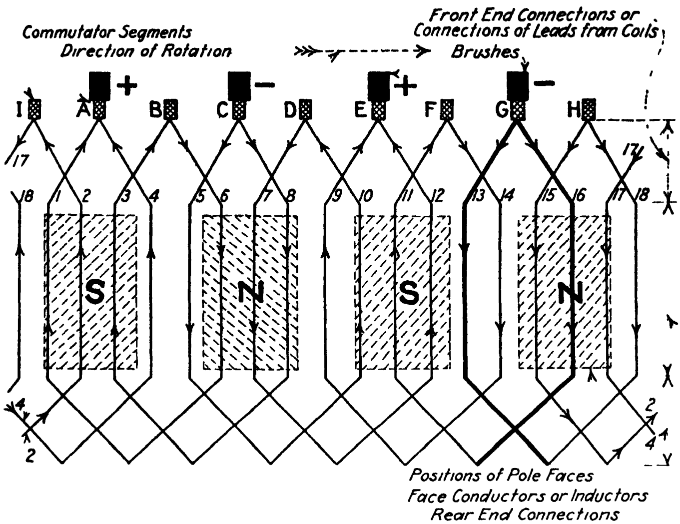

FIG. 332.—Lap winding. A typical winding developed.

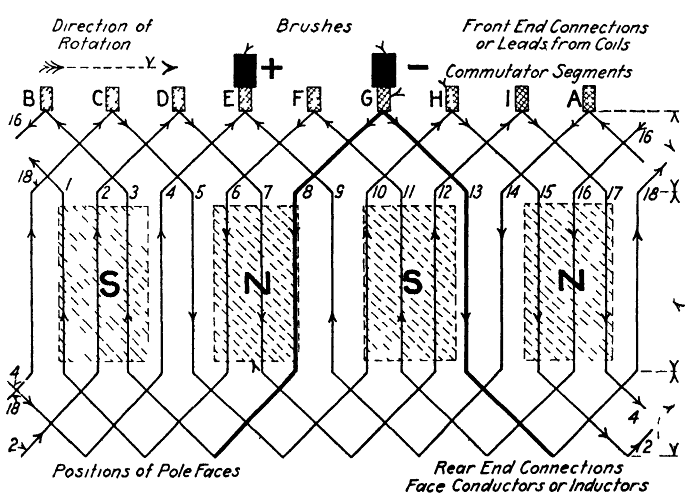

FIG. 333.—Wave winding. The development of a typical winding.

THE WAVE WINDING is used for direct-current, drum armatures where a lap winding would not provide a sufficiently high e.m.f. or would require an excessive number of fine-wire turns per coil. With a wave winding (Fig.

333) only two brush studs are necessary but more up to a number equal to the number of poles can be used. There are always only two circuits in multiple irrespective of the number of poles. This type of winding can be distinguished from the fact that the rear-end connections form geometric "waves" from coil to coil as indicated by the black-line coil in Fig. 333. A coil for a wave winding is shown in Fig. 309.

599. Developments or Diagrams of Armatures Windings may be either *plane developments* which represent the winding as it would appear (Figs. 332 and 333) if it were unrolled from the core and laid out flat on a plane surface, or *star developments* (Figs. 334 and 335) which delineate the winding somewhat as it

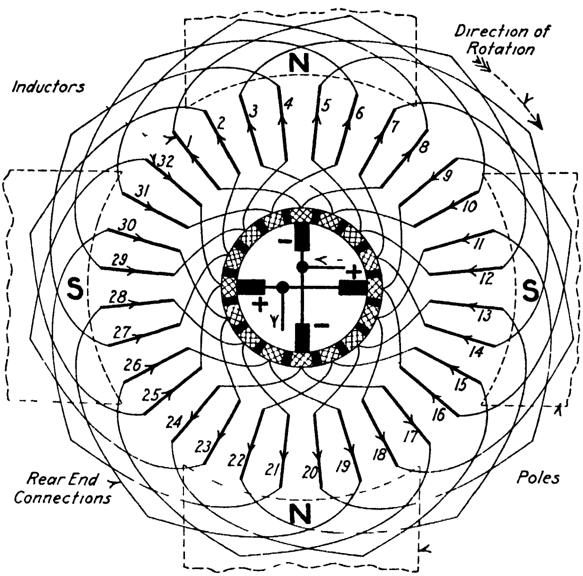

FIG. 334.—Lap-winding for drum armature having 32 inductors (16 coils).

might appear if it were possible to strip it from the core and flatten it out by a pressure, applied along the axis (shaft) of the armature core. A plane development has the disadvantage that it does not show the armature-conductor circuits as entirely closed. In the star development this disadvantage is corrected.

600. E.m.f. and Current in the Lap Winding (Art. 598).—The plane development of a lap winding for a four-pole generator is shown in Fig. 332. There are 18 inductors or 9 coils. With the direction of rotation as given, the e.m.f. directions will be as shown (hand rule, Art. 427) by the arrow heads. One side of

each coil is under a N pole and the other side under a S pole. If the winding is traced out, starting at inductor 1, it will be found that all of the inductors are in series and, in effect, constitute a closed helix. The e.m.f. directions determine the brush locations. Wherever the e.m.fs. in the two conductors connecting to a segment are directed into or out of that segment, a brush should be located at that segment. The star development of Fig. 334 shows a lap winding, having 32 inductors (16 coils), for a four-pole machine. The e.m.f. directions will be as shown determining the brush locations at the segments indicated. From this dia-

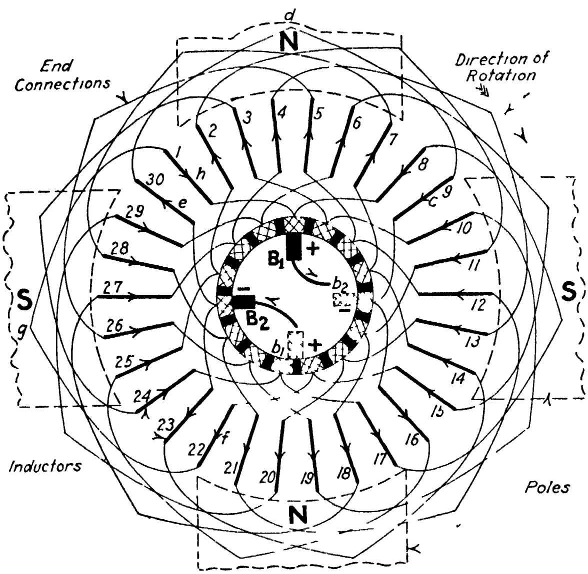

FIG. 335.—Wave winding for a drum armature having 30 inductors (15 coils).

gram it is readily apparent that there are as many current routes through the armature as there are poles (Art. 595).

601. E.m.f. and Current in a Wave Winding.—Consider the diagrams of Figs. 333 and 335. The directions of the e.m.fs. in the inductors (shown by the arrow heads) directly under the poles can be readily determined (hand rule, Art. 427) and those in the other conductors must correspond. Only two brushes (B_1 and B_2, Fig. 335) are necessary but brushes may be and are, where the total current is large, located at each neutral plane. All of the conductors are in series and there are really, regardless of the

number of brushes used, only two current paths through a wave winding.

NOTE.—Why the addition of two more brushes b_1 and b_2 at the neutral planes in Fig. 335 will not affect the situation may be explained thus: The added brush b_2 is, at the instant shown, in metallic connection with B_2 through the low-resistance path b_2cdeB_2 provided by this armature coil. The effect is the same as if B_2 and b_2 were the same brush. Furthermore, b_1 and B_1 are connected directly by the coil b_1fghB_1, producing a similar condition. The generator would also operate as well if b_1 and b_2 were provided and B_1 and B_2 omitted as when only B_1 and B_2 are used. In practice, where only two brushes are used, they are usually located above the horizontal diameter of the commutator in which position they will be most available.

SECTION 35

DIRECT-CURRENT-GENERATOR VOLTAGES, RATINGS AND EFFICIENCIES

602. To Compute the Voltage Induced in the Armature of a Bipolar, Direct-current Generator, the formula shown below can be used. The formula has already been given (Art. 542) for the computation of the e.m.f. induced in any rotating coil. That which follows is merely a modification of the rotating-coil one to specificially adapt it to the armature of a bipolar-frame machine. The formula of Art. 542 for any rotating coil is:

$$E = \frac{\phi_T \times C \times T}{100{,}000{,}000 \times 60} \qquad \text{(volts)}$$

But in the new formula, let ϕ_P = flux per pole. Also let C_T = the *total* number of inductors or sides of coils on the armature. Obviously, in a two-pole machine $\phi_T = 2\phi_P$. And, in a two-pole machine, the inductors per pole, $C = C_T \div 2$. Then substituting these new symbols in the above equation:

$$(118) \quad E = \frac{2\phi_P \times C_T \times T}{100{,}000{,}000 \times 2 \times 60} = \frac{\phi_P \times C_T \times T}{100{,}000{,}000 \times 60} \qquad \text{(volts)}$$

Wherein E = average e.m.f. induced in the armature of any direct-current bipolar generator. ϕ_P = total useful flux or lines of force per pole. C_T = total number of inductors or sides of coils on the armature; in drum armatures C_T is equal to twice the number of loops. T = number of revolutions per minute.

Example.—What average e.m.f. would be induced in the loop of Fig. 273, if it were rotated at the rate of 1,800 r.p.m. and the flux per pole was 400 kilolines, that is 400,000 lines? **Solution.**—Obviously the total number of inductors is 2. Now substituting in the formula (118):

$$E = \frac{\phi_P \times C_T \times T}{10^8 \times 60} = \frac{400{,}000 \times 2 \times 1{,}800}{100{,}000{,}000 \times 60} = 0.24 \textit{ volt.}$$

Example.—The armature of a bipolar generator has 104 loops on its armature, and the flux per pole is 2,270,000 lines. What e.m.f. will be induced in its armature if it is driven at the rate of 1,500 r.p.m.? **Solution.**—

If the number of loops is 104 the number of inductors is $104 \times 2 = 208$. Now, substitute in the formula (118):

$$E = \frac{\phi_P \times C_T \times T}{10^8 \times 60} = \frac{2{,}270{,}000 \times 208 \times 1{,}500}{100{,}000{,}000 \times 60} = 118 \text{ volts.}$$

603. To Compute the E.m.f. Induced in Any Direct-current Generator Armature the following formula, the derivation of which follows from that of Art. 602, can be used:

$$E = \frac{P \times \phi_P \times C_T \times T}{10^8 \times 60 \times m} \qquad \text{(volts)} \tag{119}$$

Wherein the symbols have the same meanings as in Art. 602, except : P = the number of field poles. m = the number of parallel conducting paths between the positive and negative brush sets; that is, $C_T \div m$ is the number of inductors or armature conductors in series between positive and negative brush sets.

EXAMPLE.—A four-pole, lap-wound armature has 200 inductors. The flux per pole is 3,000,000 lines. It is rotated at the rate of 1,200 r.p.m. What e.m.f. is induced in it? SOLUTION.—In a lap-wound armature there are as many conducting paths as poles (Art. 598). Substitute in the formula (119):

$$E = \frac{P \times \phi_P \times C_T \times T}{10^8 \times 60 \times m} = \frac{4 \times 3{,}000{,}000 \times 200 \times 1{,}200}{100{,}000{,}000 \times 60 \times 4} = 120 \text{ volts.}$$

EXAMPLE.—The armature of a four-pole generator has a wave winding and comprises 200 inductors. It is rotated at a speed of 1,200 r.p.m. in a field of 3,000,000 lines per pole. What e.m.f. is induced in this armature? SOLUTION.—There are always 2 conducting paths through a wave winding (Art. 598). Substituting in the formula (119):

$$E = \frac{P \times \phi_P \times C_T \times T}{10^8 \times 60 \times m} = \frac{4 \times 3{,}000{,}000 \times 200 \times 1{,}200}{100{,}000{,}000 \times 60 \times 2} = 240 \text{ volts.}$$

Note that the e.m.f. with this wave or two-circuit series winding is twice that of the similar generator of the preceding example with the lap winding.

604. Armature Resistance is the resistance of an armature as measured between brushes of opposite polarity. The *brush contact resistance* (Art. 605) is not included in "armature resistance." The conductors of an armature constitute a divided circuit (Art. 197) as shown in Fig. 336. Hence, the resistance of an armature may be computed by treating it as a divided circuit. In an armature for a two-pole generator there are two paths in parallel. In the armature of a four-pole generator (lap winding only) there are four paths in parallel and so on.

EXAMPLE.—If a coil of wire AC (Fig. 336,I) having a resistance of 1 ohm is formed into a winding on a bipolar-machine armature, the resistance of each of the two paths will be: $1 \div 2 = 0.5$ *ohm.* Then the resistance of these two paths in parallel—there are two paths in parallel through a bipolar-machine armature—would be: $0.5 \div 2 = 0.25$ ohm. Hence the armature resistance of this armature is 0.25 ohm. Similarly, if this 1 ohm coil is formed into the winding for a four-pole armature as at II, the "armature resistance" from the positive brush terminal K to the negative brush terminal L would be 0.063 ohm.

EXAMPLE.—If the "armature resistance" of a four-pole generator is 0.20 ohm, what would be the resistance of the conductor comprising it if it were in one continuous length (like FJ, Fig. 336,II)? SOLUTION.—Since there are four paths in parallel *each* section of conductor constituting *one* of these paths must have a resistance of: $0.20 \times 4 = 0.80$ ohm. Then these four sections if arranged in series would have a resistance of $0.80 \times 4 = 3.2$ *ohms*

605. Brush Resistance and Brush-contact Resistance are always present. The brush resistance is the resistance of the

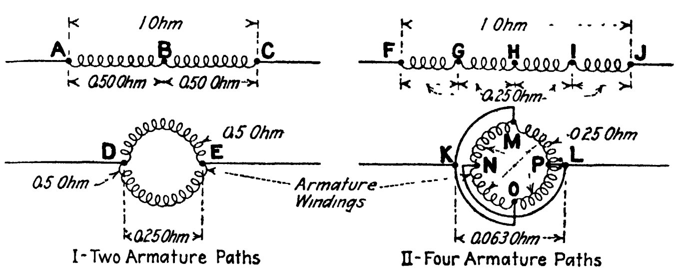

FIG. 336.—Illustrating the term "armature resistance."

brush itself and obviously varies with the volume and shape of brush just as does the resistance of any conductor. Brush-contact resistance is the resistance which occurs at the plane of contact between brush and segment. The drop in voltage due to the combined effects of brush and brush-contact resistance may be about 1 or 2 volts in average machines under normal load.

EXAMPLE.—Experience indicates that the brush resistance and brush-contact resistance combined will vary from about 0.2 to 0.4 ohm per sq. in. of each brush contact. The resistance varies with the current density (Art. 239) through the brush and contact and with the circumferential speed of the commutator.

606. Terminal E.m.f. The Brush E.m.f. of a Generator is the e.m.f. that the machine impresses on its brushes as distinguished from the e.m.f. induced in the armature which is the armature

e.m.f. The brush e.m.f. may differ from the armature e.m.f. due to $I \times R$ drop (Art. 134) in the armature winding. When the generator is not carrying load, the brush e.m.f. and the armature e.m.f. will be practically the same. *The terminal e.m.f.* is that which the machine impresses on the line. In a shunt-wound generator the brush e.m.f. is the same as the terminal e.m.f. In compound- and series-wound machines the terminal e.m.f. will be slightly lower, at any instant, than the brush e.m.f. because of IR drop in the series windings.

607. The E.m.f. Impressed on an External Circuit by a Generator, that is, the *terminal e.m.f.*, is the same as that induced *only when there is no current in the armature,* that is, only when all external paths from the brushes are open. When there is a current in the armature there will always be an internal IR drop (Art. 134) in voltage in it. The drop is due to the armature resistance (Art. 604) and the brush and brush-contact resistances (Art. 605). Numerically this internal drop, in volts, will be equal to (*current in amperes*) $\times$ (*ohms armature resistance* + *ohms brush and brush-contact resistance*). The e.m.f. impressed on the line will be smaller than the induced e.m.f. by the amount of this $I \times R$ drop.

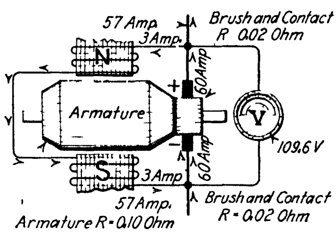

Fig. 337.—Effect of Armature resistance on terminal e.m.f.

Example.—Assume the armature of Fig. 337 to be that of the 1,500 r.p.m. bipolar generator above (Art. 602). Then the e.m.f. induced by the armature is 118 volts. If the armature resistance is 0.1 ohm and the brush and contact resistance for each brush is 0.02 ohm, what will be the terminal e.m.f. with a current of 60 amp. in the armature? Solution.—The drop in the armature will be 0.1 *ohm* $\times$ 60 *amp.* = 6.0 *volts.* The drop due to brush and contact resistance will be: 2 $\times$ 0.02 *ohm* $\times$ 60 *amp.* = 2.4 *volts.* Then the total drop is: 6.0 *volts* + 2.4 *volts* = 8.4 *volts*. Hence the terminal voltage at this load will be: 118 volts − 8.4 *volts* = 109.6 *volts.*

608. The Resistance of the Entire Circuit Through Which Its E.m.f. Impels Current Determines the Current in an Armature.—This follows from Ohm's law (Art. 134 also 528). The e.m.f. induced in the armature forces the current to circulate. The resistance determines the rate of flow or amperes. Consider an example:

Example.—The armature of Fig. 338 is inducing an e.m.f. of 125 volts. It is impelling a current through a circuit comprising the following compo-

nents: (1) Armature winding, 0.04 ohm. (2) Two brushes and contact surfaces, 0.03 ohm each. (3) Two line wires, 1 ohm each. (4) Incandescent lamp, 100 ohms. What is the current in the circuit? SOLUTION.—The total resistance in circuit is:

Armature	0.04 ohm
Two brushes @ 0.03 ohm each	0.06 ohm
Two line wires @ 1 ohm each	2.00 ohms
One incandescent lamp	100.00 ohms
Total resistance in circuit	102.10 ohms

Then applying Ohm's law: $I = E \div R = 125\ volts \div 102.1\ ohms = 1.22\ amp.$, which is the current I in the armature, line wires and lamp.

EXAMPLE.—The voltage at V_1 or the terminal voltage in the above problem, would be the induced e.m.f. less the armature and brush resistance. IR drops or: $125\ volts - [1.22\ amp. \times (0.04\ ohm + 0.06\ ohm)] = 125\ volts - 0.12\ volts = 124.88\ volts$. Hence voltmeter V_1 should read 124.88 volts.

EXAMPLE.—The voltage V_2 at the lamp would be the voltage V_1 less the IR drop in the line wires. Or: $V_2 = 124.88\ volts - (1.22\ amp. \times 2\ ohms) = 124.88\ volts - 2.44\ volts = 122.44\ volts$. Hence V_2 would read 122.44 volts.

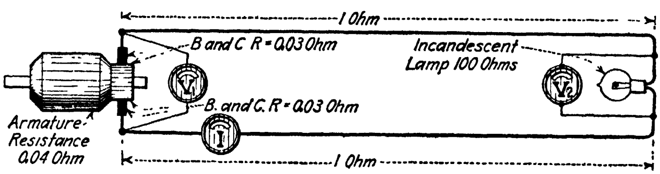

FIG. 338.—Illustrating how the resistance in the circuit determines the current in the armature of the generator.

609. Capacity of a Direct-current Generator.—If the prime mover driving a generator has ample power, the current which the machine will impel may be materially increased over that for which the machine is rated. However, such overloading is possible only to a certain extent because the capacity of a machine is limited by one or all of three considerations:

1. EXCESSIVE SPARKING at the commutator. If the current through the brushes is much greater than that which they were designed to carry, they will probably spark badly and they may "flash over," that is, an arc may form between brushes of opposite polarity.

2. EXCESSIVE INTERNAL DROP or excessive $I \times R$ voltage drop in the armature. Since $I \times R$ drop in the armature decreases the terminal voltage of the generator, if the armature current is too great, the voltage which the machine impresses on its line terminals will become so low that it will not impel a sufficiently large current through the devices connected to the external circuit to operate them properly; the speeds of the motors will become too low and the incandescent lamps will become too dim.

(3) EXCESSIVE HEATING.—The heating varies directly as the square of the current (Art. 167). If the current in a machine is too great, its conductors will become so hot that their insulations will be injured or "burnt out."

NOTE.—*Permissible Temperature Rises in Electrical Machinery* are determined by the temperatures the insulating materials will safely withstand, since the other materials in a machine are metals which may be subjected to very high temperatures without damage. Ordinary combustible insulations such as silk, cotton, paper and the like should never be subjected to temperatures greater than that of the boiling point of water—212 deg. F. or 100 deg. C. Mica, asbestos and the non-combustible materials used for insulation may, without injury, be subjected to temperatures as great as 257 deg. F. (about 125 deg. C.).

In establishing temperature-rise ratings for electrical machinery, it has been arbitrarily assumed that the temperature (Ambient temperature) in the rooms where the machinery is installed will be 104 deg. F. (40 deg. C.).* This gives for the ordinary combustible installations, with the conditions as stated, a permissible rise of: 212 − 104 = 108 *deg. F.* (60 deg. C.). For the non-combustible insulating materials the permissible rise is 257 − 104 = 153 *deg. F.* (85 deg. C.).

610. The Rating of a Direct-current Generator is the kilowatts —power (Art. 159)—load that the machine will carry continuously without excessive (1) *heating*, (2) *sparking* or (3) *internal voltage drop*, Art. 607. If a load greater than that for which it is rated is imposed on a machine for an extended period, it will probably give trouble, due to one of the three above-mentioned causes. Nearly all generators are so designed and rated that they will carry some overload for an hour or so without injury. Every constant-potential (Art. 544), direct-current generator is designed to maintain practically constant some certain terminal voltage when operated at its rated speed. It follows that the current circulated by the machine will vary directly with the kilowatts load on the machine—and inversely as the resistance of the external circuit. Because of the electrical losses in the machine—$I^2 \times R$ (Art. 167), eddy-current (Art. 504) and hysteresis losses (Art. 303)—a dynamo always generates more power than it delivers to the external circuit; see efficiency, Art. 176.

EXAMPLE.—An 80-kw., 220-volt, 1,800-r.p.m. generator is one which will, when its armature is rotated at 1,800 r.p.m., maintain 220 volts at its line terminals if the connected load does not greatly exceed 80 kw. This means that this machine can continuously and satisfactorily circulate a current of: 80,000 *watts* ÷ 220 *volts* = 364 *amp.* In other words it will operate satisfactorily when impelling current in an external circuit having a resistance of ($R = E \div I = 220 \div 364 =$) 0.61 *ohms* or more.

* A. I. E. E. Standardization Rule 305, December 28, 1916.

611. The Losses in a Direct-current Generator—or in any generator—may be divided into: (1) *mechanical losses* and (2) *electrical losses.* Furthermore, the electrical losses may be broadly subdivided into *iron losses* and *copper losses.*

1. THE MECHANICAL LOSSES include *journal* or *bearing friction, brush friction*—the friction of the brushes on the rotating armature—and *windage.* Windage is the friction of the rotating armature on the air in which it is moving. These mechanical losses may vary with the speed of the machine but the rotational speed of a constant-potential generator is practically constant. It follows that mechanical losses are practically constant at all loads. A portion of the power input to the machine is consumed in supplying these losses and its net output is decreased accordingly.

2. THE ELECTRICAL LOSSES.—Under *iron losses* may be grouped the hysteresis (Art. 303) and eddy-current (Art. 504) losses in the armature and field cores. Under copper losses may be grouped the $I^2 \times R$ losses (Art. 167) in the armature and field windings, the eddy-current losses in the copper conductors and the losses in the controlling rheostats. Brush contact

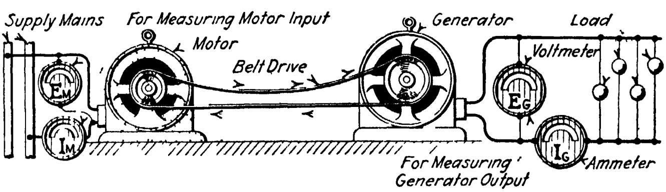

FIG. 339.—Illustrating method of determining the efficiency of a generator.

(Art. 605) $I^2 \times R$ loss is an electrical loss and is sometimes classed in with the copper losses. Most of the electrical losses increase with the load.

NOTE.—In practical testing it is often difficult or impossible to segregate the iron and the copper losses.

612. Determination of the Efficiency of a Direct-current Generator.—Efficiency, see Art. 176, is the ratio of: *the electrical power delivered by the generator to the mechanical power received by it.* Hence, the efficiency of a generator is also:

$$(120) \qquad Efficiency = \frac{(Input) - (Losses)}{Input} \text{ or } = \frac{Output}{Input}.$$

Probably the most convenient method of determining the efficiency of a generator consists in driving the generator (Fig. 339) with an electric motor of known efficiency. The electrical power output of the generator in watts can be readily ascertained by multiplying the current and voltage readings obtained from an ammeter (I_G) and a voltmeter (E_G) connected in the generator

circuit. The mechanical power input to the generator can be found by measuring the electrical input to the driving motor (by multiplying together the ammeter, I_M, and voltmeter, E_M, readings) and then multiplying this result by the motor efficiency at this load. Their product will be the electrical input to the generator in watts, it being assumed that the belt losses are negligible which they usually are in practical work. The efficiency will be the ratio of *the watts output to the watts input.*

EXAMPLES.—Large generators have higher efficiencies than small ones and the efficiency of any generator will vary with the load, because the losses (Art. 611) vary with the load. A good 5-kw. generator will have a full-load efficiency of about 82 per cent., a 100-kw. generator 91 per cent. and a 1,000 kw. generator 93.5 per cent. As an example of how efficiency varies with the load: For a 100-kw. machine the efficiency at full-load is 91 per cent.; ¾ load, 90.5 per cent.; ½ load, 89 per cent. See the author's AMERICAN ELECTRICIANS' HANDBOOK for complete table showing full and partial load efficiencies for generators ranging from 5 kw. to 1,000 kw. in capacity.

613. The Output of Any Direct-current Generator, is equal in watts (power output), to the product of the voltage and current delivered by the machine. This follows from the statements of Art. 164. That is:

(121) $$P_W = E \times I \qquad \text{(watts)}$$

hence

(122) $$E = P_W \div I \qquad \text{(volts)}$$

and

(123) $$I = P_W \div E \qquad \text{(amp.)}$$

Wherein P_W = power output of the generator, in watts. E = e.m.f. impressed by the machine on its external circuit, in volts. I = current, in amperes, flowing through the line terminals of the machine. It follows that:

(124) $$P_K = \frac{E \times I}{1{,}000} \qquad \text{(kw.)}$$

hence

(125) $$E = \frac{P_K \times 1{,}000}{I} \qquad \text{(volts)}$$

(126) $$I = \frac{P_K \times 1{,}000}{E} \qquad \text{(amp.)}$$

Wherein the symbols have the same meanings as above except: P_K = power output of the generator in kilowatts.

Example.—If a voltmeter (Fig. 340,*I*) connected across the terminals of a direct-current generator reads 220 volts and an ammeter cut in the line reads 40 amp., the watts power output of the machine is: $P_W = E \times I = 220 \times 40 = 8{,}800$ *watts*. In kilowatts the power output would be: $P_K = (E \times I) \div 1{,}000 = (220 \times 40) \div 1{,}000 = 8{,}800 \div 1{,}000 = 8.8$ *kw.*

Example.—If a generator (Fig. 340,*II*) is developing 44,000 watts and the current through its terminals is 200 amp., the terminal voltage of the machine will be: $E = P_W \div I = 44{,}000 \div 200 = 220$ *volts*. If the wattmeter reads 44 kw., then the voltage would be computed thus: $E = (P_K \times 1{,}000) \div I = (44 \times 1{,}000) \div 200 = 44{,}000 \div 200 = 220$ volts.

Example.—The current through the terminals of the direct-current generator of Fig. 340,*III*, which is developing 312 kw. at a pressure of 110 volts would be: $I = (P_K \times 1{,}000) \div E = (312 \times 1{,}000) \div 110 = 312{,}000 \div 110 = 2{,}840$ *amp.*

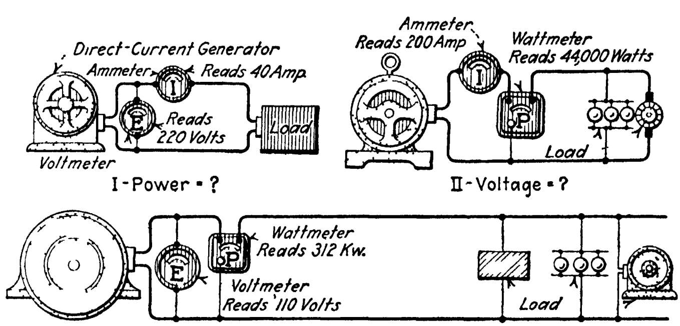

Fig. 340 — Examples in the computation of power output of direct-current generators.

614. The Output of a Direct-current Generator in Horse Power equals its output in watts divided by 746 (Art. 166) or its output in kilowatts divided by 0.746. Thus:

(127) $$HP = P_W \div 746 \quad \text{(h.p.)}$$

and

(128) $$P_W = HP \times 746 \quad \text{(watts)}$$

or

(129) $$HP = P_K \div 0.746 \quad \text{(h.p.)}$$

and

(130) $$P_K = HP \times 0.746 \quad \text{(kw.)}$$

Example.—If a generator is developing 2,000 watts, it is developing: $HP = P_W \div 746 = 2{,}000 \div 746 = 2.68\ HP$. Or if a generator is developing 85 kw., it is developing: $HP = P_K \div 0.746 = 85 \div 0.746 = 114 HP$.

EXAMPLE.—A generator developing 24 h.p. is producing: $P_W = HP \times 746 = 24 \times 746 = 17{,}900$ *watts*. Or this same generator is producing $P_K = HP \times 0.746 = 24 \times 0.746 = 17.9$ *kw.*

615. The Power Input of a Generator, that is the mechanical power required to drive the generator at any load is equal (Art. 612) to the power output of the generator at the given load divided by the efficiency of the generator at that load. The efficiencies of the different kinds of generators at different loads are tabulated in the author's AMERICAN ELECTRICIANS' HANDBOOK. Stated as a formula:

(131) $$P_I = \frac{P_O}{\mathsf{E}} \qquad \text{(input)}$$

or

(132) $$\mathsf{E} = \frac{P_O}{P_I} \qquad \text{(efficiency, in per cent.)}$$

and

(133) $$P_O = P_I \times \mathsf{E} \qquad \text{(output)}$$

Wherein P_I = the power input to the generator. P_O = the power output of the generator. E = the efficiency of the generator which is usually expressed as a percentage. NOTE that P_I and P_O must always be expressed in the same units, which may be either watts, kilowatts or horse power.

EXAMPLE.—If the power output of a generator is 90 kw. and its efficiency at that output is 80 per cent., the corresponding input is: $P_I = P_O \div \mathsf{E} = 90 \div 0.80 = 112.6$ kw.

Where the electrical power output in kilowatts is known and it is desired to ascertain the mechanical power input in horse power necessary to drive the machine at the given load, the following formula may be used:

(134) $$HP_I = \frac{P_{OK} \times 100}{0.746 \times \mathsf{E}} = \frac{P_{OK} \times 134}{\mathsf{E}} \qquad \text{(h.p.)}$$

or

(135) $$P_{OK} = HP_I \times \mathsf{E} \times 0.00746 \qquad \text{(kw.)}$$

and

(136) $$\mathsf{E} = \frac{P_{OK} \times 134}{HP_I} \qquad \text{(efficiency, in per cent.)}$$

Wherein HP_I = the power input, in horse power, necessary to drive the generator at the given load. P_{OK} = the given load output of the generator, in kilowatts. E = the efficiency of the generator at the given load output, in per cent.

EXAMPLE.—What horse power is required to drive the 150-kw. generator of Fig. 341, when it is developing its full rated load of 150 kw.? The machine has a full-load efficiency of 91.5 per cent. SOLUTION.—Substitute in the formula (134):

$$HP_I = \frac{P_{OK} \times 134}{E} = \frac{150 \times 134}{91.5} = 219\ h.\ p.$$

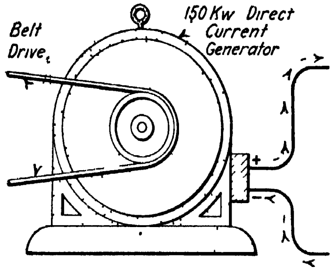

FIG. 341.—What power necessary to drive generator.

EXAMPLE.—If the power input to a generator which has an efficiency of 80 per cent. is 70 h.p., the output of the machine will be: $P_{OK} = HP_I \times E \times 0.00746 = 70 \times 80 \times 0.00746 = 41\ 8\ k.w.$

EXAMPLE.—If a generator is developing an output of 246 kw. and its power input is then 364 h p., what is its efficiency at this load? SOLUTION — Substitute in the formula: $E = (P_{OK} \times 134) \div HP_I = (246 \times 134) \div 364 = 32{,}964 \div 364 = 90\ 5$ *per cent. efficiency.*

SECTION 36

DIRECT-CURRENT-GENERATOR CHARACTERISTICS

616. Characteristic Curves of Generators are graphs which show how the e.m.fs. developed by the machines vary with the load—that is with the current output. Fig. 348 which follows indicates typical examples for machines of the different classes. A characteristic graph of a machine offers a graphic statement of its peculiar qualities or traits. It is therefore obvious that a general familiarity with the typical characteristics of the generators of the different classes is valuable.

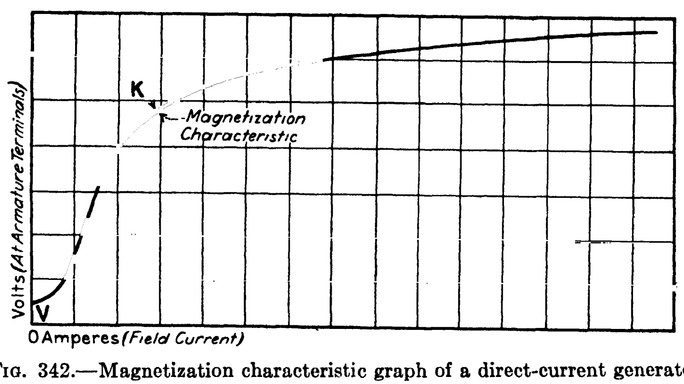

FIG. 342.—Magnetization characteristic graph of a direct-current generator.

617. The Magnetization Characteristic of a Generator is shown for a typical case in Fig. 342. This graph shows how the terminal e.m.f. of the generator changes as the field current is varied. In taking data for plotting a graph like this, the field should be separately excited, Fig. 292. The current values, plotted horizontally, are the readings of an ammeter connected in the field circuit. The voltage values, plotted vertically, are the readings of a voltmeter connected across the armature terminals as the armature is rotated at the rated speed of the machine.

NOTE.—With zero current in the field, there is a small voltage OV developed. This is due to residual magnetism (Art. 556). As the field current

is increased the voltage increases. For low field currents the voltage rises rapidly as the field current is increased. However, beyond the knee K of the curve a considerable increase in field current is required to effect a small increase in voltage. At about the point K the magnetic circuit becomes saturated (Art. 248). Obviously the magnetization characteristic curve is merely a reproduction of the magnetization graphs of Fig. 147 which should be reviewed.

618. The Characteristics of a Series Generator (review Art. 558 on the "Series Generator").—The graph is shown in Fig. 343. Fig. 344 indicates how the instruments might be connected in the circuit to obtain data for plotting the graphs; current and voltage values are plotted as in the preceding article. It is assumed that the armature is being rotated at constant speed. With the external circuit open at switches S_1 and S_2, there would

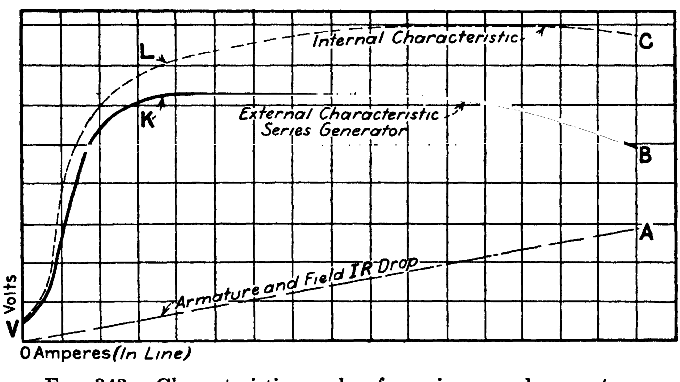

FIG. 343.—Characteristic graphs of a series-wound generator.

be no field current and the voltage impressed on the external circuit across S_1S_2 would be only (OV) due to residual magnetism. If now S_1 and S_2 be closed, a current will be circulated in the external circuit. As more load—lamps or other devices—is connected across the line, the external-circuit resistance is decreased. This permits a greater current to flow, which strengthens the series field, effecting the induction of a greater e.m.f. The voltage impressed on the external circuit rises, as VK. However, with too great a current, the internal IR drop becomes excessive and as the current is further increased due to decreased external-circuit resistance, the terminal voltage will drop off as from K to B. The graph VKB is the *external characteristic* of the machine.

The graph OA shows the internal IR drop in the field coils and

armature. The resistance of these members remains constant; hence the drop due to them increases directly with the load. This drop, in addition to that in the external circuit, must be overcome by the e.m.f. induced in the armature of the machine. Therefore, if the internal *IR* drop graph *OA* be added vertically to the external $I \times R$ drop graph (for that is what it is) *VKB*, their sum will be the graph *VLC*, which indicates the internal characteristic of the generator—that is, how the e.m.f. induced in the armature varies as the current in the circuit changes. The internal-characteristic curve of the machine would be identical with its magnetization curve were there no armature reaction (Art. 583).

If the resistance of an external circuit fed by a series generator is too low, an excessive current will circulate and the terminal e.m.f. of the machine will decrease excessively due to great internal *IR* drop and armature reaction. On the other hand, if

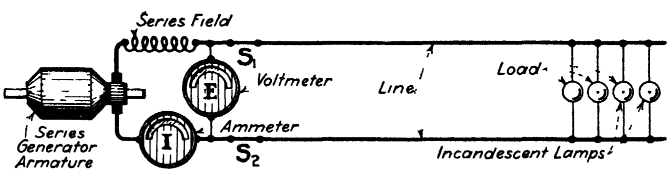

FIG. 344.—Series generator serving load, with instruments for taking characteristic graph data in circuit.

the external-circuit resistance is too high a current sufficient to magnetize the fields will not circulate and hence the generator will not operate.

NOTE.—Series generators are sometimes called *constant-current generators*, not because they inherently produce constant currents but because they are inherently adapted for the development of the e.m.f. for constant-current series circuits. See Art. 558 on the "Series Generator" and Art. 544 on "Constant-current Generators."

619. The Characteristics of the Shunt-wound Generator.—A typical characteristic curve is shown in Fig. 345. Fig. 346 indicates how the measuring instruments should be connected for obtaining the data which is plotted into curves as described in preceding articles.

When a shunt-wound generator (Art. 559) is rotated at its rated constant speed, its field immediately "builds up" (Art. 556) even though switches S_2 and S_3 are open and there is no

current in the external circuit. This is shown in the graph. If the external circuit now be closed (by closing S_2 and S_3) the armature e.m.f. will circulate a current in it. As the external-circuit resistance is decreased the line current increases, hence the armature current increases. This causes a drop in the voltage impressed on the brushes because of three conditions: (1)

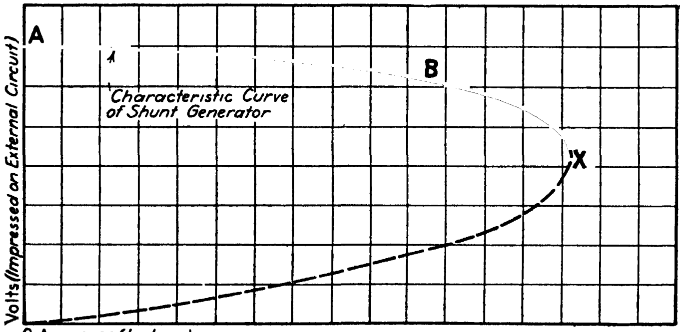

FIG. 345.—Characteristic graph of a shunt-wound generator.

The internal armature $I \times R$ drop (Art. 134) is increased. (2) The back ampere-turns (Art. 586), causing armature demagnetization, are increased. (3) These two effects decrease the brush e.m.f. which decreases the field current, causing a still further lowering of the terminal or brush e.m.f.

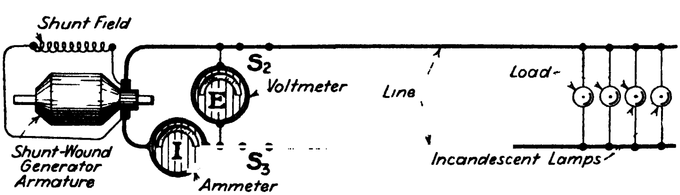

FIG. 346.—Shunt generator serving load, with instruments for taking characteristic graph data in circuit.

As the load on the generator is increased, the voltage impressed on the external circuit continues to drop as shown in Fig. 345. If the external-circuit resistance is so greatly decreased that the line current exceeds the value X, the voltage decreases very rapidly and the impressed e.m.f. falls to zero. This is because the field-core magnetization falls to a value below the knee of the curve (Art. 248) when the voltage falls to the value X so that, as the field-magnetization current decreases, the flux and e.m.f.

diminish very rapidly. That external-circuit resistance which is low enough to cause a shunt generator to thus rapidly demagnetize is the *critical resistance* for that machine.

NOTE.—The point x, Fig. 345, where the voltage of a generator drops off rapidly with increase in load is beyond the operating range AB of the machine and hence is of theoretical interest only.

EXAMPLES.—A shunt-wound generator may be considered a constant e.m.f. generator and hence under certain conditions may be applied for the services outlined in the "Example" under Art. 544. But although shunt-wound generators develop fairly constant terminal voltages even under varying loads they do not operate as satisfactorily as compound-wound machines (Art. 560) nor do they produce as constant voltages. Shunt-wound generators have, in practice, been almost entirely superseded by the compound-wound machines.

620. Characteristics of the Compound-wound Generator.—As outlined in Art. 560, a compound-wound machine is a combi-

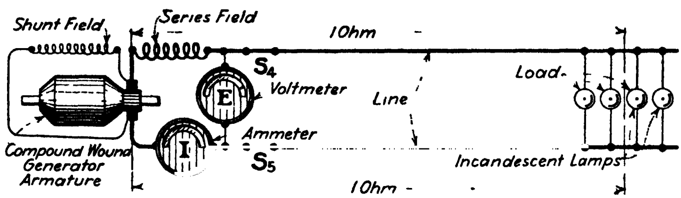

FIG. 347.—Compound-wound generator serving load, with instruments in circuit for taking characteristic graph data.

nation of a series and a shunt machine. Hence, its characteristic is the resultant of the characteristics of the shunt and series generator. Fig. 347 shows the arrangement of instruments for taking the data for plotting the external characteristic graph and Fig. 348 delineates the graph. The dashed line shows how the terminal voltage impressed on the line would vary with the load if the machine had only its shunt-field winding.

If, however, a series-field winding of the proper number of turns be incorporated, then the terminal voltage will be as indicated by the full line. The dot-dashed line shows how that portion of the terminal voltage due to the series winding increases as the load increases. The height of the full-line graph above the horizontal reference line is at any point equal to the sum of the heights of the dashed and the dot-dashed graph. This series-winding graph rises very gradually (instead of abruptly as does the series-generator curve of Fig. 343) because the series winding

of a compound machine has relatively few—or the equivalent of relatively few—turns. The magnetization due to the series winding can be so proportioned that it will compensate or more than compensate for the tendency toward decrease in voltage due to internal $I \times R$ drop (Art. 606) and armature reaction (Art. 583).

EXAMPLES.—Compound-wound machines are now used almost invariably where a constant-e.m.f. direct-current generator is required. They are utilized in railway work and for electric lighting and power transmission where it is not necessary to distribute the energy over great distances. A compound-wound generator is a good example of a constant-e.m.f. (Art. 544) generator.

621. A Flat-compounded Generator is one having the number of its series turns so proportioned that the terminal voltage is

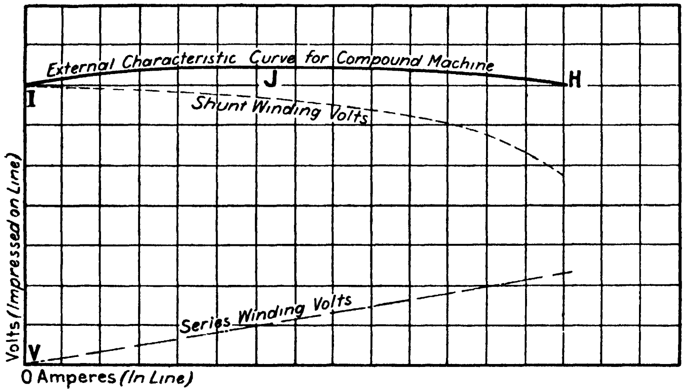

FIG. 348.—Characteristic graphs of compound-wound generator.

maintained (without field rheostat regulation) practically constant at all loads within its range. Usually in these machines the full-load voltage *H*, Fig. 348, is the same as the no-load voltage, *I*; and the voltage *J* at intermediate points is a trifle higher. It is difficult if not impossible to so proportion the field windings that the characteristic graph will be absolutely "flat" throughout the entire range of the machine.

622. An Over-compounded Generator is one having the number of its series turns so selected that its full-load voltage is greater than its no-load voltage. Over-compounding is necessary where it is desirable to maintain a practically constant voltage at some specified point out on the line distant from the

generator. Over-compounding compensates for line drop. Generators are usually so "compounded" that their full-load terminal e.m.fs. are from 3 to 10 per cent. greater than their no-load e.m.fs.

EXAMPLE.—Assume that each of the line wires of Fig. 347 has a resistance of 1 ohm, that each of the lamps constituting the load should have 100 volts impressed across its terminals and that each lamp takes 1 amp. Then with one lamp burning the line drop would be: 1 *amp.* $\times$ 2 *ohms* = 2 *volts.* Hence, with only one lamp burning the generator terminal e.m.f. would have to be: 100 + 2 = 102 *volts.* But with four lamps burning the line drop would be: 4 *amp.* $\times$ 2 *ohms* = 8 *volts;* with four lamps burning the generator terminal e.m.f. would have to be 100 + 8 = 108 volts to insure 100 volts across the lamps at the end of the line. Note that the generator e.m.f. must be increased as the load connected to it increases if the voltage at a given point out on the line is to be maintained constant. That is, the generator must be over-compounded. In the imaginary case just described, assuming that 4 amp. (or: 4 *amp.* $\times$ 100 *volts* = 400 *watts*) is the rated capacity of the generator, it should be so over-compounded that its no-load terminal e.m.f. would be 100 volts and its full-load e.m.f. would be 108 volts.

623. Voltage Regulation.—The voltage regulation of a generator is the percentage of the full-load terminal e.m.f. that the terminal e.m.f. decreases, from no load to full load or:

$$(137)\quad \textit{Voltage Regulation} = \frac{(\textit{No-ld Voltage}) - (\textit{Full-ld Voltage})}{\textit{Full-load Voltage}}$$

The voltage values should be taken with the speed of the machine constant. "Voltage regulation" refers only to the changes in voltage that occur when the load on it changes, because of conditions within and brought about by the machine itself.

EXAMPLE.—The "regulation" of a shunt-wound generator is poor, Fig. 345 because its terminal voltage "drops off" considerably at full-load. The regulation of a flat-compound generator (Fig. 348 is practically perfect because its terminal voltage is the same at full as at no-load. A generator having a no-load voltage of 110 and a full-load voltage of 103 would have a regulation of: (110 − 103) ÷ 103 = 6.8 *per cent.*

624. "Voltage Control" of Generators refers to changes in impressed voltage effected by attendants manipulating field rheostats or to changes effected by antomatic *control apparatus.* "Regulation" (Art. 623) is due to the machine itself. "Control" is due to some external source.

EXAMPLE.—The terminal voltage of a shunt or compound generator can

be controlled by adjusting the shunt-field rheostat. If the resistance in the shunt-field circuit (Art. 559) is diminished the shunt-field current increases and the terminal voltage is correspondingly increased; the reverse is also true. If the speed of the prime mover—engine, water wheel or the like—changes, the terminal voltage of the generator it is driving will change. Where such speed changes are not too frequent the terminal voltage of the machine can frequently be maintained reasonably constant by suitable adjustments of the field rheostat. Automatic *voltage regulators* (the operation of which is explained in the author's Central Stations) can be purchased which will maintain the voltage of a machine practically constant regardless of wide variations in speed and load.

SECTION 37

DIRECT-CURRENT-MOTOR PRINCIPLES

625. An Electric Motor (Fig. 349) is a device for converting electrical energy into mechanical energy. Note that a motor is just the "opposite" of a generator as defined in Art. 509. A motor will supply mechanical power when a current due to some external source of e.m.f. is forced through it. The motion of a motor is due to the reaction between: (1) *the current flowing in a set of conductors mounted on an armature;* and (2) *a magnetic field in which the conductors and their armature rotate.* The motor exerts its mechanical effort or torque as a pull on a belt or thrust on a gear or a twisting force on a shaft.

626. How a Direct-current Motor Converts Electrical Energy into a Mechanical Energy is explained in detail in articles which follow. Briefly the situation is this:

EXPLANATION.—A direct-current motor is the same as a direct-current generator in construction. When a motor is connected to a source of e.m.f. —for example to a generator as in Fig. 350—the e.m.f. developed by the generator will impel a current through the motor armature and a current through the motor field windings. Certain electromagnetic reactions (Art. 434) then occur between the armature and the field which cause the motor armature to rotate and pull its load.

The motor will itself, in a way to be described (Art. 635), tend to govern the intensity of the current which flows through it. When pulling a small load, the current taken by a motor will be small. When pulling a large load, the current taken by a motor will be correspondingly large. That is, with the impressed e.m.f. remaining steady, the current taken at any instant will be almost exactly proportional to the load which the motor is pulling at that instant.

It has been shown that: *power in watts = volts × amperes* (Art. 164). This holds true for a motor. The amperes input to the motor multiplied by the voltage at the motor terminals equals the electric power taken by the motor. The *mechanical power delivered* by the motor is somewhat smaller than the *input power* because there are certain losses in motors similar to those in generators (Art. 611). But if the motor power input be multiplied by the efficiency (Arts. 176 and 676) of the motor, the product will be the power available at the motor pulley for doing mechanical work. See following Arts. 675 to 677 for formulas and examples further explaining this principle.

627. The Principle of Operation of an Electric Motor is based on the phenomena described in Art. 434 where it is shown that

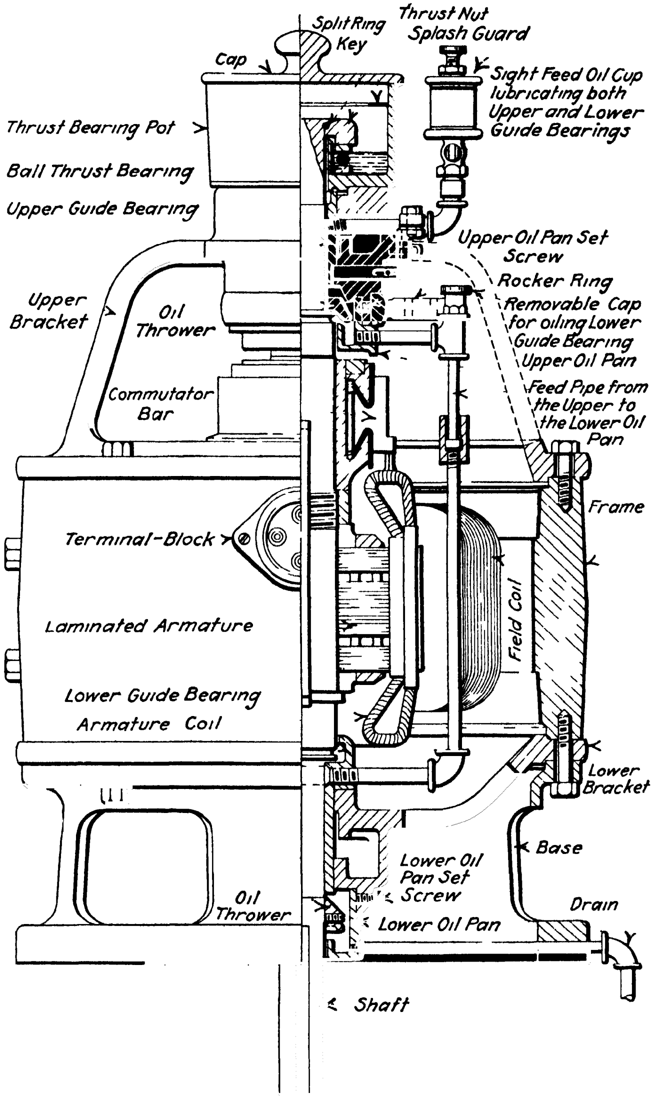

FIG. 349.—Typical sectional view of a vertical direct-current motor.

there is always an electromagnetic force acting on any current-carrying conductor located in a magnetic field. This can be

readily demonstrated with the simple apparatus shown in Fig. 352. Figs. 214 and 351 picture this action on one conductor in a field. Now if a conducting loop like that of Fig. 353 be mounted on an axis O (Fig. 354) in a magnetic field and current be forced through the loop, electromagnetic forces develop tending to cause the loop to rotate.

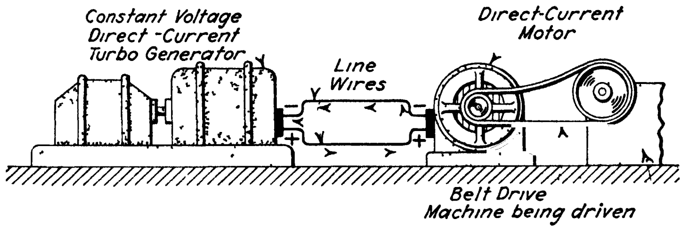

FIG. 350.—Illustrating the conversion of electrical energy into mechanical energy by the use of an electric motor.

EXPLANATION.—As explained in Art. 434, when a conductor carrying current is placed in a magnetic field, the resultant field—that due to a combination of the original field and the field due to the current in the conductor—is stronger on one side of the conductor than on the other. This produces an excess of flux lines (field strength) on one side of the conductor and a deficit on the other side as shown in Fig 354. The flux on one side of the

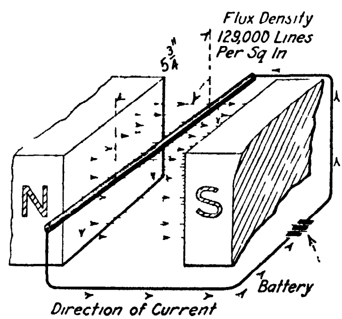

FIG 351 —Illustrating action on a current-carrying conductor in a magnetic field. (The conductor would be forced *up* out of the field by the magnetic reaction)

conductor is distorted—"bunched." The distorted flux lines, displaying their characteristic taut-rubber-band-like tendency (Art. 56) endeavor to straighten out, thus forcing the current-carrying conductor to move in a direction at right angles to the field. Another way of stating this same fact is: The current-carrying conductor tends to move from the strong field to a weaker one.

With a conducting loop like that of Figs. 353 and 354 in a field, a current

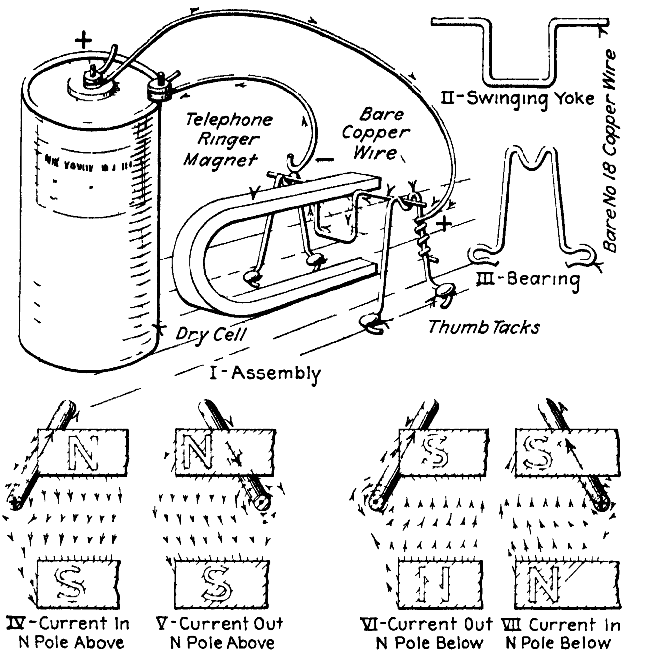

FIG 352 —Showing how a current-carrying conductor is thrust from a magnetic field

With the arrangement shown (which can be assembled in a few minutes) and one dry cell the thrusting effect of a magnetic field on a current-carrying conductor can be readily demonstrated Try it first as at *I*, the yoke will be forced from the field as at *IV* Then reverse the direction of current through the swinging yoke and the yoke will be thrust in the opposite direction as at *V*, in the opposite direction Now turn the magnet over and repeat the experiment as at *VI* and *VII*

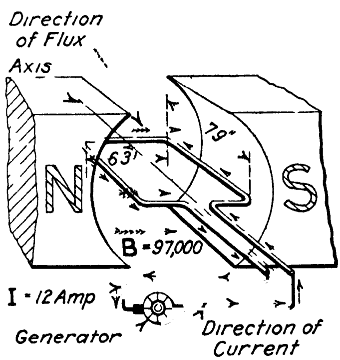

FIG 353 —Illustrating action on a current-carrying loop in a magnetic field (Loop would rotate in clockwise direction. Right-hand side would go down and left-hand side would go up)

in the loop will produce magnetic interactions, as indicated in the illustration, between each of the sides of the loop and the field. Both sides of the loop act in unison and tend to produce rotation of the loop around its axis *o*. The directions of the flux about the sides of the loop of Fig. 354 may be verified by applying the hand rule of Art. 427.

When the loop has rotated until it has assumed a position at right angles to the direction of the field, the tendency to cause its rotation becomes zero as shown in Fig. 355,*I*. However, in an actual motor, when a given loop

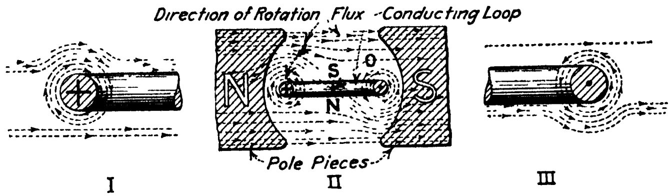

FIG. 354.—Showing how flux distortion tends to produce rotation of a loop, which carries current, located in a magnetic field.

rotates into the position of *I*, its commutator (Art. 529) reverses the current in it and the flux is then distorted somewhat as shown at *III*. With the conditions as shown at *III* there would be no tendency for the loop to rotate in either direction. It would be on a "dead-center." But in actual motors there would be other loops on the armature in about the position of Fig. 354 which would force the loop of *III* to move in the direction of the arrow from the "dead-center" position; once displaced from this position, it would continue to rotate as hereinbefore described.

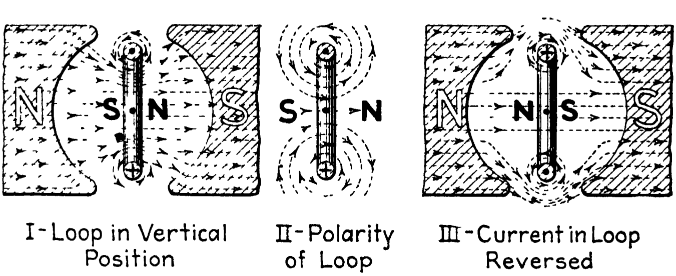

FIG. 355.—Flux about a loop in a magnetic field through which current (electrons in motion in the conductor) is being forced.

ANOTHER EXPLANATION (sometimes offered that does not, however, tell the entire story) which accounts for the rotation of a current-carrying loop in a magnetic field is this: Consider that each loop (Fig. 355,*II*) constitutes an electromagnet (Art. 215) having at one face a N pole and at its other face a S pole. Then the N pole of the loop is always attracted by the S pole of the motor-field magnet and repelled by N pole of the field magnet. In Fig. 354,*II*, the *S* pole of the loop is attracted by the N pole of the field magnet and repelled by the S pole of the field magnet; thus the loop is forced into the position of Fig. 355,*I*.

It would remain held in this position, but at this instant the commutator (Art. 529) reverses the current through the loop as shown in Fig. 355,*III*. Now the S pole of the loop is repelled by the S pole of the field magnet and attracted by the N pole of the field magnet, insuring a continuation of the rotation. This explanation follows from Maxwell's law (Art. 211) which states that: *Any current-carrying coil will tend to assume such a position that the flux enclosed by it will be a maximum and all in the same direction.*

A Simple Electric Motor (Fig. 356) which will operate on a dry cell can be constructed in a few minutes. This little machine will afford a splendid demonstration of the principles hereinbefore outlined.

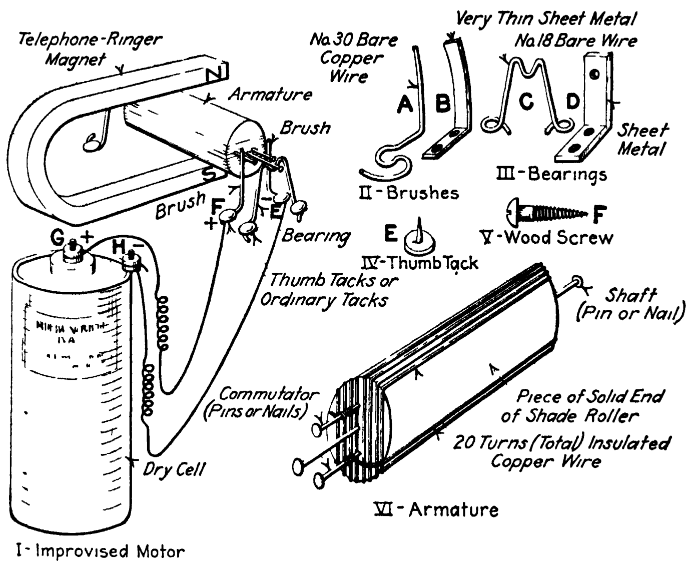

Fig. 356.—Showing how an electric motor can be quickly made.

This experimental motor can be easily made from materials usually available. It will operate on one dry cell. It demonstrates the fundamental principles. Reverse the direction of current through the armature by transposing the connecting wires at *E* and *F* or at *G* and *H* and note that the direction of rotation of the armature is thereby changed. Reverse the direction of the field by turning the magnet upside-down and note that this will also reverse the direction of rotation.

628. Any Electric Generator Will Operate as a Motor.—It follows therefore that the constructional details of motors are in general the same as those of generators as described in preceding sections. For certain special motor applications, such as those in railway, mining and hoisting work, it is necessary that very compact and sturdy construction be used and that the motors be of the enclosed type to insure against the entrance of dirt. Such motors are different in mechanical construction from ordi-

nary generators but electrically they are essentially the same as generators and could be used as such. Many motors have two-circuit or series-armature windings (Art. 598) which require only two sets of brushes regardless of the number of poles. This type is used in railway and automobile service since it permits of the two sets of brushes being located in readily accessible positions.

629. The Force Exerted on a Current-carrying Conductor in a Uniform Magnetic Field (at right angles to the direction of the field) which tends to move the conductor in the direction indicated by the left-hand rule of Art. 513, may be computed by using the following formulas:

$$F_g = 0.000{,}62 \times I \times L \times \mathbf{B} \qquad \text{(grains)} \tag{138}$$

or

$$F_{\text{lb.}} = 0.000{,}000{,}088 \times I \times L \times \mathbf{B} = \frac{I \times L \times B}{11{,}300{,}000} \qquad \text{(lb.)} \tag{139}$$

Wherein F_g = force, in grains, exerted on the conductor (NOTE that there are 7,000 grains in 1 lb.). $F_{\text{lb.}}$ = force, in pounds, exerted on the conductor. I = current flowing in the conductor, in amperes. L = length of the conductor, in inches, *in the uniform magnetic field,* at right angles to the direction of the field. $\mathbf{B}$ = flux density of the magnetic field, in lines per square inch.

EXAMPLE—What force, in grains, is exerted on the conductor shown in Fig. 351? The current in the conductor is 40 amp., the flux density is 129,000 lines per sq. in. and the length of the conductor in the field is $5\frac{3}{4}$ in. SOLUTION.—The conductor in this case would be forced down (hand rule, Art. 427 and Fig. 254) and with a force of:

$$F_g = 0.00062 \times I \times L \times \mathbf{B} = 0.00062 \times 40 \times 5.75 \times 129{,}000 = 18{,}400 \textit{ grains.}$$

The force exerted, *in pounds,* on the conductor would be:

$$F_{\text{lb.}} = 0.000000088 \times I \times L \times \mathbf{B} = 0.000000088 \times 40 \times 5.75 \times 129{,}000 = 2.6 \textit{ lb.}$$

EXAMPLE.—What is the torque (Art. 173) or tendency to rotate developed by the loop in Fig. 353? Each side of the loop is 7.9 in. long. The diameter of the loop is 6.3 in. The flux density of the field is 97,000 lines per sq. in. and the current flowing in the loop is 12 amp. SOLUTION.—Substitute in the formula (139) to determine the force exerted on each side of the loop:

$$F_{\text{lb.}} = \frac{I \times L \times \mathbf{B}}{11{,}300{,}000} = \frac{12 \times 7.9 \times 97}{11{,}300} = 0.82 \textit{ lb.}$$

This force is exerted on each side of the loop hence the force exerted on both sides of the loop is: $2 \times 0.82 = 1.64$ lb. The force is exerted: 3.2

in. ÷ 12 = 0.26 *ft.* from the axis. Hence, the torque is: 0.26 *ft.* × 1.64 *lb.* = 0.43 *lb.-ft.* torque.

630. The Total Torque Exerted by All of the Conductors on a Motor Armature (Fig. 100) may be ascertained by using the formula given below. The force acting on *one* current-carrying conductor tending to move it in a magnetic field may be computed as suggested in Art. 629. When a number of conductors are arranged into an armature winding and current is forced through the winding by an external e.m.f., a magnetic force is exerted by the field on each conductor. The forces on all of the conductors act in unison and thus produce a torque (Art. 173) on the armature shaft. This total torque is—with a given current flowing—independent of the speed of the armature and may be expressed thus:

$$(140)\quad T = \frac{P \times \phi_P \times C_T \times I}{85{,}200{,}000 \times m} = 0.1175 \times P \times \phi_P \times C_T \times I \div m$$

(lb. at 1 ft. radius)

Wherein T = total torque exerted by the armature, in pounds, at 1 ft. radius (Art. 173). P = number of poles of motor. ϕ_P = total useful flux per pole, *i.e.*, number of lines of force per pole that cut armature conductors. C_T = number of active conductors or coil sides on the armature. I = total current, in amperes, forced through the armature from the line. m = number of armature paths between brushes.

EXAMPLE.—What would be the torque exerted by a four-pole motor armature having 80 armature conductors with a current of 20 amp. in the armature circuit? The flux per pole is 400,000 lines. The armature has a multiple winding. SOLUTION.—In a multiple-wound armature there are as many armature paths as poles; hence in this case there are four paths. Now substituting in the formula:

$$T = \frac{P \times \phi_P \times C_T \times I}{85{,}200{,}000 \times m} = \frac{4 \times 400{,}000 \times 80 \times 20}{85{,}200{,}000 \times 4} = 7.5\ lb.\text{-}ft.\ \text{torque.}$$

631. Counter E.m.f. is a phenomenon which occurs in electric motors and which should be thoroughly understood. When *any* conductor cuts flux there is an e.m.f. induced in the conductor, Art. 416. This is just as true of the conductors of a machine when it is operating as a motor as when it is running as a generator, because in both cases the armature conductors are rotating in a magnetic field. Furthermore, the e.m.f. *induced in the armature conductors* in a motor is in the same direction as that induced in the machine if operated as a generator. But this

e.m.f. induced in a motor armature is in a direction opposing (but is never as great as) the e.m.f. which is impressed on the armature and which causes it to rotate. The hand rules, Figs. 208 and 209 confirm this statement. It is also in conformity with Lenz's law (Art. 435). Since this induced e.m.f. is in a direction opposing the impressed e.m.f. and since it *tends* to impel a current in a direction opposite to that of the current which causes the motor armature to turn it is termed the *counter* or back *e.m.f.* of a motor.

632. The Effect of Counter E.m.f. is to limit the current in a motor armature. The resistance of any motor armature is always very small. Frequently it is much less than an ohm. Hence, it is evident that if the normal line e.m.f. is impressed directly on an armature of a motor, an excessive current will flow unless there is something to limit the current to a reasonably safe value. As will be shown, resistance is inserted in series with an armature to limit the current while the armature is being started (Art. 633). The counter e.m.f. developed by the armature limits the current after the armature has commenced rotating. Obviously, when a motor armature is at rest the counter e.m.f. developed by it is zero. When the armature starts to turn it commences to induce a counter e.m.f. and as it "speeds up" the counter e.m.f. increases.

When an armature is at rest the intensity of the current that will flow in the armature is determined solely by Ohm's law ($I = E \div R$)—by the impressed voltage and the resistance in ohms of the armature as shown in the following example. When the armature is rotating, the counter e.m.f. induced, since it is in a direction opposite to that of the impressed e.m.f., has the effect of decreasing the effectual impressed e.m.f. That is, the e.m.f. which is actually effectual in forcing current through the armature is the *difference between the impressed e.m.f. and the counter e.m.f.* As shown in the following example, this difference—that is the effectual e.m.f.—may be but a few volts, even when the impressed e.m.f. is several hundred volts.

NOTE.—The action of motor counter e.m.f. may be better understood by a consideration of Fig. 357. Where a 100 volt generator is shown, diagrammatically, forcing current through a rotating armature. It is assumed that the generator and motor are very close together and that the connecting leads are large so there is practically no voltage drop in them. The voltage impressed on the armature is, then, as represented by the length of the heavy dashed line, 100 volts. The counter e.m.f. induced by the rotating arma-

ture is (dot-and-dashed line) 90 volts and is in the opposite direction to the impressed e.m.f. Then obviously the *effectual e.m.f.*, the pressure available to force current through the armature, is: 100 *volts* − 90 *volts* = 10 *volts* as represented by the short black line.

EXAMPLE.—A 500-volt motor armature (Fig. 358) has a resistance of 0.2 ohm. (1) What current will flow in the armature if it is restrained from

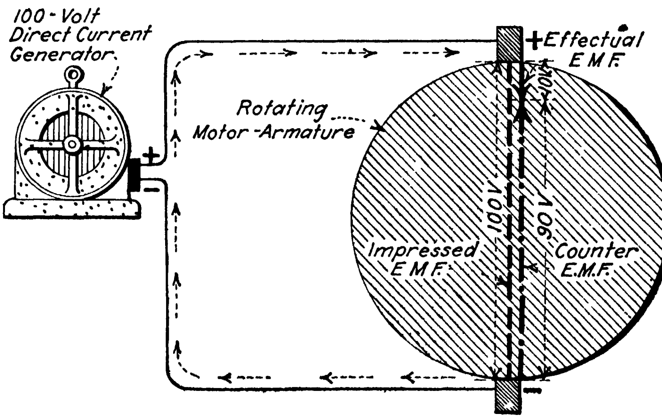

FIG. 357.—Illustrating impressed, counter and effectual e.m.fs. of a motor.

rotating and the rated voltage of the machine, 500 volts, is impressed across it? (2) If, when rotating at its full-load speed the armature induces a counter e.m.f. of 490 volts, what current will then be forced through the armature? SOLUTION.—(1) With the armature at rest its resistance alone limits the current through the armature and in this case the current would then be: $I = E \div R = 500\ volts \div 0.2\ ohm = 2{,}500\ amp.$ (2) With the

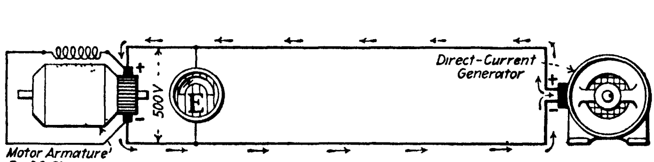

FIG 358.—Direct-current generator forcing current (electrons in motion) through a direct-current motor armature.

armature inducing a counter e.m.f. of 490 volts, the effectual e.m.f. would be: 500 *volts* − 490 *volts* = 10 *volts*. Then the current through the armature would be: 10 *volts* ÷ 0.2 *ohm* = 50 *amp.*

633. The Function of a Starting Resistance (this matter is more fully treated in Art. 654) is to limit the armature current while the motor is being started and before it has attained its rated speed. Such resistances are usually arranged in series

with motor armatures about as shown in Fig. 359. When the motor is connected to the line, all of the resistance is in series with the armature. The resistance may be great enough that the starting current is limited to any desirable value. However, as the armature attains speed, the resistance is cut out, step by step, until, when the armature is running at the speed at which it was designed to operate, all of the starting resistance will be cut out of the circuit and the armature will then be connected directly across the line.

634. The Counter E.m.f. Induced in Any Direct-current Motor Armature Is the E.m.f. Which the Same Armature Would Develop If Operated in a Generator at the Same Speed and in the Same Flux.—It follows from this that the formulas of Arts. 602 and 603 may be applied for computing the counter e.m.f. of a motor. Thus, the e.m.f. induced in the armature of

FIG. 359.—Showing a motor-starting resistance.

any direct-current motor may be computed by using the following formula which is identical with the generator formula of Art. 603:

$$E_B = \frac{P \times \phi_P \times C_T \times T}{10^8 \times 60 \times m} \quad \text{(volts)} \tag{119}$$

Wherein E_B = the counter or back e.m.f. induced in the motor armature in volts. P = the number of field poles of the motor. ϕ_P = the flux or number of lines of force per pole entering or leaving the armature. C_T = the total number of inductors or sides of coils on the armature (see Art. 541). m = the number of parallel conducting paths between positive and negative brush sets (see Art. 598). T = the number of revolutions or turns per minute of the armature.

635. The Speed of Any Direct-current Motor is, at any instant, determined largely by the counter e.m.f. which it develops. The tendency of a direct-current motor is always to operate at a speed such that the sum of the armature IR_A *drop and the counter e.m.f.* will just equal the impressed e.m.f. (Art. 632). At light loads the current, I will be small. Armature resistances

are small. Hence, at light loads, the armature drop, $I \times R_A$, will be small. Therefore, at light loads, E_B will be large and may almost (but can never quite) equal the impressed e.m.f., E. Therefore, when a motor is pulling a light load its armature will tend to rotate at a higher speed than when it is pulling a heavy load. How the speeds of series-, shunt- and compound-wound motors vary, more or less, with their loads in accordance with this principle is discussed in Arts. 647, 662 and 669.

636. The Essential Formulas Relating to Counter E.m.f. are, it follows from the preceding discussion:

$$(141) \qquad I_A = \frac{E - E_B}{R_A} \qquad \text{(amp.)}$$

and transposing:

$$(142) \qquad E_B = E - (I_A \times R_A) \qquad \text{(volts)}$$

and

$$(143) \qquad E = E_B + (I_A \times R_A) \qquad \text{(volts)}$$

Wherein I_A = current, in amperes, through motor armature. E = e.m.f., in volts, impressed on the motor brushes. E_B = counter or back e.m.f., in volts, induced in the motor armature. R_A = armature resistance (Art. 604) of motor, in ohms.

EXAMPLE.—What will be the current in a motor armature that has a resistance of 0.6 ohm, if it is inducing a back e.m.f. of 95 volts and the impressed e.m.f. is 100 volts? SOLUTION.—Substitute in the formula (141): $I_A = (E - E_B) \div R_A = (100 - 95) \div 0.6 = 5 \div 0.6 = 8.3$ *amp.*

EXAMPLE.—What counter or back e.m.f. is being induced in a motor operating on an impressed e.m.f. of 220 volts and having an armature resistance of 0.3 ohm when the armature current is 20 amp.? SOLUTION.—Substitute in the formula (142): $E_B = E - (I_A \times R_A) = 220 - (20 \times 0.3) = 220 - 6 = 114$ volts.

637. The Relations Between the Counter E.m.f. of a Motor and the Power Developed by It are shown by the following formulas. As stated in Art. 636, equation (141), $I_A = (E - E_B) \div R_A$. Hence:

$$(144) \qquad I_A \times R_A = E - E_B$$

Now if every term of the equation be multiplied by I_A

$$(145) \qquad I^2_A \times R_A = (E \times I_A) - (E_B \times I_A)$$

therefore

$$(146) \qquad I_A \times E = (I^2_A \times R_A) + (E_B \times I_A).$$

Equation (146) shows that the total power input to a motor, $I_A \times E$, comprises two components: (*a*) The $I_A{}^2R_A$ power loss in the armature winding due to the current flowing through the winding. (*b*) The power, E_BI_A, due to the counter e.m.f. of the armature. (The power required to excite the field windings and the power required for other losses are here disregarded.) The power, $I_A{}^2R_A$, wasted in the armature winding is a total loss. Hence the power available for driving the motor shaft must be E_BI_A, hence it may be written that:

$$P_M = E_B \times I_A \qquad \text{(watts)} \tag{147}$$

Wherein P_M = the mechanical power, in watts, developed by the motor, including that available at the motor shaft for driving the load and that expended in friction, windage (Art. 611), eddy-current and hysteresis (Art. 611) losses.

It is obvious, then, from formula (147) that the power developed by a direct-current motor is determined by two factors: (1) *The armature current.* (2) *The counter e.m.f.* If, then, the counter e.m.f. of a motor is increased, the current remaining constant, the power of the motor is increased. The power increases directly as the counter e.m.f.

638. The Relation Between the Direction of Rotation of a Motor, the Direction of Flux and the Direction of E.m.f. is the same as for a generator (Art. 513) except that in the case of a motor it is the *impressed* e.m.f. which is usually considered. The impressed voltage is that which causes the motor to operate. This impressed e.m.f. is opposite in direction to the e.m.f. induced if the machine were operated as a generator. Hence, the left hand, as shown in Fig. 254,*II* is used in determining the relations of the directions for a motor. The *direction of the armature current in a motor* is always in the direction of the impressed e.m.f. It follows that the direction of current in a motor is opposite to that of the current in the same machine if operated as a generator.

639. The Direction of Rotation of a Motor and the Method of Reversing the Direction of Rotation can be ascertained by applying the left-hand rule of Fig. 254,*II*. If a machine operating as a generator is being rotated in a certain direction, clockwise for example, the same identical machine will, if a current is forced through it making it operate as a motor, rotate in the same clockwise direction (Art. 521). That is, if a machine is to be operated as a motor, the current must be forced through its armature in a

direction opposite to the direction in which the current flows in the armature when the machine is operating as a generator.

Note.—A consideration of the hand rule of Art. 513 will indicate that the direction of rotation is determined by the relative directions of armature

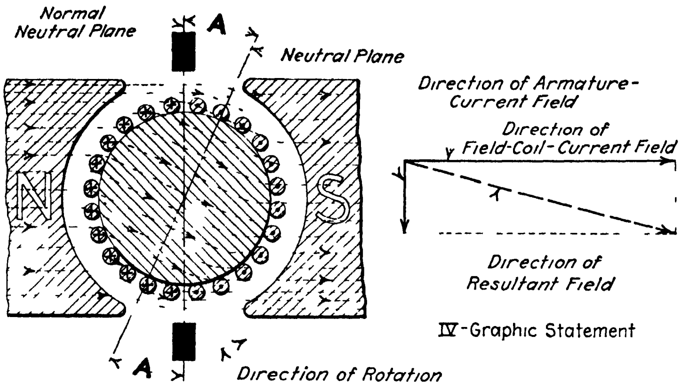

Fig. 360.—Field distortion caused by armature reaction (motor).

current and flux. If follows that to change the direction of rotation the direction of *either* the flux or of the armature current should be reversed. Hence, *to reverse the rotational direction of a motor reverse EITHER the field-coil connections or the armature connections.* Do not reverse both. Obviously, if *both* flux and armature current are reversed, their directions as related to one another remain the same and the change will have no effect on the rotational direction of the armature. Verify this statement by applying the left-hand rule (Fig. 254,*II*).

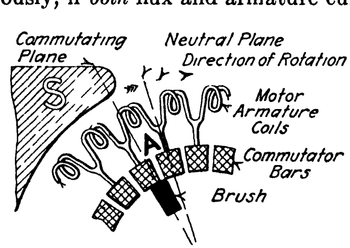

Fig. 361.—Showing how the brushes of a motor should be shifted behind the neutral plane to promote sparkless commutation.

640. Commutation in Motors involves the same principles as does commutation in generators (Art. 580). In this respect motor commutation is similar to motor armature reaction. But with motor armature reaction the field distortion is *back* (Fig. 360), instead of ahead as in generators. Hence, in a motor the neutral plane is *back of* or *behind* the normal neutral plane (Art. 581). The brushes of a motor then are given a *backward lead.* The commutating plane of a motor is a trifle back of the neutral plane (Fig. 361), when the brushes are set in the plane of spark-

less commutation (Art. 588). The motor brushes are thus located so that when any armature coil rotates into the location where commutation occurs there will then be no current flowing in the coil. That is, the brush short-circuits the coil (Fig. 361) before it rotates into the neutral location where it will cease to cut flux.

In a motor, the cutting of flux by an armature conductor induces a counter e.m.f. opposite in direction to the current which is flowing in the armature and which is causing the motor to rotate. Hence, if the brushes are shifted back of the neutral plane just the right distance, the *counter e.m.f. induced by the armature conductors cutting flux* will be just equal to the e.m.f *of self-induction* developed when the short-circuited coil circuit (*a*, Fig. 361) is opened by the commutator bar *B* rotating from under the brush. As noted in Art. 464, the direction of an e.m.f. of self-induction is always such that it opposes any change. Therefore, in an armature coil, the e.m.f. of self-induction at the instant of commutation tends to maintain the current in the coil in the same direction as that in which it was flowing prior to the instant of commutation. For sparkless commutation, the motor counter e.m.f. should neutralize the e.m.f. of self-induction in the coil at the instant of commutation.

641. Commutating Poles Insure Sparkless Commutation in Motors because their effect in a motor is essentially the same as in a generator and as described in Art. 591. Where motors are subject to extreme changes in load or to sudden reversal in rotational direction they are, if of ordinary construction, prone to spark. Furthermore, with motors of ordinary construction, it is usually necessary to shift the brushes as the load changes to insure minimum sparking. With well-designed commutating-pole motors sparking does not occur even when the motors are operating under heavy overloads or when they are suddenly reversed under load. It is not necessary nor desirable to change the brush setting—the brush position—of a motor of this type after the best brush location has once been determined (see Art. 591). Fig. 321 shows the appearance of a commutating-pole motor frame—the construction of the motors and generators of this type is practically identical. See ELECTRICAL MACHINERY.

642. Armature Reaction in Motors is, as might be inferred from the fact that any generator will operate as a motor (Art. 628), similar to armature reaction in generators. However, the cur-

rent in a motor armature, which is rotating, say, clockwise is in the opposite direction from the current in the armature of the same machine when it is operating as a generator and rotating clockwise. This renders motor armature reaction different in detail from generator armature reaction. However the essential principles involved are those outlined under GENERATOR ARMATURE REACTION in Art. 583.

EXPLANATION.—Consider the generator armature which is shown diagrammatically in Fig. 318 and which is rotating counterclockwise: The current direction in the inductors on the right of the neutral plane is *in*, while that in the inductors on the left of the plane is *out*. Now if this same generator be operated as a motor by forcing a current through it, obviously the direction of current in the motor armature will be reversed. But, the direction of rotation of the armature will remain the same (hand rule, Fig. 254, *II*). Then, the current in those inductors on the motor armature on the right of

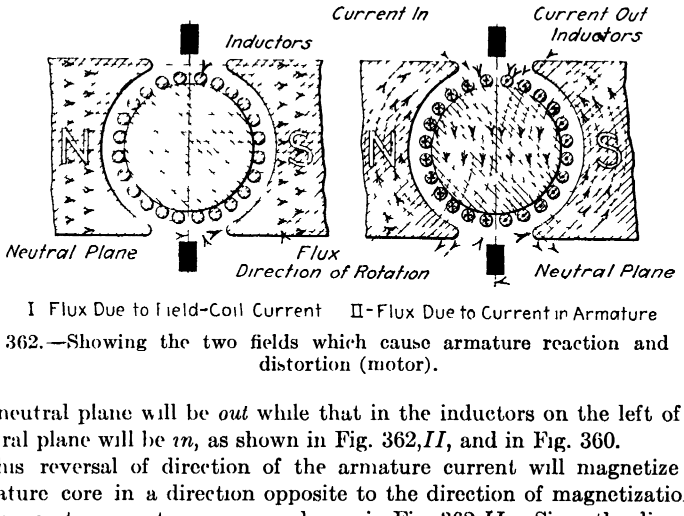

FIG. 362.—Showing the two fields which cause armature reaction and field distortion (motor).

the neutral plane will be *out* while that in the inductors on the left of the neutral plane will be *in*, as shown in Fig. 362,*II*, and in Fig. 360.

This reversal of direction of the armature current will magnetize the armature core in a direction opposite to the direction of magnetization of the generator armature core as shown in Fig. 362,*II*. Since the direction of the field-current field remains the same (Fig. 362,*I*) in a motor as in a generator the field of the motor will be distorted as shown in Fig. 360,*III*. Note that the field distortion in a motor is in the opposite direction to that in a generator. Then the neutral plane will be shifted *back* (against the direction of rotation) as shown in *III*.

The reader should compare carefully the illustrations showing *generator* armature reaction with those showing *motor* armature reaction. Note particularly that, in the diagrams illustrating these conditions which are given in this book, the direction of rotation of generator armature and of the motor armature is the same. With the generator the field is distorted *ahead;* in the motor the field is distorted *back*. Hence, the neutral plane is *ahead* of

the normal neutral plane in a generator and *back* of the normal neutial plane in a motor.

643. A Speed-torque Classification of Loads That May Be Driven by Direct-current Motors should be considered. Motors of the different types described herein (Art. 644) have different operating characteristics which determine their adaptabilities for different classes of work. The different kinds of loads which motors may be called upon to drive may be divided into three classifications thus:

Characteristics of motor loads	1. Constant torque at variable speed. 2. Variable torque at constant speed. 3. Variable torque at variable speed.

LOADS OF CONSTANT TORQUE AT VARIABLE SPEED are those where the load—torque—(Art. 173) is always practically constant but where the speed at which the load is raised or driven must be varied. Cranes, hoists and elevators offer loads of this character. Series motors (Art. 661) have characteristics which render them adaptable for driving such loads.

LOADS OF VARIABLE TORQUE AT CONSTANT SPEED are those imposed by machine tools such as lathes, planers and saws by line shafts and by all other machinery the speeds of which must be maintained practically constant but the loads of which may be almost zero at one instant and considerable a few instants later or may vary more or less at different times. Shunt (Art. 646) and compound motors (Art. 669) are adapted for such services.

LOADS OF VARIABLE TORQUE AT VARIABLE SPEED are those such as imposed by railway traction loads. For example, in starting a car, the torque required is a maximum while the speed is a minimum. After the car has been started the torque required decreases but the speed increases. Series motors (Art. 661) have characteristics which adapt them for such service.

644. Speed Characteristics. The Three General Types of Motors, classified in accordance with the method whereby the field windings are connected, are the same as the three general types of generators, viz.: (1) SHUNT, (2) SERIES and (3) COMPOUND. Each of the three types is discussed in following articles. Just as the generators of the different types have different characteristics, the motors of the different types also have different characteristics. When one speaks of the characteristic of a *generator* it is ordinarily its *voltage characteristic* which is referred to, the speed of the machine remaining constant and the load varying. In speaking of the characteristic of a *motor*, it is the *speed characteristic* which is referred to, the impressed voltage remaining constant and the load varying. This follows because motors

practically always operate from constant-voltage circuits while generators are practically always driven by constant-speed prime movers.

NOTE.—As will be shown, the speed characteristics of motors of the different types (shunt, series or compound) are similar to the voltage characteristics of generators of the corresponding types. *For example:* A generator of a type which will maintain a practically constant voltage while the load which it is serving changes, its speed remaining constant, will, when operated as a motor, maintain a practically constant speed when its load varies, the impressed voltage remaining constant.

644A. "Speed Regulation" and "Speed Control" are two terms that have certain specific meanings just as do the terms *"voltage regulation"* and *"voltage control"* defined in Art. 623 in connection with generators. *Speed regulation* refers to changes in speed caused by the interactions inherent to and within the motor itself as the load driven by it decreases or increases. *Speed control* refers to changes in speed effected by the hand or by the automatic manipulation of some device, usually external to the motor itself, whereby the speed of the motor is changed.

645. The Speed Regulation of a Motor may be expressed as a percentage calculated thus:

$$\text{(148)} \qquad \textit{Speed regulation} = \frac{\textit{No-load r.p.m.} - \textit{Full-load r.p.m}}{\textit{Full-load r.p.m.}}.$$

In words: The speed regulation of a motor is the percentage of the full-load speed that the speed of the motor decreases from no load to full-load.

EXAMPLE.—If the speed of a motor at no-load is 1,800 r.p.m. and its speed at full-load is 1,720 r.p.m., its speed regulation is (1,800–1,720) ÷ 1,720 = 80 ÷ 1,720 = 0.047 = 4.7 *per cent. speed regulation.*

EXAMPLE.—If a motor has a speed regulation of 5 per cent. and its no-load speed is 1,000 r.p.m. its full-load speed will be 1,000 ÷ 1.05 = 952 r.p.m.

EXAMPLE.—If a motor has a speed regulation of 4 per cent. and its full-load speed is 800 r.p.m., its no-load speed will be: 800 + (800 × 0.04) = 832 r.p.m.

SECTION 38

THE SHUNT MOTOR AND ITS STARTING AND CONTROL

646. The Shunt Motor (Fig. 363) will be considered first because its characteristics adapt it to so many applications that it is used more frequently than are direct-current motors of any of the other types. Its most important characteristic is that, with a constant (unvarying) impressed voltage it will maintain practically-constant speed under wide variations in load. Why this is true will be explained. Commercial shunt motors are practically identical in construction with shunt generators as described in Art. 559. Any shunt motor may be operated as a generator and *vice versa.* The characteristic graphs of a shunt motor and of a series motor

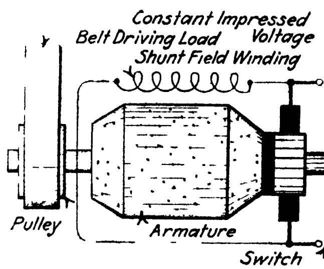

FIG. 363.—The elements of a shunt motor.

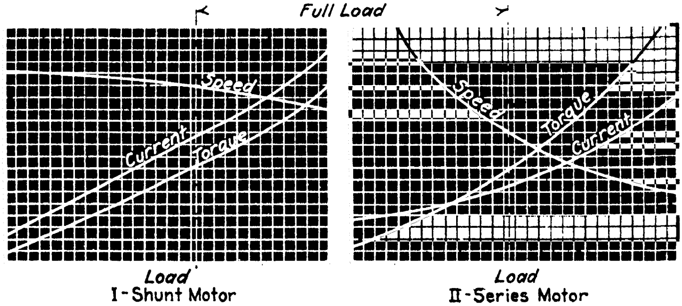

FIG. 364.—Characteristic graphs of shunt and series motors.

are shown side by side in Fig. 364 for comparison. In Fig. 365 the characteristic graphs of a direct-current compound-wound motor are given.

647. The Explanation of the Speed Regulation of the Shunt Motor, that is, why a shunt motor always maintains a practically constant speed even when its load varies widely is this:

EXPLANATION.—Consider the diagrammatic illustration of a shunt motor shown in Fig. 363, which indicates only the essentials. Assume that the switch is closed, impressing a constant voltage across *AB*. This will excite the field winding and its excitation current and field strength will remain constant as long as the impressed e m.f. remains constant. Furthermore, current will be forced through the armature. (In practice, resistance is inserted in the armature circuit when a motor is started, as described in Art. 633 but this feature will, for the present, be disregarded.) Due to the interaction of the shunt-winding field and the field developed by the current in the armature conductors (Art. 627), the armature will start to rotate and it will "speed up" until it attains some certain speed such that the *counter e.m.f. induced in it* (Art. 632) plus *the* $I \times R_A$ *drop in it* just equals the impressed voltage. Obviously the speed thus attained will be determined by the design—field strength, number of armature conductors, etc.—of the motor in question.

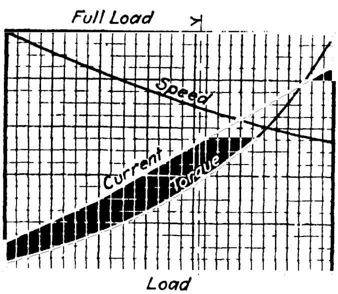

FIG. 365.—Characteristic graphs of a compound-wound motor.

Practically, it may be assumed that the counter e.m.f. when the motor is operating at no load is equal to the impressed e.m.f. because, at no-load, the $I \times R_A$ drop is very small. Hence, at no-load, practically no current is forced through the motor by the voltage of the line. The power then developed by the motor, assuming for simplicity that it is 100 per cent. efficient will be, in watts (Art. 14), equal to *the product of the impressed voltage and the current.* Since the current, at no-load, is very small, the power developed at no-load will be correspondingly small. All that is then produced will be lost in overcoming the internal friction and windage of the machine (Art. 611) and electrical losses. None will be available at the motor shaft for pulling the load.

Now assume that the load which the motor is turning is increased considerably. The motor will "slow down"—not much, only a very little. The armature conductors will cut a smaller number of lines per second and hence will induce a smaller counter e.m.f. This will permit the forcing of the greater current, necessary to pull the load, from the line through the armature. However, as will be shown (Art. 648), a very slight decrease in armature speed (counter e.m.f.) allows a considerably greater current to flow through the armature, so that the motor will maintain practically the same speed when pulling this considerable load as it did when it was pulling no load at all. Again, the power developed by the motor will be equal to *volts* $\times$ *amperes,* but now the current is considerable and the power developed will be correspondingly great.

If the load on a shunt motor decreases, the armature will "speed up" a trifle, thus inducing a greater counter e.m.f. and effecting a material decrease in the current being forced through the motor. Furthermore, the power in watts developed by the motor will also equal *volts* × *amperes*, but since the current is now smaller than before the power developed will be proportionately less.

Thus, it is apparent that a shunt motor will maintain practically constant speed as the load it is pulling varies and that it automatically regulates the intensity of the current forced through it at any instant in proportion to the load it is pulling at that instant. Numerical examples further illustrating this condition are given under Art. 648.

648. The Automatic Speed Regulation of the Shunt Motor May Be Explained Numerically thus: From Art. 629, formula (139), the force tending to thrust a current-carrying conductor from a magnetic field is:

$$F_{\text{lb}} = 0.000000088 \times I_A \times L \times B \qquad \text{(lb.)} \tag{149}$$

In a shunt motor operating on a constant-voltage circuit the flux density, B, remains practically constant because the current through the shunt-field winding is always about the same. Furthermore, the length, L, of the armature conductors in the field always remains constant in a given armature. Obviously, if the torque of the motor is to vary at different loads, the armature current, I_A, must vary.

It has been shown in Art. 637 that the current through an armature is: $I_A = (E - E_B) \div R_A$. Hence, the armature current I_A can be increased only by increasing E, decreasing E_B or decreasing R_A. But on a constant voltage circuit E is constant and in any given motor R_A is constant. It follows that if the armature current, hence torque, of any given operating motor is to be increased to drive a greater load, its counter e.m.f. E_B must decrease. The counter e.m.f. of a motor will be decreased when its armature rotates at a lower speed—this counter e.m.f. varies directly as the speed of rotation (Art. 634).

When then, the load on a motor is increased, its speed and hence counter e.m.f. decrease somewhat and a greater current is forced through the armature from the line. Thereby the torque of the motor is materially increased and it pulls its load at a very slightly decreased speed. Consider the following example:

EXAMPLE.—A shunt motor is connected across a constant-voltage, 220-volt main. Its armature resistance is 0.5 ohm. It is operating at such a

speed that its counter e.m.f. is 215 volts. Then the armature current will be: $I = (E - E_B) \div R_A = (220 - 215) \div 0.5 = 5 \div 0.5 = 10$ amp. Now assume that an additional load is thrown on the motor which decreases its speed 1 per cent., then the counter e.m.f. will be decreased 1 per cent. and will be: $0.99 \times 215 = 213$ *volts*. Under these conditions the armature current will be: $(220 - 213) \div 0.5 = 7 \div 0.5 = 14$ amp. Thus, with a decrease of only 1 per cent. in speed the armature current has been increased by: $(14 - 10) \div 10 = 4 \div 10 = 0.4$ or 40 *per cent*.

The power being developed by the armature when the armature current was 10 amp. was: 10 *amp.* $\times$ 215 *volts* = 2,150 *watts* or 2.88 *h.p.* (Art. 164). But with the 14-amp. armature current the power is: 14 *amp.* $\times$ 213 *volts* = 2,982 *watts* or 3.99 *h.p.* Obviously in this case the power has been increased by $(3.99 - 2.88) \div 2.88 = 1.11 \div 2.88 = 0.38$ or 38 per cent. (The increase in power is not quite proportional to the increase in current shown in the above article because the I^2R_A watts power loss in the armature increases as the square of the armature current). Thus, with a decrease in speed of only 1 per cent. the power output of the armature has been increased 38 per cent.

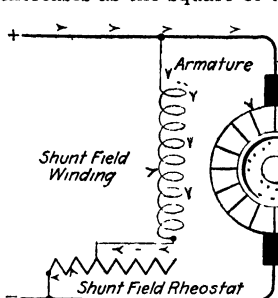

FIG. 366.—Showing a shunt-field rheostat for speed control of a shunt motor.

649. Speed Control of Shunt Motors may be treated under two headings: *field control*, and *armature control*. Field control is the most efficient and effective for reasons which will be given, hence will be considered first.

650. Field Control of Shunt-motor Speed is usually effected with a rheostat as shown diagrammatically in Fig. 366. By varying the resistance of this shunt-field rheostat, which is connected in series with the shunt-field winding across the constant-voltage main, the current through the shunt-field winding may be varied accordingly. The field of the motor may thereby be strengthened or weakened. WEAKENING the field of a shunt motor or of any other kind of a motor causes its speed to INCREASE. STRENGTHENING the field of a motor causes the speed to DECREASE. This state of affairs is precisely the opposite to that which the student usually anticipates. Note the following explanations:

EXPLANATION.—The simplest explanation is, probably, this: Consider the case of a motor armature, which is developing a constant power watts output at different speeds and which is fed from a constant voltage source. The current input to the motor armature must then be the same at all speeds. Likewise the counter e.m.f. must remain constant (Art. 634) if the current

remains constant. Now if the field strength or flux is weakened, the armature must *rotate more rapidly*—must "speed up"—in order to cut the same number of lines per second, that is, to induce the same counter e.m.f., as before. (NOTE.—With conditions as just described that is with constant power output and varying speed, the torque of the armature must vary inversely as the speed. Such a condition is seldom encountered in practice. The explanation of the condition which is usually encountered, that is, one where the driven torque remains practically constant and the horse power output of the motor therefore varies with the speed, follows.)

EXPLANATION.—Where a shunt motor is driving a load of practically constant torque, a decrease in field current, and hence flux, will cause the counter e.m.f. to decrease just as above described. Thereby, a greater current will be forced from the line through the armature. Now the increase in armature current is much greater proportionately than the decrease in flux. Hence, the torque (Art. 176) will be increased and the armature speed will increase proportionately. The following specific example illustrates numerically how the thing works out.

EXAMPLE.—*First.*—Consider the case of a 15-h.p., 220-volt shunt-wound motor, having an armature resistance of 0.2 ohm, which is pulling its full rated load at a speed of 1,000 r.p.m. and taking at this load and speed an armature current of 55 amp.

Second.—Then note how the speed increases when the field flux is decreased by 10 per cent.

WITH THE MOTOR OPERATING AT RATED LOAD AND SPEED AS ABOVE: The torque developed by the motor (Art. 176) will be:

$$(a)\quad T = \frac{\text{h.p} \times 33{,}000}{2 \times \pi \times \text{r.p.m.}} = \frac{15 \times 33{,}000}{2 \times 3.14 \times 1000} = 78.6 \text{ lb. at } 1\ \textit{ft. radius.}$$

The counter e.m.f. (Art. 636) is:

$$E_B = E - (I_A \times R_A) = 220 - (55 \times 0.2) = 220 - 11.0 = 209\ \textit{volts.}$$

WITH THE FLUX DECREASED BY 10 PER CENT.: At this instant the counter e.m.f. will then (Art. 636) obviously be 0.9 of what it was formerly, or it will be: 0.9 × 209 = 188 volts. Then the armature current will be (Art. 636):

$$I_A = \frac{E - E_B}{R_A} = \frac{220 - 188}{0.2} = \frac{32}{0.2} = 160\ \textit{amp.}$$

That is, the armature current is: 160 ÷ 55 = 2.9 times or 290 per cent. as great as it was. Now since the torque is proportional to the product of the flux and the armature current (Art. 630) the torque has now been increased to:

$$(d)\qquad 78.6 \times (0.9 \times 2.9) = 205\ \textit{lb. at } 1\ \textit{ft. radius.}$$

Thus, the torque developed by the motor has been increased: 205 ÷ 78.6 = 2.6 times. It follows that the motor will speed up accordingly. It will run 2.6 times as fast as it did originally.

651. Speed Variation by Field Control Is, in Practice, Limited. —Non-commutating-pole motors are designed to operate at some certain speed and with the iron of their magnetic circuits then

practically saturated (Art. 248). Hence it is not feasible to materially strengthen their fields and thus decrease the speed much below normal. Conversely, if the field flux of such a motor is decreased too greatly, excessive sparking at the commutator results because the field is then too weak to induce a counter e.m.f. that will neutralize the e.m.f. of self-induction of the armature coils (Art. 588) which produces sparking. The self-inductive action of the armature coils is obviously increased as the armature speed is increased. Non-commutating-pole motors can not, ordinarily, be operated at speeds greater than 65 to 70 per cent. above normal. Commutating pole motors (Art. 641) may be obtained whereby a speed variation as great as 6 to 1 can be readily secured.

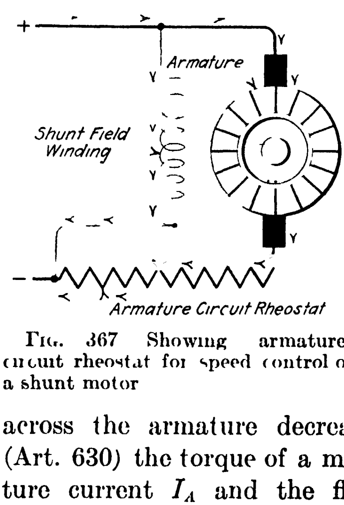

FIG. 367 Showing armature-circuit rheostat for speed control of a shunt motor

652. "Armature Control" of Shunt-motor Speed is effected by inserting a rheostat in series between the source of e.m.f. and the armature, as shown in Fig. 367 and then by varying the effective resistance of this rheostat. Thus the voltage impressed on the armature may be varied (decreased) but the flux due to the shunt-field winding is not affected. Obviously, a decrease in the pressure across the armature decreases the armature current. Now (Art. 630) the torque of a motor varies as the product of armature current I_A and the flux ϕ_P. Hence, since ϕ_P remains practically constant, a variation in the armature current will produce a corresponding variation in torque developed and speed. Therefore, the speed of a shunt motor can be decreased by inserting resistance in its armature circuit.

653. There Are Two Objections to Armature Control and because of these it is seldom used except for the starting of shunt motors (Art. 655) and for certain special applications.

First.—Armature control is wasteful as with it there is a considerable $I^2 \times R$ power loss (Art. 167) in the control rheostat. This loss varies as the square of the armature current.

Second.—The speed regulation of an armature-controlled shunt motor operating at any speed less than maximum is poor. The speed regulation of a field-controlled shunt motor is good because

with such a motor the armature is connected directly across a source of a constant e.m.f. The e.m.f. across an armature-controlled armature varies as the current through the armature and the voltage drop in the control resistance varies.

654. Resistance Is Required in Series with the Armature in Starting a Shunt Motor, to prevent an excessive current from flowing through the armature before the armature has had time to commence rotation and thereby induce a counter e.m.f. This device and its function were described briefly in Art. 633. Resistors of practically the same construction and connected in practically the same manner are used with series and compound

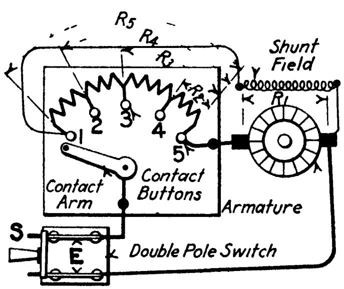

FIG. 368.—Simplified connections of shunt motor and starting rheostat.

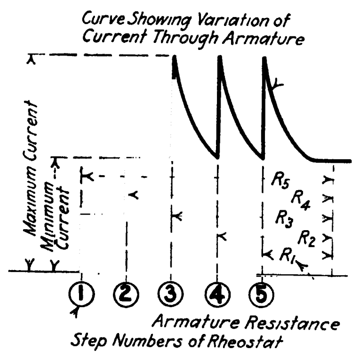

FIG. 369.—Indicating how the current through an armature may be limited with a starting rheostat.

as with shunt motors. With a resistance in circuit the following formula will be true:

$$I_A = \frac{E - E_B}{R_A + R_R} \qquad \text{(amp.)} \tag{150}$$

Wherein I_A = current, in amperes, through the armature. E = e.m.f. impressed on motor and rheostat, in volts, Fig. 368. E_B = counter e.m.f., in volts, induced in the motor armature. R_A = resistance of armature circuit, in ohms. R_R = resistance of starting rheostat, in ohms.

NOTE.—At the instant when the motor is started, there is no counter e.m.f., that is, E_B is then zero. Therefore, the current through the armature at this instant is: $I_A = E \div (R_A + R_R)$. Hence, at the instant of starting, enough starting resistance, R_R, must be cut in series with the armature to limit the starting current to a reasonable value, one which will not dangerously heat the machine and cause excessive sparking at the commutator. The maximum (Fig. 369) starting current permissible is usually assumed as

that intensity which if exceeded would injure the motor windings, cause excessive sparking and pronounced line voltage drop. In practice the maximum starting current is limited to about 1.5 times normal full-load current. The minimum starting current permissible is that necessary to develop the required starting torque.

Frequently the total resistance of the rheostat is made such that with all this resistance (R_5, Figs. 368 and 369) in circuit with the armature, 50 per cent. of the normal starting current will flow through the armature. That is, when the contact arm of Fig. 368 is on contact button "1," one-half the normal starting current will flow as shown in Fig. 369. This current is, since it is only one-half normal, insufficient to start the motor, assuming that is must develop full-load torque to start.

When the arm is moved to button "2," the rheostat resistance then in series is such that the normal starting current flows through the armature as shown in Fig. 369. This may not be a great enough current to start the load. But when the arm is moved to button "3" the maximum permissible current will be forced through the armature from the line and the load should

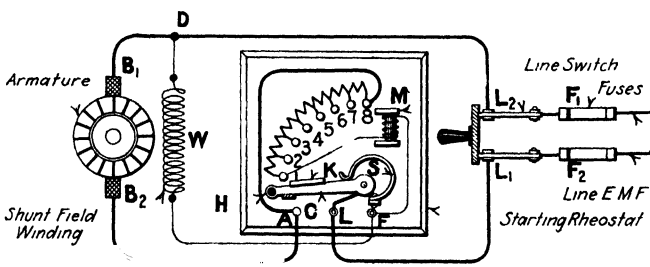

FIG. 370 —Showing the connections between a starting rheostat and the shunt motor which it serves.

start. As the motor increases its speed its counter e m.f. increases and the current decreases, as shown in the diagram of Fig. 369.

The contact arm is then shifted successively over the other contact buttons until it rests on button "5." Now all of the rheostat resistance is cut out, the only resistance in the armature circuit being that (R_1) of the armature itself, because the motor armature is now inducing a sufficiently high counter e.m f. to itself limit the current through the armature.

655. A Starting Rheostat for a Shunt Motor or Compound Motor or as it is sometimes called "a starting box" is a device wherein are assembled the devices essential for the starting of the motor and frequently those for the protection, of the motor after it is in operation, and of the circuits which feed it. (The term "starting rheostat" as it is commonly used implies more than merely a starting adjustable rheostat.) Fig. 370 shows diagrammatically a "starting rheostat" of the simplest type. The variations in practice to satisfy the many possible condi-

tions are almost endless. See the author's AMERICAN ELECTRICIAN'S HANDBOOK for detailed descriptions. A commercial shunt or compound motor starting rheostat usually comprises:

FIRST.—An adjustable resistor or starting rheostat pure and simple, the function of which is to limit the armature current at starting as described in Arts. 652 and 654.

SECOND.—A no-voltage release, described below.

THIRD.—A no-field release, described below.

FOURTH.—An overload release, described below; this device is frequently omitted.

656. The Operation in Starting a Shunt Motor is as follows: The line switch is closed, impressing the line voltage across brush B and metallic swinging arm C (Fig. 370). Then the arm is moved up to contact with button 1. This connects both the armature and the field circuits to the line and current is forced therefrom through each. The current through the armature, which is small, being limited by the starting resistance as described in preceding Art. 655, flows via the route: L_1, L, C, 1, 8, A, B_2, B_1, D, L_2. The path of the field current is: L_1, L, C, 1, M, F, W, D, L_2.

NOTE then, that when the arm is on the first button, a small current flows through the armature and that the field winding is connected directly across the line. (Magnet winding M is in practice of very low resistance as compared with the resistance of the shunt-field winding.) So the field is strong—a maximum—and the armature current weak. Now the arm is moved successively over buttons 2, 3, 4, etc., with a pause at each button. The current through, and the torque of, the armature will be thereby gradually increased as described in preceding Art. 633. The field is weakened slightly as the arm advances because of the insertion thereby of the armature resistance in the field circuit. This weakening of the field tends to assist in "speeding up" the armature (Art. 650). From 15 to 30 sec. should ordinarily be consumed in starting a motor.

Finally, the arm is moved to contact with button 8. It is held there by the attraction of magnet M on iron keeper K, which is attached to the arm. The field current actuates M, the functions of which are treated in detail below. With the arm on 8, the armature is connected directly across the line and the field winding is across the line in series with M and the starting resistance. The combined resistance of the *starting resistance* and M in series is small as compared with the resistance of the motor-field winding. Hence the shunt-field strength is little affected thereby.

657. To Stop the Motor, pull the line switch. Magnet M will then be demagnetized and spring S will force the arm back to its original position shown in the illustration.

658. A No-voltage Release Device is provided on motor-starting rheostats to protect the motor and the circuits feeding it against an excessive rush of current in case the line e.m.f. is discontinued and is then again impressed on the motor without the starting-rheostat handle having first been returned to the starting position.

EXPLANATION.—Assume that the starting rheostat of Fig. 370 is not equipped with the no-voltage release magnet *M* or the spring *s*. Then, if lever arm *c* were moved into contact with button 8, the arm would remain in that position, even if the line e.m.f. were discontinued because of an accident or otherwise. Assume that the arm is in contact with 8, that the line e.m.f. is discontinued and the motor armature slows down or ceases to rotate in consequence. Now if the e.m.f. is again impressed on the line, *c* still on 8, there would be a very large current forced through the armature because it would be turning slowly or be at rest and therefore inducing a small or no counter e.m.f. This excessive current would cause sparking at the commutator and might damage it or "burn out" the motor windings or the wiring.

But with the no-voltage release magnet *M*, arranged as in Fig. 370, when the line e.m.f. is discontinued and the armature ceases rotating, *M* is demagnetized and *c* is forced back into the "off" position by spring *s*. Then the circuit through the motor is open and the resumption of line e.m.f. can do no harm.

659. A No-field Release Device is provided on motor-starting rheostats to prevent the armature remaining connected to the line in case the motor field becomes weak or ceases to exist. Frequently, as in the diagram of Fig. 370, the same magnet, *M*, answers for a no-voltage and a no-field release.

EXPLANATION.—With a weak field the armature speed and sparking will become excessive as suggested in Art. 651. An armature rotating at a speed much in excess of that at which it was designed to operate may have its windings torn off by centrifugal stresses. With no field at all, the armature will cease rotating and then, there being no counter e.m.f., a dangerous current will be forced through it. However, with the holding magnet *M* (Fig. 370) connected in the shunt-field circuit, if this circuit opens or the current through it becomes too weak, iron keeper *K* will be released and arm *c* will be forced back to the "off" position, thus opening the circuit.

660. An Overload Release on a Starting Rheostat is provided to prevent excessive current flowing through the motor or through the motor-circuit wiring, if the motor is overloaded or if a short-circuit or similar trouble occurs on the motor circuit. Circuit breakers and fuses perform functions similar to that of the overload device and are frequently substituted for it, because they may protect both sides of the circuit (F_1 and F_2, Fig. 370).

Usually an overload device like that to be described protects only one side of the circuit.

EXPLANATION.—Fig. 371 illustrates the principle of an overload release device of one type. The holding magnet, M, is the same one as the magnet M in Fig. 370. The overload release magnet is shown at S. The low-resistance winding of this magnet is in series in one of the line wires of the rheostat so that the motor current actuates it. This magnet S is so pro-

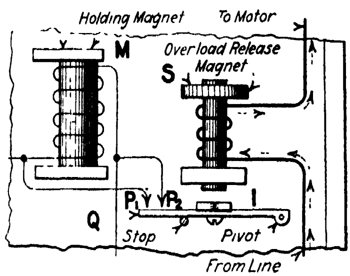

FIG. 371.—Diagram of overload release device.

portioned that if a current greater than that which the motor and circuits can safely carry flows through it, it will attract the iron keeper I which is attached to the metal arm Q. This will raise Q into connection with contact points P_2 and P_2. Thus a shunt or short-circuit is established around the winding of M which is thereby demagnetized. Then the keeper K (Fig. 370) which M was holding will be released and arm K will be forced, by the spring S, back to its "off" position opening the motor circuit.

SECTION 39

SERIES AND COMPOUND-WOUND MOTORS

661. The Series Motor is shown diagrammatically in Fig. 372. It consists merely of an armature and series-field winding, connected in series, and almost invariably operated from a constant e.m.f. source. A constant impressed e.m.f. is assumed in the following discussion. Obviously the current in the armature and in the field winding must, at any given instant, be the same.

662. The Characteristics of the Series Motor are shown graphically in Fig. 364,*II* where they may be conveniently compared with those of the shunt motor of *I*.

THE TORQUE OF A SERIES MOTOR, when the magnetic-circuit iron is being worked well below the saturation point (Art. 248) varies nearly as the square of the current. Thus, motors of this type have splendid starting torque. Torque (Art. 173) varies as the product of flux, ϕ_P and armature current I_A. Now in a series motor, if the current is doubled, the flux is also doubled (approximately) since the armature current traverses the field coils. Hence the torque (proportional to: *current* × *flux*) is increased four times. When the load being driven by a series motor increases, the motor slows down, hence its counter e.m.f. decreases. Then its current increases and its torque increases as the square of the current tending to assist the motor in pulling the load.

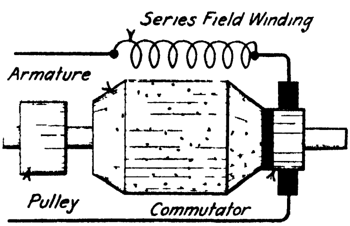

FIG. 372.—Series motor diagram.

THE SPEED OF A SERIES MOTOR varies (Fig. 364,*II*) with the load. For any given horse-power load which a certain series motor is driving, there will be a certain definite speed. With a heavy load, a series motor will operate at relatively-slow speed. With a light load it will operate at a high speed. With no-load it will, theoretically, operate at an infinite speed, that is, it will "run away." The explanation is this:

EXPLANATION.—Any armature always tends to operate at such a speed that the sum of the volts lost in its armature, in IR_A drop, and the volts counter e.m.f. induced will equal the impressed voltage. At heavy loads the current, and consequently the IR_A drop in a series motor armature, will be large and it is not necessary for the armature to rotate very rapidly to induce the required counter e.m.f. But suppose the load is thrown off of

the motor. There now being no resisting torque, the motor speeds up, inducing a higher counter e.m.f. and decreasing the current through the field coil and armature. This weakens the field causing a further increase in armature speed (Art. 650) which again decreases the counter e.m.f. With a series motor operating without load, this process continues until the armature "bursts" due to great centrifugal stresses.

663. A Comparison of the Characteristics of Series and Shunt Motors is given graphically in Fig. 364. Since in the shunt motor the field is practically constant, its torque varies directly with the current (approximately). As shown in Art. 662 the torque of a series motor varies almost as the square of the current. The series motor has, then, a much greater starting torque (ability to rotate something) than has the shunt motor. Hence a series motor of a given size (that is, weight) will start a much greater load than will a shunt motor of the same weight. The shunt motor has a practically constant speed for all loads within its capacity (Art. 646) while the series motor has a certain speed for each load—its speed varies with the load.

664. The Applications for Which the Series Motor Is Adapted are limited because of its speed and torque characteristics which are described above. The motor is not adapted to drive loads the power required by which varies, and which should be driven at a constant speed. The reason is that the speed of a series motor varies with the load.

NOTE.—Hence, a series motor is not, in general, adapted for machine-tool or line-shaft drives. If it is used for such, the speed will be low when the load driven is great and high when the driven load is light. Furthermore, since a series motor will "run away" at no-load, it should be applied only where it can be geared or otherwise permanently connected to its load. Crane, hoist and railway traction applications are good examples of services for which the series motor is well adapted. In other words, the series motor is fitted for conditions where constant speed is not necessary but where great torque at low speeds is desirable.

665. In Starting Series Motors, an adjustable resistance is connected, in series with the motor across the source of impressed e.m.f. (Fig. 373), to prevent an excessive rush of current at the instant of starting, while the armature is stationary and inducing no counter e.m.f. As the armature attains speed, the effective resistance of the rheostat may be decreased by turning the handle. The function of this starting resistance is the same as that of the armature resistance used in starting a shunt motor and which is described in Art. 654.

666. The Speed Control of Series Motors is effected by changing the resistance in circuit between the motor and the source of constant e.m.f. The same rheostat Fig. 373 may be used for this as is used for starting the motor, providing its resistors have sufficient current-carrying capacity to prevent their overheating in speed-control service. The method is somewhat similar to armature-speed control (Art. 652) for shunt motors. It is subject to the same disadvantages. When the effective resistance of the rheostat is varied, the voltage across the armature, E_A, Fig. 373, is varied accordingly and the armature then "speeds up" or "slows down" correspondingly until its counter e.m.f. equals (approximately) the e.m.f., E_A, impressed across the armature. Thus while it is true that a series motor has a definite speed when driving any given load with a given e.m.f. impressed across its armature (Art. 662) its speed when it is pulling a given load may be altered by changing the e.m.f. across the armature.

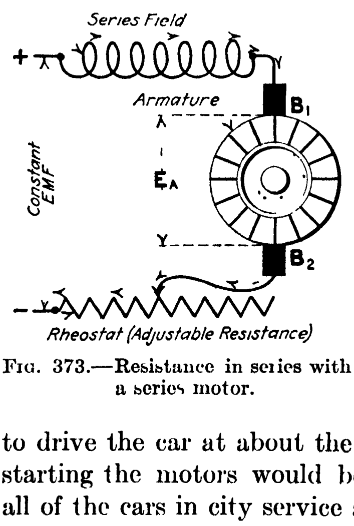

FIG. 373.—Resistance in series with a series motor.

667. Series Motors Are Extensively Used for Driving Electric Cars because their torque and speed characteristics (Art. 662) render them especially fitted for this service. Shunt motors would not be well adapted for propelling a car as they would tend always to drive the car at about the same speed and on grades and in starting the motors would be greatly overloaded. Practically all of the cars in city service are driven by series direct-current motors. Many of the cars and locomotives in suburban and trunk line service are also driven by direct-current series motors although alternating-current motors have, recently, been applied, to some extent, for this work. Cars are equipped with two or four motors so that the torque may be applied at both trucks and so that series-parallel control (Fig. 374) may be used.

668. Series-parallel Car Control is used principally because it is economical. The *controller* located on the car platform and operated by the motorman is merely a rotary switch whereby the different series and parallel connections of the motors may be effected. Briefly, the operation is this:

EXPLANATION.—Refer to Fig. 374. Advancing the controller handle to the first "notch" connects both motors, 1 and 2, in series with all the starting resistance *AC* in series also. As the controller handle is swung around the starting resistance is gradually cut out, the car accelerating meanwhile, until both of the motors are connected directly across the line without any intervening starting resistance. Throwing the controller around another notch connects the motors in parallel between trolley and rail but again in series with all of the starting resistance *A'C'*. As the car accelerates the controller handle is swung on around until the motors are connected directly in parallel between trolley and rail.

The notches whereat the motors are connected without resistance: (1) *in series between trolley and rail*, and (2) *in parallel between trolley and rail* are termed running notches and are shown by longer raised marks on the controller top casting than are the others. When the controller handle pointer is over either of these marks no power is being wasted in starting resistance —all power taken from the line (except motor losses) is being expended by the motors in propelling the car.

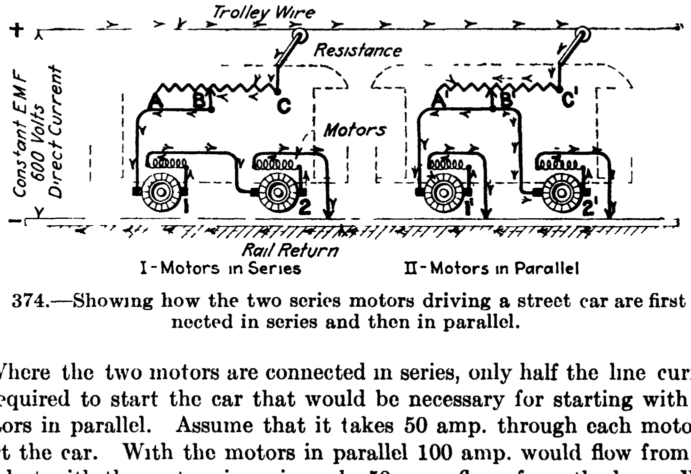

FIG. 374.—Showing how the two series motors driving a street car are first connected in series and then in parallel.

Where the two motors are connected in series, only half the line current is required to start the car that would be necessary for starting with the motors in parallel. Assume that it takes 50 amp. through each motor to start the car. With the motors in parallel 100 amp. would flow from the line but with the motors in series only 50 amp. flows from the line. With the motors in parallel, resistance in series with them would be necessary to limit the current through each to 50 amp. But with the motors in series each motor takes the place of a starting resistance for the other. Thus the economy of series-parallel control is obvious.

669. Compound-wound Direct-current Motors partake of the characteristics of both shunt and series motors just as compound-wound generators (Art. 560) partake of the characteristics of both series and shunt generators. Fig. 365 shows typical characteristic graphs for a compound-wound motor. A compound-wound motor may be made to approximate more or less the characteristics of a shunt or a series motor respectively by making

the ampere-turns of its shunt or series windings more or less powerful proportionately. The elementary connections of a compound-wound motor are the same as those of Fig. 296 for a compound-wound generator.

670. A Differential-compound Motor has its windings connected as shown in Fig. 375,*I*. Its series-field winding "bucks" or opposes its shunt-field winding. A motor of this type can be arranged to operate at an almost exactly constant speed under varying load. As the load increases and the motor tends to "slow down" the series-field magnetization increases and since it opposes the shunt-field magnetization (which is normally greater than the series magnetization) the resultant field is weakened and the motor maintains the same speed as before. Of course, the proportion of shunt-field turns and series-field turns

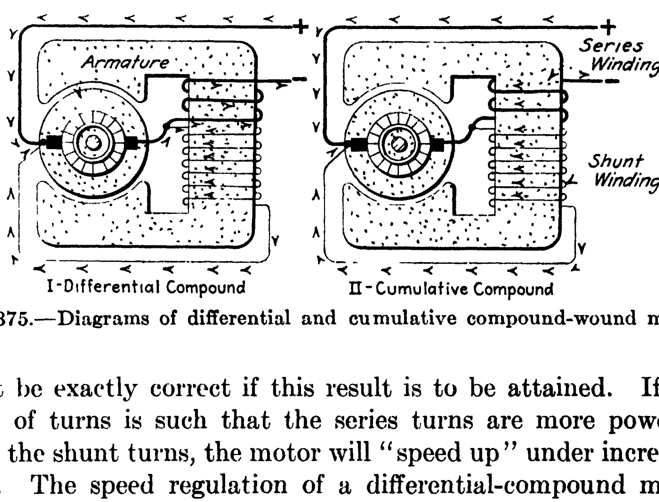

FIG. 375.—Diagrams of differential and cumulative compound-wound motors

must be exactly correct if this result is to be attained. If the ratio of turns is such that the series turns are more powerful than the shunt turns, the motor will "speed up" under increased load. The speed regulation of a differential-compound motor may be closer than that of a shunt motor but its starting torque is not as great.

671. A Cumulative Compound Motor is so connected that its series turns augment the shunt-turns magnetization as shown in Fig. 375,*II*. A motor thus connected will have strong starting torque—stronger than that of a shunt motor but its speed regulation will not be as good. Motors of this type are used for shears, rolls and similar applications where close speed regulation is not necessary but where great torque may be desirable at starting and while running. Frequently motors of this type are merely

series machines provided with sufficient shunt turns to prevent the motor from "running away" when operating at no load.

672. The Starting and Control of Compound Motors is, in general, similar to that for shunt motors as hereinbefore described. The same starting rheostats can be used for ordinary compound as for shunt motors. Ordinarily the series winding has no special connection to the starting rheostat. Sometimes provision is made whereby the series winding may be shunted out of circuit after the motor has been started. The possible special control arrangements are too numerous to describe.

SECTION 40

DIRECT-CURRENT MOTOR POWER, CURRENT AND VOLTAGE RELATIONS

673. The Commercial Voltages Used for Direct-current Motors are in North America 110, 220, 500 and 600, these being the nominal constant-potential-circuit voltages (see the author's AMERICAN ELECTRICIAN'S HANDBOOK). A voltage of 110 is ordinarily used only for the smaller motors fed from incandescent lighting circuits. A voltage of 220 is common for motors used in factories. Street railway circuits operate at a pressure of 600 volts.

674. The Rating of a Motor is determined by the same factors which determine the rating of a generator as outlined in Art. 610. For example, a 10-h.p. motor is one which will, when the voltage for which it was designed is impressed across its terminals, pull a 10-h.p. load continuously without injury to itself. That is, the motor will carry *continuously* the current [which may be determined from the formula (158) of Art. 676] necessary to develop 10 h.p. at the motor shaft without excessive heating, armature reaction or sparking at the brushes. Motors, like generators, are usually so designed that they can carry overloads for considerable periods. Most motors can carry a 25 per cent. overload for 2 hr. On this basis, a 10-h.p. motor can carry 12.5 h.p. for 2 hr. without injury. Obviously a motor can pull any load smaller than that for which it is rated.

675. To Find the Power Input, Current or Impressed Voltage of a Motor any two of these factors being known, the following formulas may be used. The power input to a motor, *in watts,* is equal to the current, in amperes taken by the motor, multiplied by the e.m.f., in volts, impressed across the motor terminals. This follows from the statements of Art. 164. Expressed as a formula:

(151) $P_{IW} = E \times I$, or, *Watts input = Impressed volts × amp.*

hence

(152) $E = P_{IW} \div I$, or, *Impressed volts = Watts input ÷ amp.*

and

(153) $I = P_{IW} \div E$, or, *amp. = Watts input ÷ Impressed volts.*

Wherein P_{IW} = the power input to the motor in watts. E = the e.m.f. or voltage impressed across the motor terminals in volts. I = the current flowing in the motor terminals, in amperes. The power in watts thus obtained may be reduced to *kilowatts* or *horse power* as directed in Art. 166.

EXAMPLE.—If an e.m.f. of 110 volts, Fig. 376 is impressed across the terminals of a direct-current motor and the current through the motor is 10 amp., how many watts is the motor consuming? How many kilowatts? How many horse power? SOLUTION.—Substitute in the formula (151): $P_{IW} = E \times I = 110 \times 10 = 1{,}100$ *watts*. Then 1,100 watts is: $1{,}100 \div 1{,}000 = 1.1$ *kw*. Also, 1,100 watts is: $1{,}100 \div 746 = 1.48$ *h.p.*

676. The Relations Between the Kilowatt Input, Horse-power Output, Efficiency, Impressed Voltage and Current of a Motor are indicated in the following formulas, all of which follow from Arts. 675, 166 and 176:

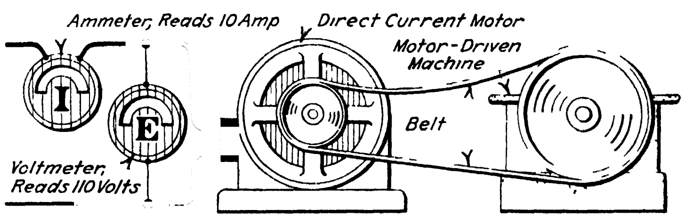

FIG. 376.—Illustrating power input to a motor.

(154) $$HP_O = \frac{P_{IK} \times \mathsf{E}}{74.6} = \frac{E \times I \times \mathsf{E}}{74{,}600} \quad \text{(h.p.)}$$

(155) $$\mathsf{E} = \frac{HP_O \times 74.6}{P_{IK}} = \frac{HP_O \times 74{,}600}{E \times I} \quad \text{(efficiency, per cent.)}$$

(156) $$P_{IK} = \frac{HP_O \times 74.6}{\mathsf{E}} \quad \text{(kw.)}$$

(157) $$E = \frac{HP_O \times 74{,}600}{\mathsf{E} \times I} \quad \text{(volts)}$$

(158) $$I = \frac{HP_O \times 74{,}600}{E \times \mathsf{E}} \quad \text{(amp.)}$$

Wherein HP_O = horse-power output of the motor. P_{IK} = kilowatt input to the motor. E = efficiency of the motor in per cent. at the output HP_O. E = the e.m.f. impressed on the motor in volts. I = current, in amperes taken by the motor for the output HP_O.

EXAMPLE.—Thus the output of a motor having an input of 2 kw. and an efficiency of 90 per cent. would be: $(P_{IK} \times \mathrm{E}) \div 74.6 = (2 \times 90) \div 74.6 = 180 \div 74.6 = 2.4$ h.p.

EXAMPLE.—If a motor at full load requires 12.2 kw. input and its efficiency at this load is 80 per cent., what is its output in horse power at this load? SOLUTION.—Substitute in the formula (154): $HP_O = (P_{IK} \times \mathrm{E}) \div 74.6 = (12.2 \times 80) \div 74.6 = 976 \div 74.6 = 13.1$ *h.p.*

EXAMPLE.—If the e.m.f. impressed on a motor is 220 volts and the current taken by it is 55.5 amp. what horse power is it delivering assuming that its efficiency at this load is 80 per cent.? SOLUTION.—Substitute: $HP_O = (E \times I \times \mathrm{E}) \div 74{,}600 = (220 \times 55.5 \times 80) \div 74{,}600 = 13.1$ *h.p.*

EXAMPLE.—A 13-h.p. motor has an efficiency of 80 per cent. and is to operate on an impressed voltage of 220; what current will it take? SOLUTION.—Substitute in the formula (158): $I = (HP_O \times 74{,}600) \div (E \times \mathrm{E}) = (13 \times 74{,}600) \div (220 \times 80) = 969{,}800 \div 17{,}600 = 55$ *amp.*

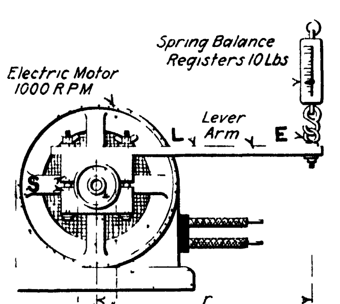

FIG. 377.—Prony brake arranged on an electric motor for a horse-power test.

677. The Horse Power Developed by Any Motor on the Basis of Its Torque and Speed can be readily computed. In Art. 676 the method of determining the power output of a motor, where its electrical input and efficiency are known, is outlined. In this article the method of directly determining mechanical power output is described. If the torque (Art. 173) being developed by the motor is known, this formula may be used:

$$h.p. = \frac{2 \times \pi \times r.p.m.}{33{,}000} \times T \qquad \text{(h.p.)} \tag{159}$$

Wherein $h.p.$ = horse power being developed by the motor. π = 3.,1416. $r.p.m.$ = revolutions per minute of motor. T = torque being developed by the motor, in pound-feet. Now T or torque equals (Art. 173) the product of *force* times *lever arm.* With a PRONY BRAKE, Fig. 377, the torque being developed by a motor can be determined by measuring (1) *the force exerted* and (2) *the lever arm length.* Then these factors being known the horse power developed may be computed with this formula:

$$h.p. = \frac{2 \times \pi \times r.p.m.}{33{,}000} \times r \times F \qquad \text{(h.p.)} \tag{160}$$

Wherein all of the symbols have the same meaning as above

except, r = distance, in feet, from center of motor shaft to point of application of the resisting force on a prony brake lever arm. F = force or pull, in pounds, exerted on the prony brake lever arm at the distance r from the shaft center. A study of the following example will assist the reader in obtaining an understanding of this situation.

EXAMPLE.—The prony brake arrangement of Fig. 377 is a common form. The brake shown comprises a lever arm L at one end of which are arranged two wooden shoes, S, to clamp the motor pulley. At the other end is provided an iron eye E whereby a spring balance or scale may be attached to measure the force with which the motor pulley tends to turn the lever arm. Bolts through the shoes provide means for tightening them on the pulley, thus making the motor do more work and increasing the force—measured by the spring balance—tending to rotate the lever arm.

Assume that the motor is started and that the brake shoes are tightened sufficiently that when the motor is running at 1,000 r.p.m. the lever arm pulls down on the scale with a force of 10 lb. as shown. The length of lever arm, r, is 3 ft. Then the torque (Art. 173) is 10 lb. at 3 ft. radius or is: *3 ft.* $\times$ *10 lb.* = *30 lb.-ft. torque.* Or the torque is *30 lb. at 1 ft.* radius or *60 lb. at 0.5 ft. radius.* Since the motor is rotating at a speed of 1,000 r.p.m., a point on the circumference of the pulley travels, in feet per min.: $2 \times \pi \times r \times$ r.p.m. $= 2 \times 3.14 \times 0.5 \times 1{,}000 = 3{,}140$ *ft. per min.* At its circumference the pulley is overcoming a resistance 60 lb. Hence, the motor is doing work at the rate of: *60 lb.* $\times$ *3,140 ft. per min.* = *188,400 ft.-lb. per min.* Since 1 h.p. is developed when work is done at the rate of 33,000 ft-lb. per min., the horse power that this motor is developing is: *188,400 ft.-lb. per min.* $\div$ 33,000 = *5.7 h.p.*

Note that although the torque at the circumference of the pulley was taken in the example just recited, it is not at all necessary to take the circumference at this location. The torque exerted by the motor *at any location* may be taken *provided* the radius to that point is used instead of the radius of the pulley. From explanation just preceding it follows that the formulas given in the article above can be applied for computing horsepower output. Thus for the example of Fig. 377, equation (160):

$$h.p. = \frac{2 \times \pi \times r.p.m. \times r \times F}{33{,}000} = \frac{2 \times 3.14 \times 1{,}000 \times 3 \times 10}{33{,}000} = 5.7\ h.p.$$

SECTION 41

CHARACTERISTICS OF ALTERNATING CURRENTS

678. Alternating Currents have been referred to briefly in preceding articles. These articles, an enumeration of which follows, should be reviewed before the reader proceeds. In Arts. 113 to 116 the difference between an alternating and a direct current was explained. It was shown, using Fig. 71 for an illustration, that a direct current always flows in the same direction (Art. 112) while an alternating current alternates in direction. For example, a 60-cycle current (Art. 113) alternates (Fig. 70) 120 times a second. It flows in one direction for $\frac{1}{120}$ sec.; the next $\frac{1}{120}$ sec. it flows in the opposite direction and it continues to thus alternate as long as it flows. A 25-cycle current (Fig. 68), alternates 50 times a second.

679. The Difference Between Alternating-current and Direct-current Phenomena is very pronounced. The fact that a direct current always flows in the same direction while an alternating current constantly alternates in direction, as hereinbefore described, is in itself a significant thing. But, as will be shown, due to the fact that an alternating current *does* constantly change in direction and in intensity, certain effects, which do not exist with steady currents, with alternating currents become of great importance. These are the effects of *inductance* (Art. 471) and of *permittance* or as it is frequently called *electrostatic capacity* (Art. 753). Just how inductance and permittance in alternating-current circuits produce these important conditions will be shown in following articles. For the present, note the following:

1. *The current in an alternating-current circuit may not be equal to the e.m.f. impressed on the circuit divided by the resistance of the circuit.* That is, Ohm's law (Art. 134) does not apply to alternating-current circuits in the way in which it applies to direct-current circuits. However, it will be shown, Art. 744, that Ohm's law is also true for alternating-current circuits or any circuit, but that it can not always be applied directly.

2. *The sum of the voltage drops across the components of a circuit carrying alternating-current current may not equal the e.m.f. impressed on the circuit.*

In a direct-current circuit the sum of the voltage drops does equal the impressed e.m.f., Art. 188.

3. *The sum of the currents in the branches of a divided, alternating-current circuit (Art. 197) may not be the same as the total current flowing into and out of the divided circuit.* With direct currents, the sum of the branch currents equals the main current, as stated in Kirchoff's law, Art. 202.

4. *Alternating current may flow in an "open" circuit.* Direct current can not flow in an open circuit—except possibly for an instant after e.m.f. is applied to or removed from the circuit. The alternating currents that flow in so-called open circuits are displacement or charging currents due to permittance.

680. The Method of Generating Alternating Currents was indicated in an elementary way in preceding Arts. 429 and 519. Attention was directed to the fact that an alternating current is precisely like any other kind of an electric current in one respect, namely: a voltage (*e.m.f.* or *difference of potential*) must be developed before an alternating current—or any sort of a current—can be forced to circulate in a circuit.

NOTE.—To impel an alternating current in a circuit an alternating e.m.f. must be provided. Then, when this alternating e.m.f. is impressed on a circuit, an alternating current, inversely proportional to the opposition ("Impedance," Art. 748) offered by the circuit, will be impelled in the circuit. Alternating e.m.fs. are induced in most cases, by moving conductors through flux. A machine whereby conductors are made to cut flux, in such a way that an alternating e.m.f. is impressed on the external circuit served by the machine, is an alternating-current generator or alternator.

It was explained in Art. 520 that the e.m.fs. induced in the armatures of all commercial direct-current generators are alternating e.m.fs. and that a commutator is required to commutate these alternating e.m.fs. into direct e.m.fs. Alternating e.m.fs. are induced in the secondaries of induction coils (Art. 463). Hence induction coils impel alternating currents in their secondary circuits.

681. A Cycle is a complete set of values through which an alternating current or e.m.f. repeatedly passes, as shown in Figs. 68 and 70. A cycle comprises: (*a*) an increase (*OA*, Fig. 68) in current or e.m.f. from zero to a maximum, and a decrease (*AB*) from maximum to zero, *with the current or e.m.f. in one (for example, the positive) direction;* and (*b*) *in the opposite (for example, the negative) direction,* an increase (B_C) in current or e.m.f. from zero to a maximum and a final decrease (*CD*) to zero. In the illustration the portion *OABCD* represents 1 cycle.

EXAMPLES.—The expression "60-cycles-per-second" means that the current or e.m.f. referred to completes 60 cycles in 1 sec., therefore $1/60$ sec. is required to complete 1 cycle; see Fig. 70. See also "Frequency," Art. 682.

With "a 25-cycle current," $\frac{1}{25}$ sec. is required to complete 1 cycle as shown in Fig. 68.

682. The Frequency of an Alternating Current or E.m.f. is the number of cycles that it completes in 1 sec. Fig. 70 shows a graph of a 60-cycle alternating current and Fig. 68 shows a graph of a 25-cycle alternating current, both being plotted to the same time scale. Thus it may be stated that the frequency of a certain current or e.m.f. is "60 cycles per sec." or merely "60 cycles" or the frequency in another circuit may be 25 cycles.

683. The Frequencies Used in Practice are, in North America, 25 cycles (Fig. 68) and 60 cycles (Fig. 70). High frequencies such as 125 and 133 cycles were formerly employed.

NOTE.—A frequency of 60 cycles is common—almost standard—in the United States for general lighting and power installations. A frequency of 60 cycles is usually considered preferable for electric lighting because, with lower frequencies, there may be a visible flickering of incandescent lamps. However, 25 cycles can be and is used for incandescent lighting. Some arc lamps do not operate well on 25 cycles. For long-distance power transmission a low frequency—for example, 25 cycles—is sometimes considered preferable to a higher one because the undesirable effects of inductance (Art. 471) and permittance (electrostatic capacity, Art. 753), increase as the frequency increases. However, it is now usually conceded that, all things considered, 60 cycles is, ordinarily, preferable to 25, even for power transmission, where the energy at the receiving end of the circuit is to be used for general power and lighting service. A few of the older central stations still generate at either 125 or 133 cycles, which were formerly in vogue. For single-phase railway work, 15 cycles has been used to a limited extent. See the author's AMERICAN ELECTRICIANS' HANDBOOK and his ELECTRICAL MACHINERY for further information regarding the adaptability of different frequencies for different services.

684. An Alternation is half a cycle, as shown in Figs. 68 and 70. There are two alternations in a cycle, one in the positive (Art. 522) and the other in the negative direction.

685. Phase.—Any point on an alternating-current wave is called a phase. To designate a certain phase of a wave, it must be specifically indicated in some way. Usually this is done by specifying the number of electrical degrees between the phase to be indicated and some reference point or phase on the wave. The reference point is, unless otherwise specified, taken as the 0-degree point or phase—the starting point of the wave.

EXAMPLE.—In Fig. 378 (taking 0-degree as the reference point or phase), A and A_2 are 30-degree phases, B_1 and B_2 are 90-degree phases, C_1 is a 180-degree phase, D_1 and D_2 are 240-degree phases, E is a 360-degree phase, etc.

It was explained in the example under Art. 528 that electrical degrees really refer to time. Hence, it follows that the term phase may also refer to time. Therefore, a phase may also be defined as the *time instant* when some maximum, zero or any other value is attained by the wave.

EXAMPLE.—When two alternating currents attain their corresponding zero, maximum and intermediate (not necessarily the same) values at exactly the same instants, they are said to be in phase. If the currents are not in phase they reach corresponding values at different instants. NOTE.—DIFFERENCE IN PHASE is obviously the difference, usually stated in degrees, between two specified phases.

EXAMPLE.—The difference in phase between points A_1 and B_1 in Fig. 378 is: $90 - 30 = 60$ *degrees*. The difference in phase between points B_1 and E is: $360 - 90 = 270$ *degrees*.

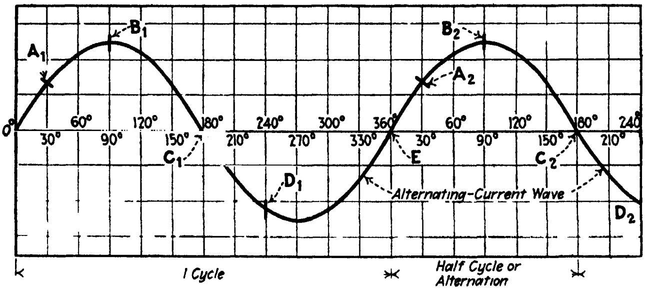

FIG. 378.—Illustrating the term "phase."

NOTE.—The above-given definitions for "phase" are the technically correct ones. However, the word "phase," as loosely used, has other meanings. Sometimes, each of the three wires of a three-phase circuit is called a "phase wire"—or for short, "a phase." Also, any pair of wires of a polyphase (Art. 793) circuit, across which the normal voltage of the circuit should exist, is sometimes referred to as a *phase* of the circuit. Two-phase and three-phase currents and circuits are treated in following Arts. 794 and 803 respectively.

686. Alternating E.m.fs. and Currents May Be Either Single-phase or Polyphase depending on the construction and arrangement of the generator used for their production. Single-phase e.m.fs. currents and principles will first be discussed and later (Art. 793) polyphase circuits and devices will be treated.

687. The Term "Single-phase" implies a single, or only one alternating e.m.f.—or current—in a circuit. That is, with a single-phase e.m.f. or current there is at any given instant only one "phase," as defined in Art. 685. With polyphase e.m.fs. or cur-

rents there is more than one "phase" at any given instant (Art. 793).

EXAMPLE.—In the graph of a single-phase alternating current, shown in Fig. 378, there is at the 90-degree instant but one "phase," B_1. In the two-phase curve of Fig. 476 there are two "phases" at the 90-degree instant. In the three-phase curve of Fig. 488 there are three phases at the 90-degree instant. There are two "phases," at every instant, in a two-phase e.m.f. or current curve and three "phases" at every instant in a three-phase curve.

NOTE.—As will be shown later, under polyphase e.m.fs. and currents (Art. 793), there may be two or more alternating e.m.fs. impressed on, or two or more alternating currents in, the same polyphase circuit. A single-phase circuit ordinarily requires but two wires and is, in this respect, similar to a two-wire direct-current circuit. A polyphase circuit always has more than two wires.

EXAMPLES.—The alternating e.m.f. induced in the loop of Fig. 271 when it is rotated in the magnetic field is a single-phase e.m.f. and the current this e.m.f. impels in the external circuit is a single-phase current. Hence, graphs like those of Figs. 269 and 379 represent graphically single-phase e.m.fs. and currents. A single-phase generator is one which impresses a single alternating e.m.f. on its external circuit. A single-phase circuit is one which has impressed on it, by its source of e.m.f., a single alternating e.m.f. Any alternating e.m.f. impressed on a two-wire circuit must be a single-phase e.m.f. and any alternating current in a two-wire circuit must be a single-phase alternating current.

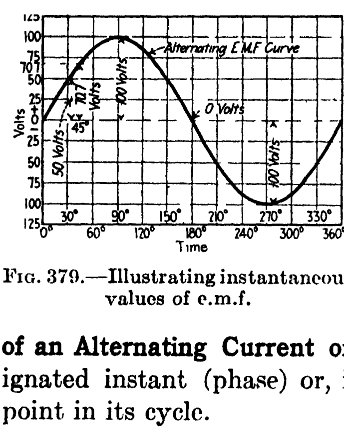

FIG. 379.—Illustrating instantaneous values of e.m.f.

688. An Instantaneous Value of an Alternating Current or Voltage is its value at some designated instant (phase) or, in other words, at some designated point in its cycle.

EXAMPLE.—In Fig. 379 the instantaneous e.m.f. at the 30-degree phase is +50 volts (see Figs. 68 and 70 which indicate the actual time elapsing between different phases for 25- and 60-cycle e.m.fs.). At the 45-degree phase the instantaneous e.m.f. is + 70.7 volts. At the ¼-cycle or 90-degree phase (Fig. 379) the instantaneous e.m.f. is + 100.0 volts, which is also the maximum or crest e.m.f. in this case. At the 180-degree phase the instantaneous e.m.f. is 0 volts. At the 270-degree phase the instantaneous e.m.f. is − 100.0 volts, etc. The symbol + means that the e.m.f. value following it is in the positive (Art. 521) direction. The symbol − means that the e.m.f. is in the negative direction.

689. The Maximum Value of an Alternating E.m.f. or Current (sometimes called the *crest* value) is the greatest value that it

attains, Fig. 380. It is an instantaneous value. For example, the maximum value of the alternating current of the graph of Fig. 70 is 100 amp.; that of the graph of Fig. 73,*I* is 1 amp. With a sine-wave form, a maximum, positive-direction current flows for only an instant during one alternation of a cycle and a maximum, negative-direction current flows for an instant during the other alteration of the cycle.

NOTE.—The insulation of alternating-current apparatus must be designed to successfully withstand the maximum e.m.f. of the circuit. In spite of the fact that this maximum is (where the e.m.f. has a sine-wave form) impressed for only an instant during each alternation, the insulation may be broken down during these "maximum" instants if it is not of sufficient strength.

690. The Average Value of an Alternating Current or E.m.f. (Fig. 380) is the average of all of the instantaneous values—of

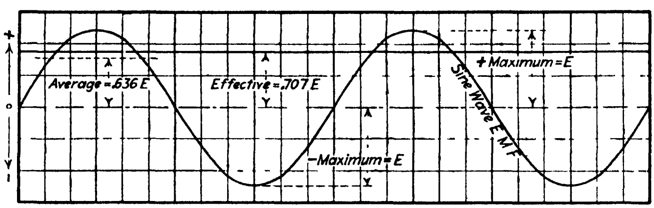

FIG. 380.—Relations between maximum, average and effective values of alternating currents and voltages.

current or e.m.f. as the case may be—of a cycle. The fact that half of the instantaneous values are positive and half negative is disregarded in computing the average value. Through the application of the higher mathematics it can be shown that: *The Average Value of an Alternating E.m.f. or Current of Sine-wave Form* = 0.636 × *Its Maximum Value.* It follows that: *Maximum Value* = *Average Value* ÷ 0.636 = 1.57 × *Average Value.* These relations are given in the formulas in Art. 697.

NOTE.—Average values are seldom used or referred to except in theoretical demonstrations, as for illustration in the proof under Art. 737. A practical man may never hear of average values in his everyday work.

691. The Effective or Virtual Value of an Alternating Current is defined as that value (Fig. 380) which will produce the same heating effect as will the same intensity of direct current. An alternating current is continually changing in intensity, within a certain range, from instant to instant. And it changes in di-

rection at each alteration. These changes occur even if the load is constant. The table under Art. 116, in the first part of the book shows how an alternating current thus varies.

Now these changes occur many times a second with the alternating currents which are used in practice. For example, with a 60-cycle current (Fig. 70) there are 120 alterations per second. Commercial ammeters could not be made which would indicate at every instant these constantly changing values—even if it were desirable to have them do so. The mechanisms of the instruments could not respond to these ever-changing variations.

When an alternating-current flows in an instrument designed for measuring alternating current intensities, the pointer of the instrument will be deflected along its scale a certain distance and will remain steady at that location so long as the current remains constant, that is, so long as the maximum value of the current does not change during successive alternations. But if the alternating-current intensity decreases—if its maximum value decreases through successive alternations—the pointer will swing back showing a smaller deflection. If the alternating current increases, the deflection of the pointer will become greater than before.

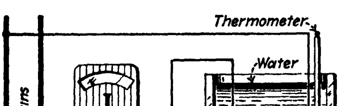

FIG. 381.—Illustrating the heating effects of direct and alternating currents.

It is evident, therefore, that by properly marking its scale, the ammeter could be "calibrated" to indicate either the average or the maximum value of the current passing through it. However, it is more convenient to have the instruments indicate effective values, as they are above defined.

EXAMPLE.—If a direct current of 10 amp. intensity (as indicated by the ammeter, Fig. 381) flowed, for say 1 hr., through the insulated resistor submerged in the water in the vessel, the heat, due to the $I^2 \times R$ loss (Art. 167) in the resistor, would raise the temperature of the water in the vessel. Assume that this direct-current heating effect raised the temperature 10 deg.—from 70 deg. to 80 deg. F. Now assume that instead of direct current, an alternating current is forced through this same resistor submerged in this same vessel. Further, assume that the intensity of this alternating current is regulated by varying the applied e.m.f., until a constant alternating current flows through the resistor which will, in exactly 1 hr., raise the temperature of the water from 70 deg. to 80 deg. F.—the same temperature rise as before. Then this alternating current would have an *effective* value of 10 amp., because it has the same heating effect as a 10-amp. direct current. In other words, it would be an alternating current of 10 amp.

NOTE.—The *ampere* was used as the unit of direct-current intensity before alternating currents were employed. When then it became necessary to measure alternating-current intensities, instead of defining some new unit for measuring these intensities, it was decided to also call the unit of alternating current an ampere and to have it, insofar as possible, equivalent to the direct-current ampere.

Now alternating-current intensities can not be compared with direct-current intensities on the basis of their electrolytic effects. Why? Because the metal deposited electrolytically by an alternating current during any one alternation would be redeposited in the other direction, during the next alternation. Obviously, the net electrolytic effect of an alternating current is nil. Hence an alternating-current intensity or rate of flow, can *not* be defined by its electrolytic effect as is the direct-current intensity. But the heating effects of alternating and direct currents can be compared readily—as shown in the above example. Hence it was decided to define the alternating-current ampere on a basis of its heating effect as hereinbefore specified.

692. Alternating-current Measuring Instruments Indicate Effective Values.—This is true of both ammeters and voltmeters. When speaking of alternating currents or e.m.fs., the values referred to are, ordinarily, unless it is otherwise specified, effective values. The practical man deals almost exclusively with effective values.

693. The Relation of the Effective to the Maximum Value of an Alternating Current or Voltage is shown in Fig. 380. It may be expressed thus: *Effective Value* = 0.707 × *Maximum Value*. It also follows that: *Maximum Value* = *Effective Value* ÷ 0.707. Equations expressing these relations are given in a following paragraph, Art. 697.

EXAMPLE.—What is the effective value of the alternating current of Fig. 70? SOLUTION.—Its maximum value, reading from the graph, is 100 amp. Hence to find its effective value:

$$\textit{Effective value} = 0.707 \times \textit{max. value} = 0.707 \times 100 \textit{ amp.} = 70.7 \textit{ amp.}$$

EXAMPLE.—What is the effective voltage of a circuit that has a maximum voltage of 156 volts? SOLUTION.—Substitute in the formula:

$$\textit{Effective value} = 0.707 \times \textit{max. value} = 0.707 \times 156 = 110 \textit{ volts.}$$

EXAMPLE.—If a voltmeter on an alternating-current circuit reads 2,200, what is the maximum instantaneous voltage? SOLUTION.—Voltmeters and ammeters always indicate effective values, Art. 692. Substitute in the formula:

$$\textit{Maximum value} = \frac{\textit{Effective value}}{0.707} = \frac{2{,}200}{0.707} = 3{,}110 \textit{ volts.}$$

694. Why the Effective Value Equals 0.707 × the Maximum Value may be explained thus: The effective value of an alter-

nating current is, as stated in Art. 691, determined by its heating effect. Now the heating effect of a current of electricity is proportional to the square of the current, in amperes (Art. 167). It follows that the heating effect of an alternating current *at any instant* is proportional to the square of the current at that instant. Now if the squares of the current intensities of an alternating current for a great number of equidistant instants over an alternation be computed and their average taken, this resulting value will be the average or mean square of all of these instantaneous currents. Then to ascertain the equivalent steady current that would produce the same heating effect as all of the instantaneous currents of different values, the square root of the mean square is taken. It will always be found, if the problem

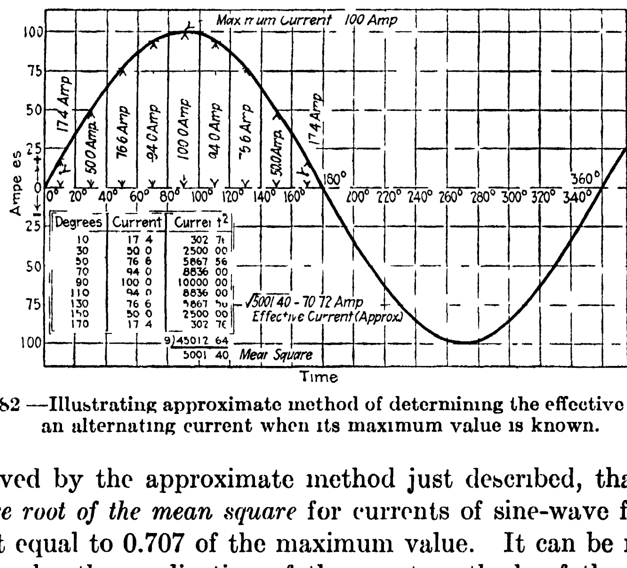

FIG. 382 —Illustrating approximate method of determining the effective value of an alternating current when its maximum value is known.

is solved by the approximate method just described, that this *square root of the mean square* for currents of sine-wave form is about equal to 0.707 of the maximum value. It can be readily shown, by the application of the exact methods of the higher mathematics, that the value 0.707 is absolutely correct for currents of sine-wave form.

EXAMPLE.—Consider the alternating-current sine curve of Fig. 382. The maximum current is 100 amp. The mean square of the current values at 9 different equidistant instants is 5,001.40. The square root of 5,001.40 is 70 72 amp. which is the effective current in accordance with this *approximate* solution. Actually the effective current, with a maximum of 100 amp. would be 70 71 amp. Hence, the above approximate solution gives a value just a trifle high. If current values are taken 2 degrees apart over an alter-

nation, instead of 20 degrees apart as in the example just solved, the effective value will come out almost exactly 70.71 amp. Obviously it is not necessary to consider values over an entire cycle, as the values for one alternation are the same as those for another, assuming that the maximum current remains the same. That is, the values for the first half cycle will be the same as those for the second half cycle.

695. The Effective Value of an Alternating E.m.f. is that value which will propagate an alternating current of corresponding intensity. The meaning of this statement can be understood from a consideration of the following numerical example.

EXAMPLE.—If a circuit (Fig. 383,*I*), comprising resistance only and having a resistance of 10 ohms, an effective alternating current of 6 amp. flows, the effective e.m.f. impelling the current is (Ohm's law): $E = I \times R = 6 \times 10 = 60$ *volts*. If an effective current of 15 amp. flows in this same circuit (Fig. 383,*II*) then the effective e.m.f. impelling it must be: $15 \times 10 = 150$ *volts*.

696. The Relation of the Effective Value of an Alternating E.m.f. to Its Maximum E.m.f. is numerically the same as the relation of effective and maximum current values. It is always the

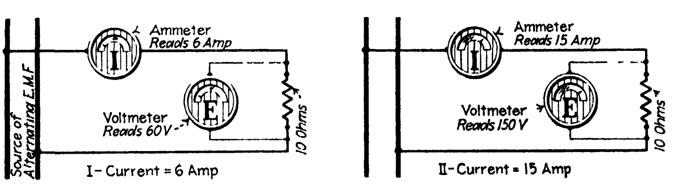

FIG. 383.—Effective e.m.fs. of 60 and 150 volts respectively impelling effective currents in alternating-current circuits.

e.m.f. which originates or impels a current and a current is always proportional to its e.m.f. If an alternating current has a sine-wave form, the e.m.f. which impels it must, obviously, have a sine-wave form. Hence, the same numerical relation must hold between maximum and effective values for both voltages and currents.

Hence, equation (166): *Effective e.m.f.* $= 0.707 \times$ *maximum e.m.f.* It follows that *maximum e.m.f.* = *effective e.m.f.* $\div 0.707 = 1.46 \times$ *effective e.m.f.*

Equations indicating these relations are given in the following article No. 697.

697. The Numerical Relations of Maximum, Effective and Average Values to one another are shown in the following equations. They have been thus collected for ready reference:

(161) *Maximum value* = *1.41 × *Effective value*

That is:

(162) $E_M = {}^*1.41 \times E_E$, or, $I_M = {}^*1.41 \times I_E$

(163) *Maximum value* = 1.57 × *Average value*

That is:

(164) $E_M = 1.57 \times E_A$, or, $I_M = 1.57 \times I_A$

(165) *Effective value* = 0.707 × *Maximum value*

That is:

(166) $E_E = 0.707 \times E_M$, or, $I_E = 0.707 \times I_M$

(167) *Effective value* = 1.11 × *Average value*

That is:

(168) $E_E = 1.11 \times E_A$, or, $I_E = 1.11 \times I_A$

(169) *Average value* = 0.636 × *Maximum Value*

That is:

(170) $E_A = 0.636 \times E_M$, or, $I_A = 0.636 \times I_M$

(171) *Average value* = 0.901 × *Effective value*

That is:

(172) $E_A = 0.901 \times E_E$, or, $I_A = 0.901 \times I_E$

* The value 1.41 is an approximate expression of the exact value $\sqrt{2}$. A more accurate expression of this value is: 1.414228.

SECTION 42

ALTERNATING-CURRENT GENERATOR PRINCIPLES AND CONSTRUCTION

698. Practical Alternating-current Generators May Be Divided into Two General Classes Thus.—(1) Revolving-armature machines. (2) Revolving-field machines. As stated in Art. 534, the armature is the portion of a generator in which the e.m.f. is induced. The field is the structure which provides the magnetic field, the flux of which cuts or is cut by the armature inductors, whereby the e.m.f. is induced. Each of the two classes is briefly treated below.

699. The Construction of Revolving-armature Alternators is shown in Figs. 384 and 385. In general they are quite similar to direct-current generators except that collector rings (Fig. 386) are substituted for the commutator. The fields must be excited with direct current, for which an independent, small direct-current generator—an exciter—is ordinarily required. Frequently the exciter-armature winding is wound on the same core with the alternator winding and a separate commutator provided for it. With such an arrangement, or when an independent exciter is direct-connected to the alternator, the revolving-armature alternator is then self-contained. Revolving-field alternators are frequently built on the same frames as are used for direct-current generators.

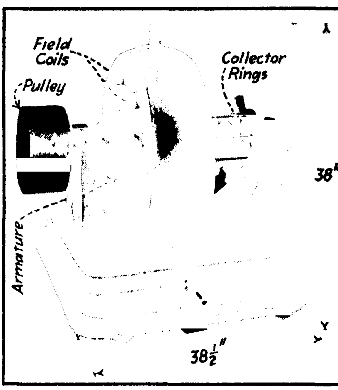

FIG. 384.—Revolving-armature, belted alternator (General Electric Co., 25-kva., three phase, 17½ kva., single phase, 4 poles, 1800 r.p.m.).

700. A Telephone Generator or Magneto Is a Revolving-armature Alternating-current Generator.—This machine is described and illustrated in Art. 543. The magnetic field in this machine is produced by permanent magnets which are described in the data on "Magnetism" in Art. 80.

701. The Application of Revolving-armature Alternators is, in practice, limited because revolving-field machines (Art. 704) are, in the larger capacities less expensive, and in any case have better inherent characteristics. It is seldom that revolving-armature alternating-current generators are made for capaci-

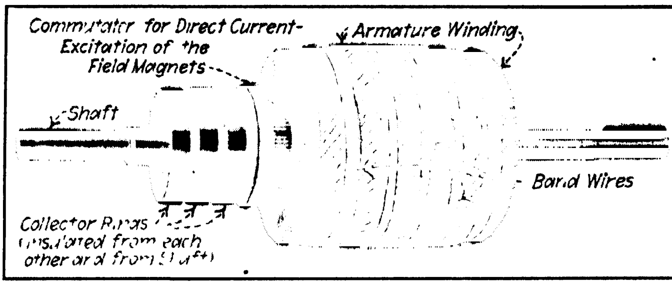

FIG. 385.—Complete armature of a revolving-armature alternator. (This, having three collector rings, is for a three-phase machine. The construction and arrangement of single-phase and two-phase armatures is similar.)

ties exceeding 25 kva. (three-phase, 17½ kva., single-phase) or for voltages exceeding 600. Their inherent regulation is poor and the alternating voltage is impressed on the armature and collector rings, which are difficult to insulate economically for

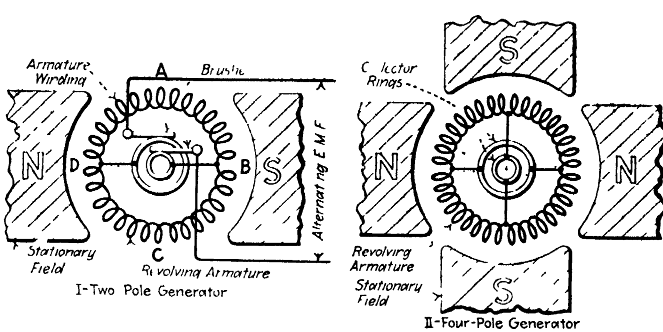

FIG. 386.—Showing how revolving-armature windings are connected to collector rings in revolving-armature, alternating-current generators (single phase).

high voltages. In revolving-field machines the revolving parts are not subjected to high or line voltages.

702. The Explanation of the Operation of a Revolving-armature Alternator is as follows:

EXPLANATION.—When the armature is in the position indicated in Fig. 386, *I*, the e.m.f. impressed on the rings is zero. The e.m.f. induced at this

instant in the inductors between B and C is equal and opposite to that induced in those between C and D. Also, the e.m.f. induced in winding BA neutralizes that in DA, so there is no e.m.f. across the rings. But when the armature is rotated ¼ revolution, then, at that instant, the e.m.fs. in windings DAB and DCB are a maximum and maximum voltage is impressed on the rings. When the armature has been rotated through another ¼ revolution the e.m.f. across the rings is again zero. When it has been rotated through still another quarter the e.m.f. is again a maximum, but it is now in the opposite direction. These statements may be verified by applying the hand rule for determining the direction of an induced e.m.f. Thus as the armature is rotated it impresses an alternating e.m.f on the external circuit connected to its collector rings.

703. A Revolving-armature Alternating-current Generator Can Be Arranged from any Direct-current Generator. (Fig. 387)—It was shown in Art. 520 that the e.m f. induced in a loop or coil rotated in a magnetic field is an alternating e.m.f. Hence, a (bipolar) direct-current generator may be modified into an alternating-current machine by separately exciting (Art. 554) its field with direct current, tapping its armature winding at two diametrically opposite points and connecting each of the taps to a collector ring, as shown in Fig. 386, *I*. When an armature thus connected is rotated in the field, an alternating e.m.f. will be impressed across the collector rings. The collector rings may be attached to (but insulated from) the commutator of the armature as shown in Fig. 387. When a machine is built as an alternator, obviously, only the collector rings are necessary; a commutator is not then required.

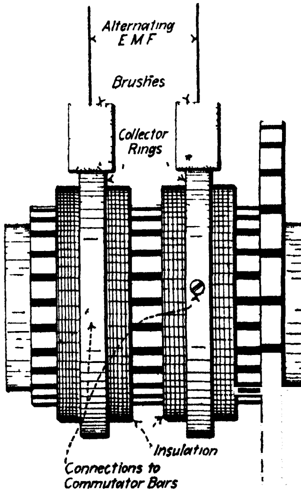

FIG. 387—Showing how the commutator of a bipolar direct-current generator can be fitted with collector rings to impress an alternating e.m f.

With a multipolar-field machine, there should be as many equidistantly spaced taps from the armature winding to the collector rings (Fig. 386) as there are poles. Alternate taps connect to the same ring. The information given in this article applies to single-phase generators. For polyphase machines it must be modified as elsewhere suggested.

EXAMPLE.—Fig. 386,*II* shows the revolving-armature winding of a four-pole generator tapped at four equidistant points and connected to two collector rings so as to impress an alternating e.m.f. across the collector rings. The brushes are, for simplicity, omitted.

704. Revolving-field Alternators are the most important commercially, for reasons suggested in Art. 701. All modern machines of large capacity are of this type. Figs. 388 and 389 show typical horizontal, revolving-field machines. Fig. 390 details the general construction of all revolving-field alternators;

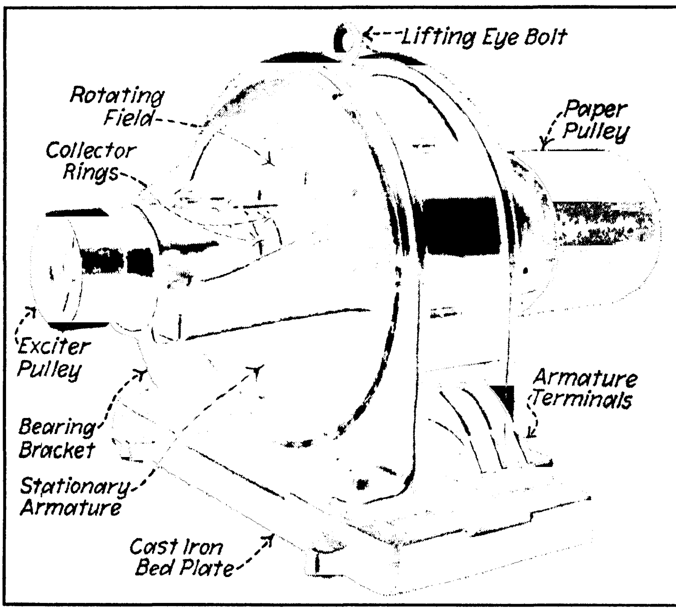

FIG. 388.—A revolving-field, belt-driven alternator (The illustration shows an 8-pole, 150-kva, 900 r.p.m., 2300-volt, three-phase, type TRB, form B General Electric Company machine.)

however, some of the mechanical features there shown are incorporated only in vertical machines.

705. The Elementary Revolving-field Alternating-current Generator is diagrammed in Fig. 391. The revolving field is produced by the rotating electromagnet M which is energized with direct current circulated by a couple of dry cells. The dry cells are equivalent to an *exciter*. The stationary conducting loop L is equivalent to a stationary armature winding. Note that this arrangement is similar to the elementary generator of

Fig. 253 except that, in that illustration, the armature is rotated and the field is stationary.

EXPLANATION.—As the field M is rotated (clockwise) its flux cuts the stationary conductor L. At the instant pictured, the flux from the S pole is cutting the top side of the loop and that from the N pole the bottom side of the loop. An e.m.f. is being induced and a current will flow (from front

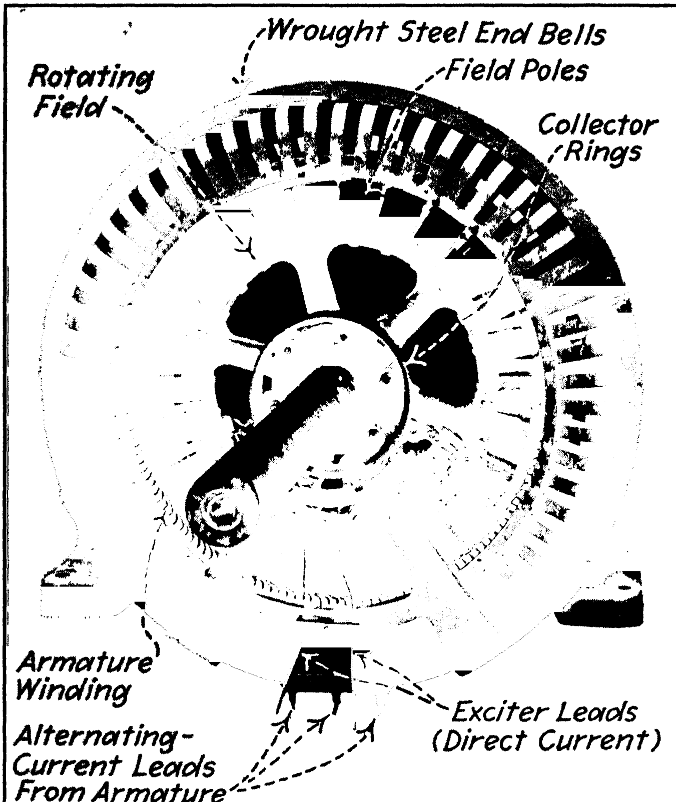

FIG. 389.—A revolving-field engine-type alternator. (This is a 26-pole Westinghouse machine.)

to rear in the top side of the loop) in the direction shown by the full-line arrows (hand rule, Art. 427). When M has been rotated $\frac{1}{4}$ revolution to a horizontal position, then at that instant, no flux will be cutting L and the e.m.f induced in it will be zero. At the instant after M has been rotated another $\frac{1}{4}$ revolution, its S-pole flux will be cutting the bottom side of the loop and the N flux the top side. Now the e.m.f. and the current it impels in the loop will be (from rear to front in the top side) as shown by the dotted

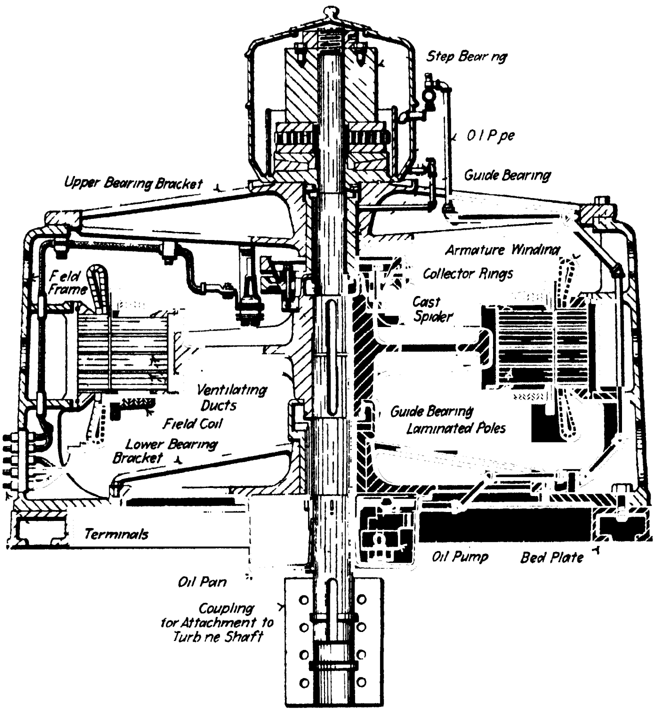

FIG. 390—Showing construction of a vertical, revolving-field, alternating-current generator for water-wheel drive.

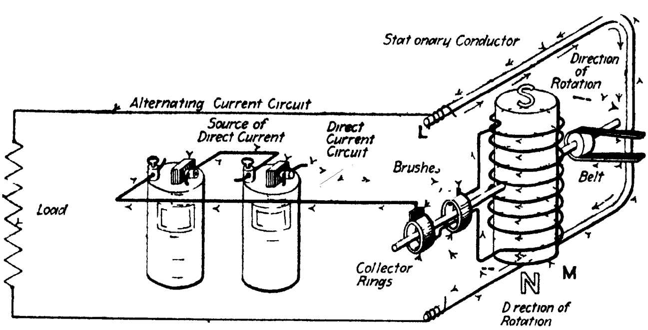

FIG 391—Alternating e m.f. being induced in a stationary conductor by a rotating field

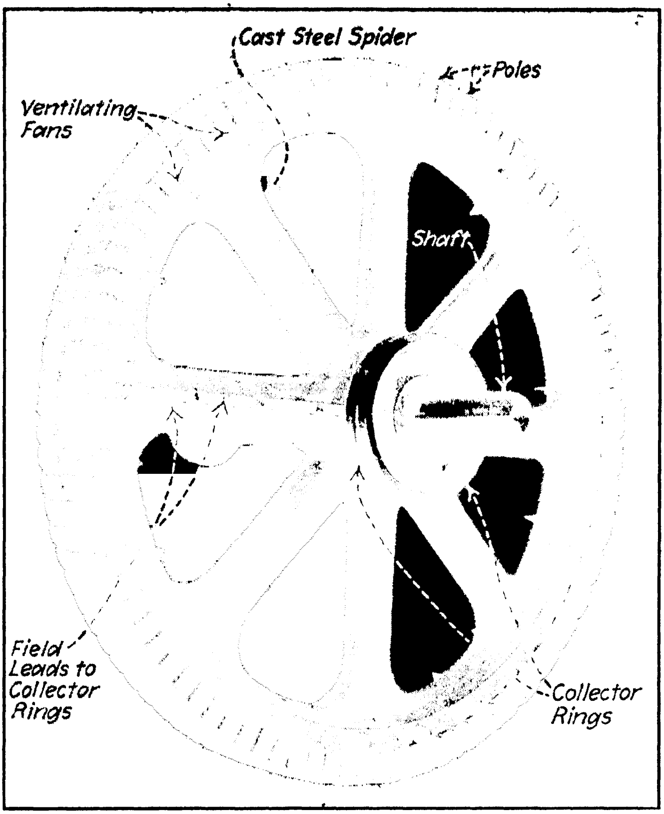

FIG. 392.—A 100-pole revolving-field for a slow-speed alternator. (Westinghouse Electric Company.)

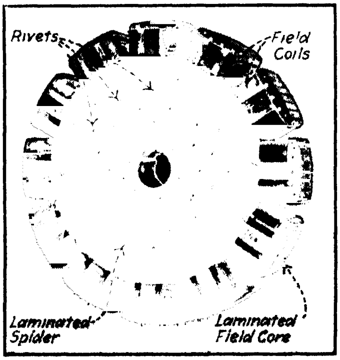

FIG. 393.—A 12-pole revolving-field structure for a high-speed alternator. (Westinghouse Electric Company.)

arrows. Thus an alternating e.m.f. has been induced in the loop as the field has been rotated. This e m f. has impelled an alternating current. So long as the rotation of the magnet M is continued, the alternating current will flow in the external circuit.

706. The Construction of Revolving Fields for Alternators is indicated in Figs. 392 and 393. Where the machine is to rotate at high speed the spider is usually built up of steel-plate laminations. Cast-steel or cast-iron spiders are used for machines for

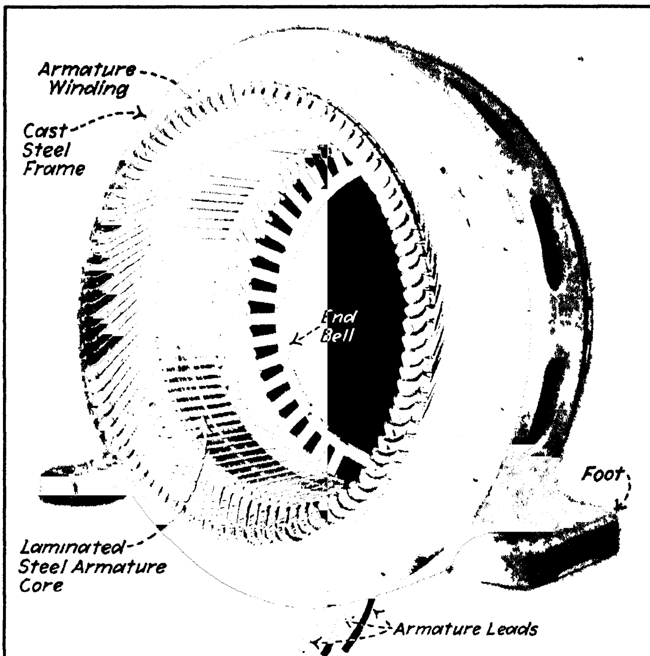

FIG. 394.—Stationary armature structure for an alternator.

the lower speeds. The field coils are wound on laminated steel cores and are frequently insulated with non-combustible materials so that they can be overloaded without damage.

707. Armature Construction and Windings of Revolving-field Alternators are shown in Figs. 394 and 395. The armature coils are, instead of being wound on projecting poles as shown in Fig. 396, arranged in slots along in the armature core as illustrated (Figs. 394 and 395). This arrangement is the most economical and effective. The armature cores are built up from

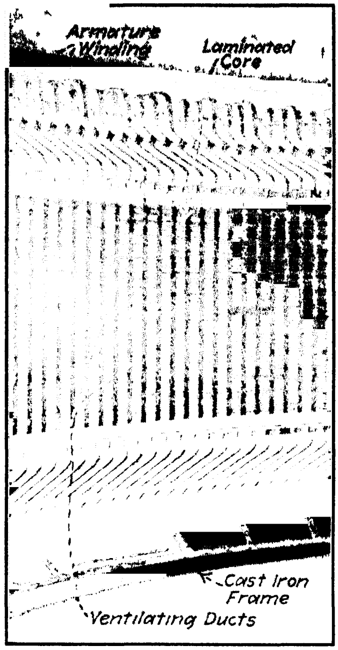

FIG 395.—Portion of the armature structure of a revolving-field alternator. (Westinghouse Electric Company.)

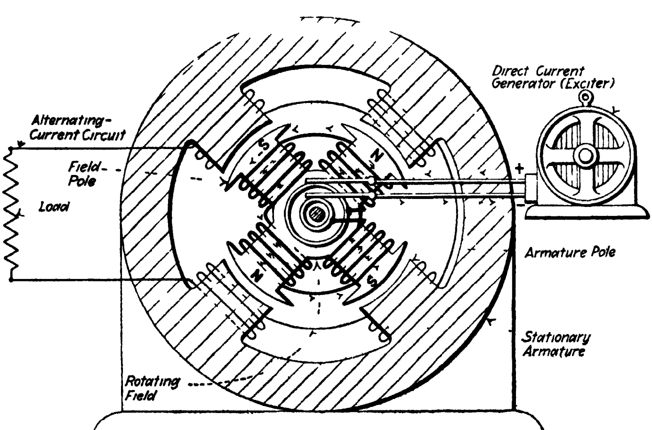

FIG 396.—Diagram of a four-pole revolving-field generator, single phase (Strictly, the pole faces should be no wider than the armature coils to insure correct generation of alternating e.m.f.)

laminations or punchings in somewhat the same way as are the armature cores (Art. 569) of direct-current generators. However, the punchings for alternating-current stationary armatures form rings similar to that of Fig. 305, except that the winding

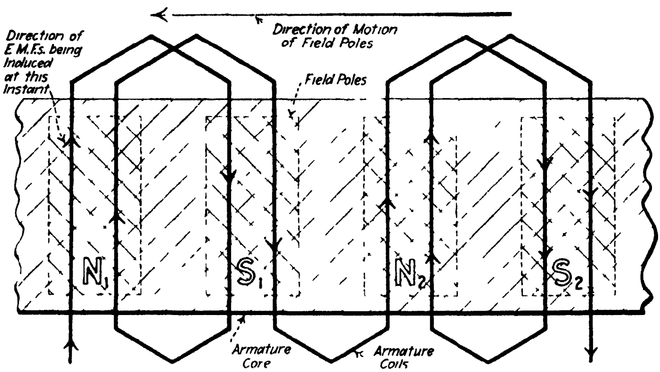

FIG. 397.—Development of an armature winding for a four-pole alternator. (This may be considered as showing either a rotating-field winding or a rotating armature winding. For a rotating-armature winding the direction of motion of the field would then be the opposite of the direction shown to produce the e.m.f. directions indicated.)

slots are on the inside of the ring for the alternating-current machines. Form-wound coils (Art. 575) are used in modern machines. Fig. 397 gives an idea as to how the armature coils are arranged.

SECTION 43

HOW ALTERNATORS DEVELOP E.M.FS.

708. The Explanation of the Induction of E.m.f. on the Basis of Cutting Flux, by an Alternating-current Generator in Which the Inductor Coils Are in Slots Distributed Along in the Surface of the Armature will now be considered in connection with Figs. 397*A* to 397*L*. The essential principles are the same as those already recited in Arts. 514 and 702. However, just what happens is not always readily apparent to the student; hence this detailed treatment. Although in the following it is assumed that the armature is stationary and that the field poles are

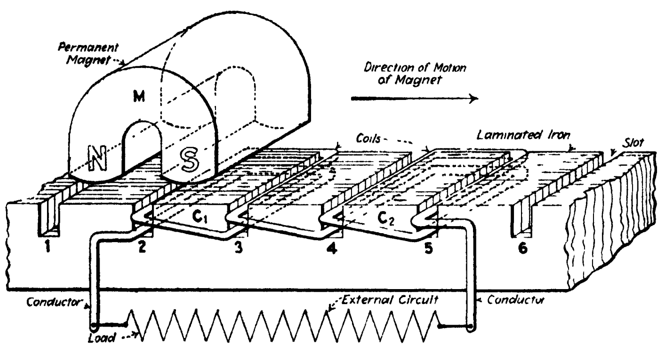

FIG. 397*A*.—Diagram to illustrate theory of the induction of an alternating e.m.f. by an alternator.

moved, a consideration of the situation will render it apparent that practically the same explanation applies also in the case of moving-armature (revolving-armature) machines.

EXPLANATION.—Consider the apparatus of Fig. 397*A*. A permanent horseshoe magnet (Art. 80), *M*, provides a magnetic field as shown in Fig. 397*B*. The magnet may be moved longitudinally, just above but not touching the stationary laminated iron slab, which has conducting coils mounted in but insulated from slots in its surface. The iron slab may be considered as the equivalent of the "developed" (Art. 599) armature core of an alternator. The arrangement of the armature coils represents a typical one for alternators but is not the only way in which the coils may be ar-

ranged. However, the general principle to be described holds for all practicable arrangements of coils.

Now if the permanent magnet, *M*,—the equivalent of field magnets of a revolving-field alternator—is pushed in either direction along over the coils, as shown in Figs 397*A* to 397*R*, a sine-wave alternating e m f will be in-

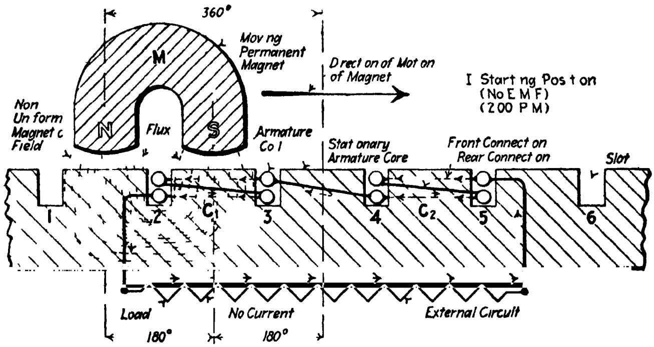

FIG 397*B* —Diagrammatic representation of apparatus for illustrating induction of an alternating e m f by an alternator (No current at this instant)

duced in the coil inductors, as will be shown Obviously, an alternating current will then be impelled in the external circuit connected to the inductors

An alternating e m f will be induced in the inductors if the permanent magnet is started from the right end and swept toward the left or if it is

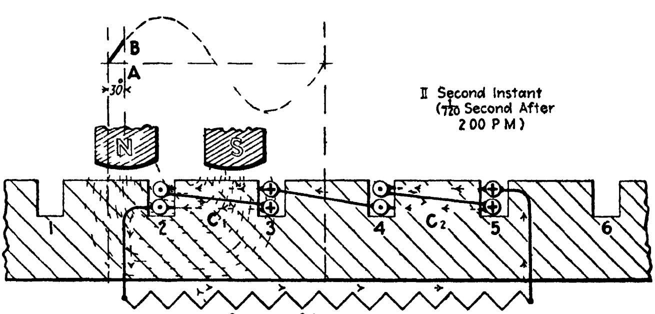

FIG 397*C* —Field magnet has been shifted through 30 electrical degrees (Current has started to flow in the positive direction in the external circuit)

started at the left end and swept toward the right In the following explanation it will be assumed that the magnet is moved from left to right

Assume that the magnet, *M*, is started from the position shown in Figs 397*A* and 397*B* At the instant just after its uniform movement toward the right is commenced, the flux of the magnet will not cut any armature-coil inductors because at this instant its flux will obviously tend to complete its magnetic circuit through the iron as shown in Fig 397*B*

Hence, at this instant, no e.m.f. will be induced in the armature inductors. However, as the movement of the magnet is continued, its flux does start to cut the inductors. The flux from the magnet poles, as they are moved toward the right, starts cutting the inductors gradually because the poles should be so proportioned as described hereinafter (Art. 709) that the flux is weak toward the sides of the poles and strongest toward their centers. The instant that the flux commences cutting the inductors then an e.m.f. will be induced in the inductors. The e.m.f. induced at any instant will, as has been shown, be proportional to the rate of cutting at that instant (Art. 438). The directions of the e.m.fs. will be as shown in Fig. 397*C* for the conditions of that instant. Verify the e.m.f. directions, using the right-hand rule for determining the direction of an induced e.m.f. of Art. 427.

At the instant just after the magnet has been moved through 30 electrical degrees (Fig. 382) a flux of considerable strength is cutting the inductors in slots 2 and 3. Therefore, an e.m.f. proportional to this rate of cutting is induced at this instant in these inductors. The e.m.f. induced at this

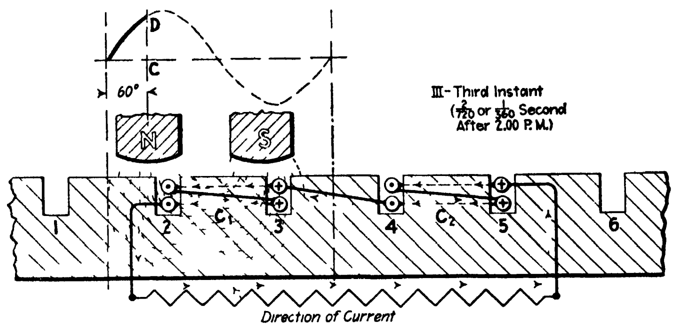

FIG. 397*D*.—Field magnet has been shifted through 60 electrical degrees. (A greater current is at this instant flowing in the external circuit in the positive direction.)

instant should be proportional to the vertical distance *AB* (Fig. 397*C*) if the device (shown in Fig. 397*A*) has been correctly designed to induce a sine-wave e.m.f.

Note that the e.m.fs. induced in the inductors in slot 2 are in series with and in the same electrical direction as those induced in the inductors in slot 3. The total e.m.f.—the sum of that induced in the inductors in slot 2 and that induced in the inductors of slot 3—impels a current through the external circuit. This current is also forced through the inductors in slots 4 and 5, since they are in series with the other inductors, although there is no e.m.f. being induced in these inductors (in 4 and 5) at this (Fig. 397*C*) instant. Note that the current in the external circuit, at this instant, is from left to right—which we will designate as the positive direction (Art. 521). The lengths of the dotted arrows, which indicate current direction, is proportional to the current intensity at this instant.

As the movement of the permanent magnet, *M*, toward the right is continued; the flux will continue to cut the inductors in slots 2 and 3. The

e.m.f. will become greater from instant to instant because, while the magnet is being shifted toward the right, the flux cutting the inductors becomes stronger as the centers of the magnet poles approach positions directly over the inductors in the slots. At the instant of Fig. 397*D* the magnet has been shifted through 60 electrical degrees. The e.m.f. being induced at

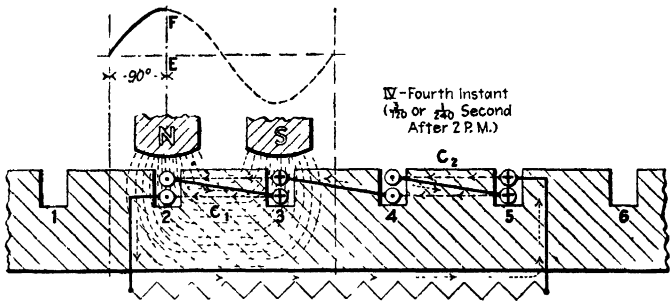

FIG. 397*E*.—Field magnet has been shifted through 90 electrical degrees. (Current has, at this instant, attained its maximum value in the external circuit in the positive direction.)

this instant is proportional to the vertical distance *CD*. The e.m.f. is still in the positive direction (hand rule, Art. 427) and the current forced by it through the circuit is in the positive direction, as represented in direction and in intensity by the dotted arrows.

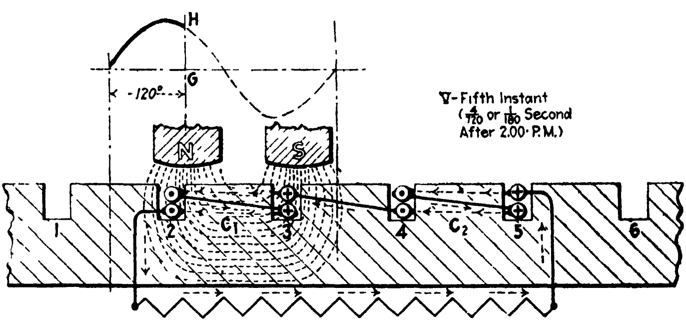

FIG. 397*F*.—Field magnet has been shifted through 120 electrical degrees. (Current is now decreasing but is still flowing in the positive direction in the external circuit.)

As the uniform movement of the magnet, *M*, toward the right is continued, the position of Fig. 397*E* is soon attained. At this instant the inductors are being cut by the field that is directly under the centers of the magnets where the field is strongest. Hence, at this instant, a maximum e.m.f., proportional to *EF*, is being induced. The current at this instant is therefore a maximum—and it is in the positive direction.

Now as the transition of the magnet, *M*, is continued, the inductors are

being cut by a weaker flux, hence the e.m.f. now decreases from instant to instant. At the instant of Fig. 397*F*, the e.m.f. is proportional to only *GH*. At the instant of Fig. 397*G* the e.m.f. is proportional to *IJ*. The current is obviously also now decreasing from instant to instant—but is still in the positive direction.

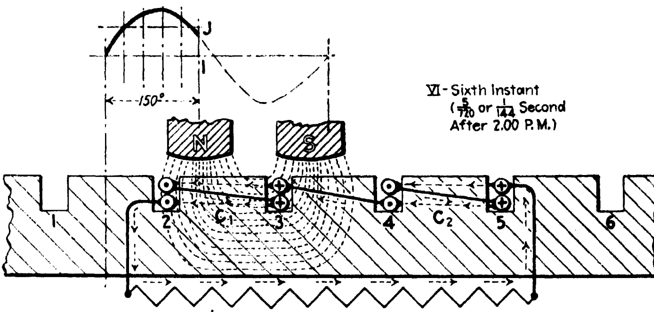

FIG. 397*G*.—Field magnet has been shifted through 150 electrical degrees. (Current at this instant is still decreasing but is still flowing in the positive direction.)

When the position of the instant of Fig. 397*H* is reached, no flux is cutting the inductors. Hence no e.m.f., as indicated at *K*, is being induced and there is no current in the circuit. Note that the e.m.f. and the current have gradually increased to a maximum and decreased gradually to zero, in the positive direction *JK*, as shown by the sine curve *XBDFHJK*, while the mag-

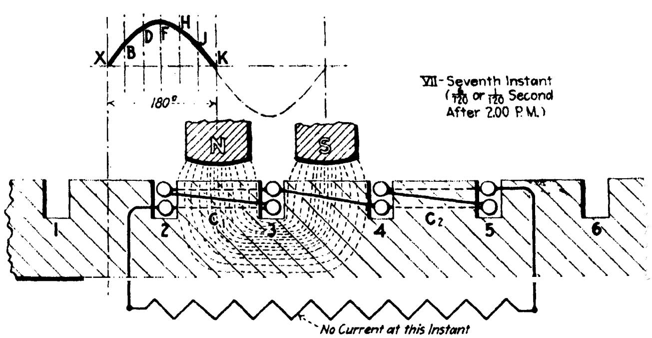

FIG. 397*H*.—Field magnet has been shifted through 180 electrical degrees (No e.m.f. is being induced at this instant, hence there is no current in the external circuit.)

net, *M*, has been shifted through 180 electrical degrees. The e.m.f. has passed through one alternation.

As the movement is continued, the flux will now cut the inductors of slots 3 and 4. But note (Fig. 397*I*) that the flux is now cutting the inductors in a direction opposite to that obtaining previously. It follows that the

e.m.f. and current will be in a direction opposite to that obtaining during the first 180 degrees. At the instant of Fig. 397*I*, after the magnet has been

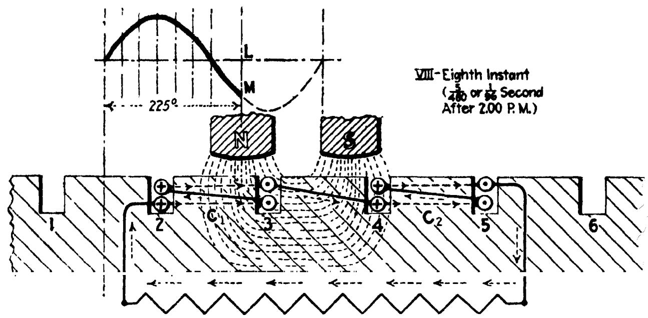

FIG. 397*I*.—Field magnet has been shifted through 225 electrical degrees. (E.m.f. and current are now in the negative direction in the external circuit and the current is, at this instant, increasing in this direction.)

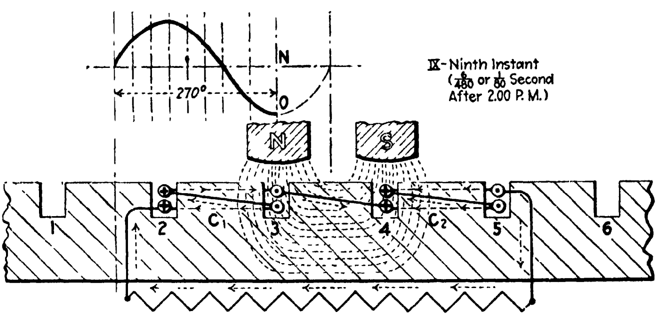

FIG. 397*J*.—Field magnet has been shifted through 270 electrical degrees

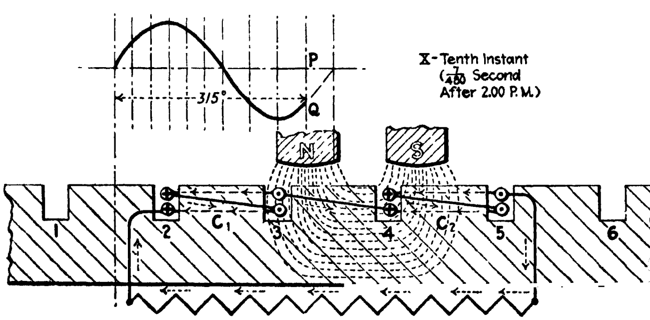

FIG. 397*K*.—Field magnet has been shifted through 315 electrical degrees.

shifted through 225 degrees, the e.m.f. has now increased in the negative direction so as to be proportional to LM. The dotted arrows represent the

direction and intensity of the current at this instant. (The pictures are now showing conditions at 45-degree intervals instead of at 30-degree intervals as during the first 180 degrees.)

Fig. 397*J* pictures conditions at the 270-degree instant. The e.m.f. and current have increased to a maximum in the negative direction. The e.m.f. at this instant is proportional to *NO*. Now as movement is continued, the e.m.f. induced will decrease gradually, as shown by the sine curve until, at the instant shown in Fig. 397*K* it will have decreased to a value proportional to *PQ*. E.m.f. and current are in the negative direction.

Fig. 397*L* diagrams conditions after the magnet has been shifted through 360 degrees. The e.m.f. and current have again decreased to zero at this instant. A cycle has been completed.

If it is assumed that the permanent magnet, *M*, was moved at such a uniform rate as to induce a 60-cycle e.m.f., it would require just $\frac{1}{60}$ sec. to move it through the 360 electrical degrees constituting a cycle. If the mag-

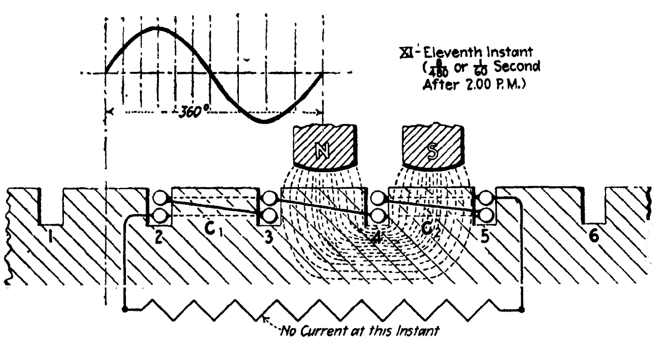

FIG. 397*L*.—Field magnet has been shifted through 360 electrical degrees.

net were started (Fig. 397*B*) at just 2 o'clock, Fig. 397*L* would (for a frequency of 60 cycles) represent conditions $\frac{1}{60}$ sec. after 2 o'clock. The intervening illustrations illustrate correspondingly the conditions at the different time instants as noted.

Consideration of the facts will then render it obvious that if a number of armature coils are arranged in an armature core and a suitable number of electromagnetic poles are arranged (as shown in Fig. 397*M*) so that the poles may be swept past the coils, alternating e.m.fs. will be induced in these coils when the poles are shifted. By forming the developed armature into a ring and by mounting the poles on a wheel (Fig. 392) to rotate within the armature ring, an arrangement whereby the poles may be continuously swept past the coils is provided. Such an arrangement would constitute a revolving-field alternator (Fig. 389).

709. The Distribution of Flux under a Field-pole Face is shown in Fig. 397*N*. Note that in this illustration the flux is much

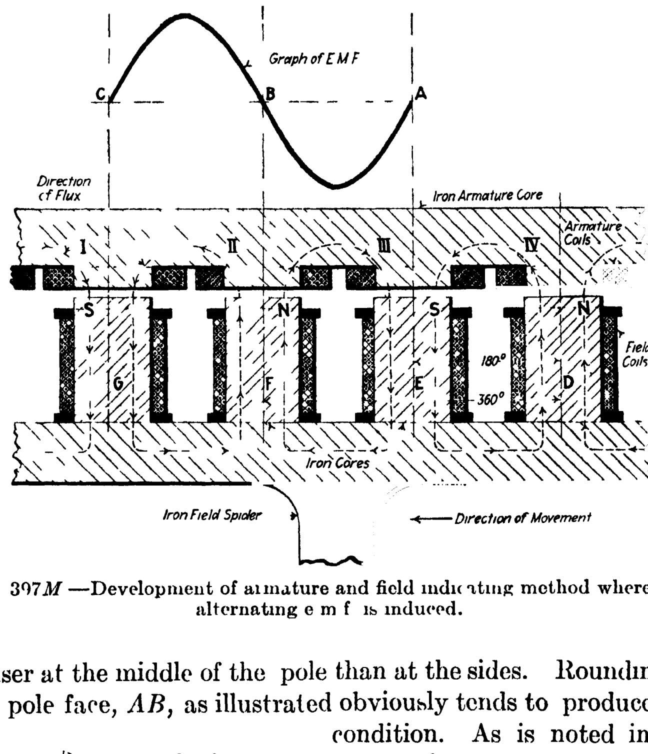

FIG. 397*M*.—Development of armature and field indicating method whereby an alternating e.m.f. is induced.

denser at the middle of the pole than at the sides. Rounding off the pole face, *AB*, as illustrated obviously tends to produce this condition. As is noted in the explanation under Art. 708, such a non-uniformity of field, if consistently planned, assists toward the production of a sine-wave form e.m.f. If the flux in the air gap were uniform all along under the pole face, the e.m.f. induced in an inductor, when the pole were swept past the inductor, would tend to have a flat-topped form somewhat like that of Fig. 73,*I* rather than a sine-wave form. In fact the wave form of the e.m.f. induced by an alternator can be controlled to a considerable extent by varying

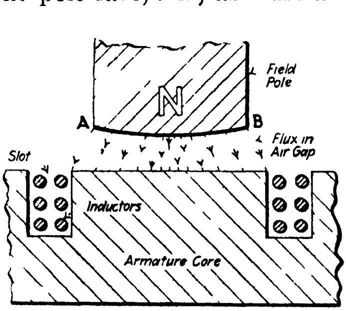

FIG. 397*N*.—Showing how the density of the flux under a field pole may vary.

the contour of the pole face. The wave form is also dependent on the arrangement of the inductors in the armature slots. *Distributing* the inductors along in the armature surface (rather than *concentrating* them at a few locations) tends toward the production of a sine-wave e.m.f.

710. The Simplest Explanation of the Induction of an Alternating E.m.f. in a Coil, Past Which a Magnet Pole Is Moved, follows from the rule of Art. 428. While a moving field pole is approaching a position just opposite a stationary coil, it is obvious that then the flux enclosed by the coil is increasing. Hence, an e.m.f. is then induced in one (clockwise or counterclockwise) direction in the coil Fig. 397*O*. When the pole is moving away from the coil, the flux is decreasing which effects

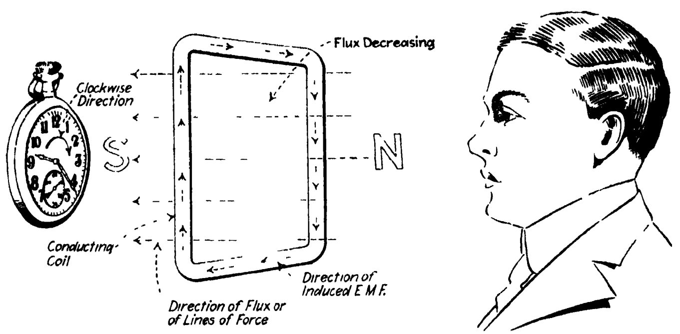

FIG. 397*O*.—Illustrating the rule: "Look through the coil in the direction of the lines of force, then a decrease in the flux enclosed by the coil induces an e.m.f. in a clockwise direction."

the induction of an e.m.f. in the opposite direction. Thus, as the pole approaches, moves past and leaves the coil, an alternating e.m.f. is induced in the coil.

711. Where a High E.m.f. Is to Be Induced in the Armature of a Revolving-field Alternator a Number of Inductor Turns Must Be Connected in Series.—That this should be true will be evident from a consideration of the statements of Art. 564 where it is explained that in direct-current generators a number of turns are connected in series where voltages sufficiently high to be used in practice are to be induced. It is obvious, therefore, that the *rate of cutting flux* (Art. 438), hence the induced voltage, obtainable with an arrangement like that of Fig. 397*L* would be very small. For the induction of usable voltages a number of armature turns of the generator must be connected in series as shown

diagrammatically in Fig. 397*P*, which illustrates a principle rather than actual construction. The greater the number of turns in series in the armature winding, the greater the rotating-field flux, and the greater the rotational speed of the field, the greater will be the rate of cutting. Hence, the effective induced e.m.f. will be increased if any one of these factors is increased and it will be decreased if any one of them is decreased.

712. An Alternation Is Completed When a Field Pole Has Been Moved a Distance Equal to the Distance Between Center Lines of Adjacent Field Poles, that is the distance *AB*, Fig. 397*M*. A study of illustrations Figs. 397*A* to 397*L* and the text accom-

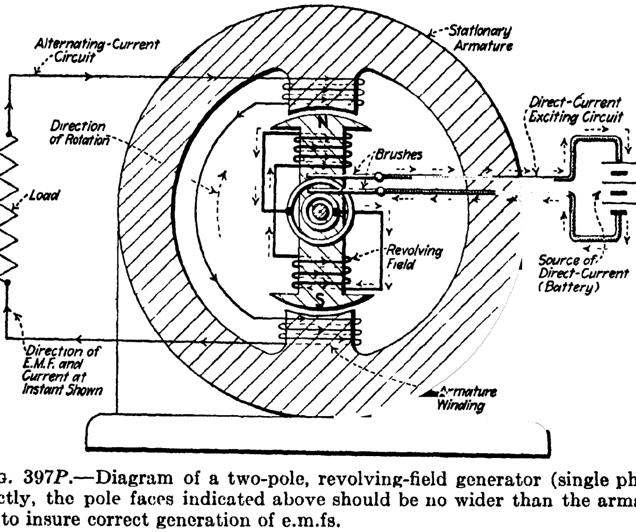

FIG. 397*P*.—Diagram of a two-pole, revolving-field generator (single phase). (Strictly, the pole faces indicated above should be no wider than the armature coils to insure correct generation of e.m.fs.

panying them will demonstrate the truth of this statement. Also, note that *a cycle is completed when any field pole has been moved a distance* (*AC*, Fig. 397*M*) *equal to the distance between the center lines of alternate field poles*—poles of like polarity.

713. To Compute the Frequency of the E.m.f. Induced by Any Alternator (the following rule and formulas are true for revolving-armature or revolving-field and polyphase or single-phase alternators): *Multiply the number of pairs of poles by the speed of the armature in revolutions per second.* That is:

$$\text{(173)} \qquad cycles = \frac{poles}{2} \times \frac{r.p.m.}{60} = \frac{poles \times r.p.m.}{120}$$

Wherein *cycles* = frequency of the machine, in cycles per second. *R.p.m.* = revolutions per minute of the rotor—either the field or the armature—of the machine. *Poles* = number of poles of the machine, either field poles or armature poles. It also follows that:

$$(174) \qquad \textit{alternations per second} = \frac{\textit{poles} \times \textit{r.p.m.}}{60}$$

$$(174a) \qquad \textit{alternations per minute} = \textit{poles} \times \textit{r.p.m.}$$

Alternations are ordinarily expressed in alternations per *minute* and unless definitely specified in alternations per second, the term "alternations" is taken to designate *alternations per minute.*

EXPLANATION.—Obviously, from the explanation of Art. 712, each time the loop of an elementary bipolar alternator, Fig. 255, is rotated one complete revolution, the e.m.f. values pass through 1 cycle (Art. 681). If the speed is 1 revolution per sec., the frequency will evidently be 1 cycle per sec. Thus, with 1 revolution per sec. and one *pair of poles,* the frequency is 1 cycle per sec. Plainly, if the speed is increased, the frequency will be increased in proportion. In the formula (173) above, "*r.p.m.*" is divided by 60 to get it into cycles per second. "*Poles*" is divided by 2 to obtain number of "pairs of poles"

A consideration of the facts will demonstrate that the e.m.f. induced by the two-pole "revolving-field alternator" of Fig. 397*P*, which has one pair of poles, will pass through a cycle per revolution; with 2 revolutions per sec., the frequency will be 2 cycles per sec., etc. Now with the four-pole "revolving-field alternator" of Fig. 396, which has two *pairs* of poles, there will, manifestly, be twice as many cycles per revolution. Hence it is apparent that the frequency is also directly proportional to the number of poles.

See the table in the author's AMERICAN ELECTRICIANS' HANDBOOK showing frequencies for different numbers of poles and speeds and *vice versa.*

EXAMPLE.—If a two-pole alternator (Fig. 397*P*) is driven at 3,600 r.p.m., what will be the frequency? SOLUTION.—Substitute in the formula (173): $\textit{cycles} = \textit{poles} \times \textit{r.p.m.} \div 120 = 2 \times 3{,}600 \div 120 = 7{,}200 \div 120 = 60$ *cycles.*

EXAMPLE.—If the four-pole alternator of Fig. 396 is to operate at 60 cycles, at what speed must it be driven? SOLUTION.—$\textit{r.p.m.} = 120 \times \textit{cycles} \div \textit{poles} = 120 \times 60 \div 4 = 7{,}200 \div 4 = 1{,}800$ r.p.m.

NOTE.—It is apparent from the above examples that, for a frequency of 60 cycles, if "7,200" be divided by the *number of poles,* the *r.p.m.* will be the result. If "7,200" is divided by the speed the number of poles will be the result For 25 cycles, the constant "3,000" may be similarly used. Thus, for a 60-cycle circuit, the 12-pole field of Fig. 393, would have to be rotated at: 7,200 ÷ 12 = 600 r.p.m. For a 60-cycle circuit, the 100-pole field of Fig. 392 would have to turn at: 7,200 ÷ 100 = 72 r.p.m.

714. Commercial Alternating-current Generators Develop E.m.fs. of Approximate Sine-wave Form.—Usually they are almost but not quite true sine waves. This matter has already been discussed in Art. 525. It follows that the alternating currents of practice also have approximate sine-wave forms. While the wave form of the e.m.f. induced in a *single* loop rotated at a *uniform speed* in a *uniform magnetic field* (as in Fig. 255) is a true sine curve, this wave form is not inherently characteristic of alternators. Practical alternators do not have single-loop inductors nor are their magnetic fields uniform. In fact, skillful designing is necessary in commercial machines to obtain the

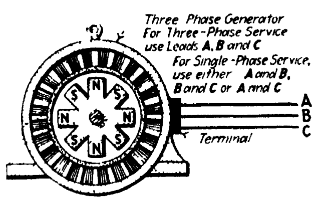

FIG. 397*Q*.—Use of a three-phase generator for single-phase service.

approximate sine-wave forms which these machines produce. The advantages of the sine-wave form are enumerated in Art. 525.

715. Single-phase Generators are seldom manufactured now, *first*, because there is little demand for them and, *secondly*, because polyphase generators are more economical in the utilization of their constructive materials. Where it is necessary that a single-phase current be generated, it is the practice to use two "phase wires" of a three-phase machine (Fig. 397*Q*). The single-phase capacity of a three-phase machine is 70 per cent. of its three-phase capacity. That is, a 100-kva., three-phase machine will deliver 70 kva., single phase.

SECTION 44

ALTERNATING-CURRENT VECTORS AND VECTOR DIAGRAMS

716. The Words "Lagging" or "Lag" as they are used in alternating-current parlance really refer to time. If an alternating current (or an alternating voltage) "lags" behind another alternating current (or voltage) the current (or voltage) which lags, attains its maximum value in each alternation a certain interval of time later than does the current (or voltage) behind which it lags. Since time may be expressed in electrical degrees (Art. 528), it may be stated that a current—or voltage—lags behind another current—or voltage—by a certain number of degrees. With a frequency of 25 cycles (Fig. 68) 1 degree is equivalent to: $1/25$ *sec.* $\div$ 360 *deg.* $= 1/25 \times 1/360 = 1/9{,}000$ sec. With a frequency of 60 cycles (Fig. 70) 1 degree is equivalent to: $1/60$ *sec.* $\div$ 360 *deg.* $= 1/60 \times 1/360 = 1/21{,}600$ *sec.*

EXAMPLE.—In Fig. 426 the current lags behind the impressed e.m.f. by 90 degrees. That is, in each alternation, the current attains its maximum value (at I_M), 90 degrees *later* than the impressed e.m.f. attains its maximum value (at A). If the current shown were a 25-cycle current, it would then with conditions as illustrated reach, in each alternation, its maximum value: $90 \times 1/9{,}000 = 1/100$ sec. later than the e.m.f. attained its maximum value. With a 60-cycle current and the conditions as shown, the current would lag behind the voltage by $90 \times 1/21{,}600$ *sec.* $= 1/240$ *sec.*

717. The Words "Leading" or "Lead" as used in reference to alternating-current phenomena also refer to time. When a current (or voltage) "leads" another current (or voltage), it attains its maximum value in each alternation a certain interval of time before the voltage (or current) which it leads attains its maximum value. The amount of lead in any case may be expressed either in degrees or in seconds as may lag, as described above.

EXAMPLE.—In Fig. 451 the current, A, leads the impressed e.m.f., B, by 90 degrees, as shown for instance at XY. In Fig. 426 the impressed e.m.f. leads the current by 90 degrees as shown for instance at MN, and the impressed e.m.f. leads the counter e.m.f. by 180 degrees as shown for instance at PQ.

718. There Are Two Graphic Methods of Representing Alternating E.m.f. or Current Values.—by Vectors and by Sine Curves.—That this statement is true might be inferred from preceding statements (Art. 526). Each of the two methods has, as will be demonstrated, its applications. Furthermore, it is sometimes desirable to use the two in conjunction with one another.

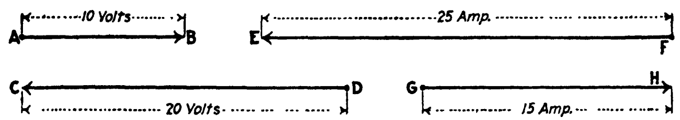

FIG. 397*R*.—Vectors representing voltages and currents.

719. A Vector (review Art. 526) is a line representing by its length the intensity of some certain alternating e.m.f. (voltage) or alternating current.

EXAMPLE.—Refer to Fig. 397*R*. The line *AB* represents, to scale, an e.m.f. of 10 volts in one, say the positive, direction (Art. 521). Then *DC* represents, to the same scale, an e.m.f. of 20 volts in the opposite or negative direction. Similarly, *FE* may represent to scale a current of 25 amp. in the negative direction while *GH* represents, to the same scale, a current of 15 amp. in the positive direction. Vectors may, obviously, be drawn to

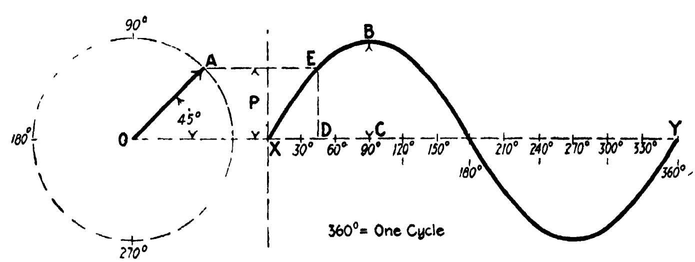

FIG. 398.—Explanation of vector conventions.

any convenient scale. Where two vectors are to be compared, both should be drawn to the same scale.

720. Sometimes Vectors Are Assumed to Rotate as in Figs. 270 and 398. When a vector rotates (Art. 526), the distance, at any given instant, of the end of the vector describing the circle, from the zero (usually taken as the horizontal) axis is proportional to the instantaneous value of the e.m.f. or current developed at the corresponding instant.

EXAMPLES.—The line OA in Fig. 398 is a rotating vector. The length of line ED is proportional to the instantaneous e.m.f. induced at the 45-degree instant.

The distance around the circle of Fig. 398 is measured in degrees, and each degree represents a certain interval of time. (The actual length of the time interval in seconds can in any case, where necessary or desirable—be determined by considering the frequency as shown in Figs. 68 and 70.) Instantaneous values—for example, AB—determined as above described, may be plotted into a sine curve as shown in Fig. 398.

NOTE.*—As ordinarily used a vector represents the magnitude of the *maximum* value of the alternating wave current or voltage but it may, if drawn to proper scale, represent the *effective* value. Fig. 398 shows two methods of representing an alternating wave. The wave may be plotted on a time axis, XY, in which case the amplitude, BC, of the wave is the maximum value, and instantaneous values are ordinates to the curve at the particular instants considered, for example DE is the instantaneous value at the 45-degree instant.

Another way of plotting the wave is by a vector OA, the length of which is proportional to the maximum value of the wave and which is supposed to rotate counterclockwise at such a constant speed that it makes one complete revolution in the time required for the wave to complete one cycle. The instantaneous values of the wave are always equal to the projection of the vector at the given instant on a vertical line. Thus, it is shown in Fig. 398 that the ordinate ED of the curve at the 45-degree instant is equal, by construction, to the projection P of the vector OA.

In the wave figure the difference of time phase is represented by the distance on the axis XY between the zero points of the waves. It is evident from the above that a rotating vector figure always represents *maximum* values. It is possible to represent *effective* values to a different scale by non-rotating vectors because they are equal to maximum values divided by the constant, 1.41. Instantaneous values can not be shown directly in a vector figure or phase diagram, but must be obtained by taking projections.

NOTE.—The lines of different kinds—open, black, dotted, dashed, etc.,—used in a number of the following illustrations, have—insofar as they are different kinds of lines—no special significance. The reason why the different kinds of lines have been used for representing different electrical quantities and values is merely so that the line representing one quantity may be readily distinguished from another line in the same diagram representing a different quantity. Plain black, thin lines could have been used for all of the lines in all of the diagrams—but the diagrams might then have been much more difficult to follow.

721. Reference Lines in Phase or Vector Diagrams are often drawn before any of the vectors are plotted so that when the vectors are plotted they can be located in some definite position:

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Thus Fig. 399,*I*, shows how such reference lines, *AA* and *BB*, are usually drawn at right angles to one another. Obviously these reference lines may be of an indefinite—any convenient—length. At *II* is shown a vector $O'E'$ plotted in the 45-degree phase position—assuming counterclockwise rotation. Usually vectors, in phase diagrams, representing current values or values

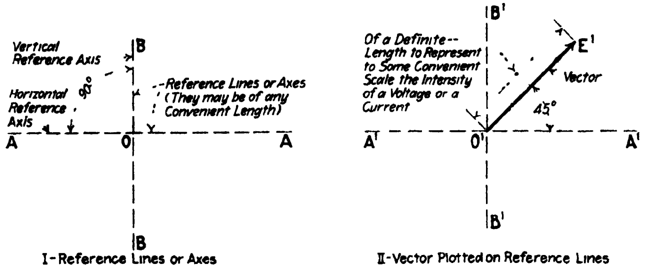


FIG. 399.—Showing reference lines or axes for vector diagrams and their application

in phase with current values are plotted along the horizontal axis or reference line, as shown in Fig. 400. Also vectors representing the energy e.m.f.—to be discussed later, Art. 744—are usually plotted on the horizontal axis. The vector *OA* in Fig. 400 represents to scale, as shown, an alternating current of 80 amp., which may be either a maximum or an effective value.

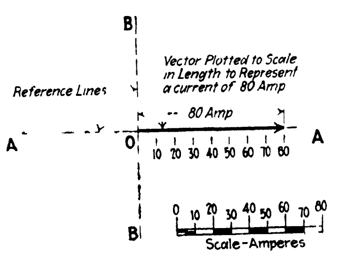


FIG. 400.—A vector representing a current of 80 amp. plotted on reference lines.

722. Phase Diagrams and Vector Diagrams are shown in the accompanying illustrations. A phase diagram* represents the magnitude and relative phase positions of electric pressures (e.m.fs. or voltages) or currents. A vector diagram represents the magnitude of the pressures or currents but *may* not represent their phase relations. A vector diagram is always a polygon and is often a triangle. Both phase and vector diagrams are composed of vectors which may (except where the vector is rotating) represent either effective or maximum values. As suggested above, rotating vectors can represent only maximum values. Since an effective value is always equal to 0.707 times its maxi-

*JACKSON'S ALTERNATING CURRENTS

mum value (Art. 697), the same vector (unless it is a rotating vector), if the proper scale is assumed, may represent either an effective or a maximum value.

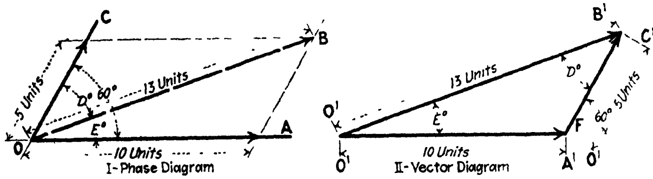


FIG. 401.—Phase and vector diagrams.

EXAMPLES OF PHASE AND VECTOR DIAGRAMS. THE PRINCIPLE OF COMPOSITION.—First consider the phase diagram of Fig. 401,*I*. If an alter-

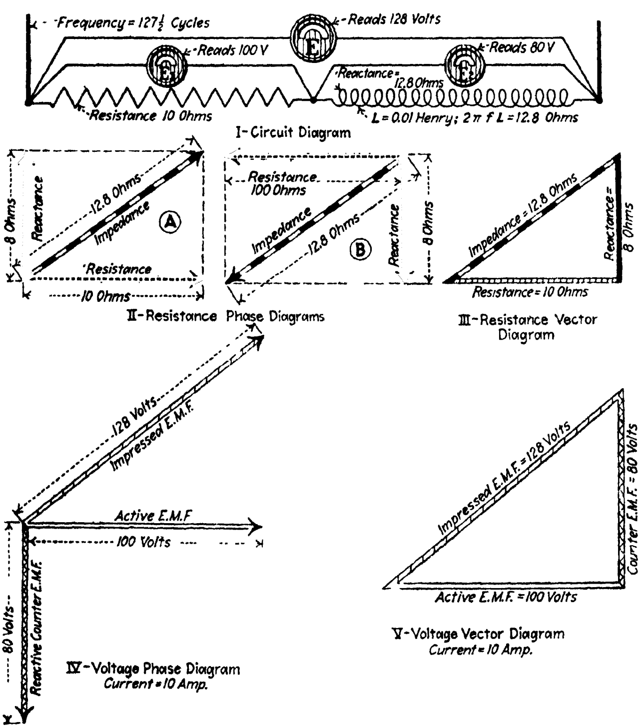


FIG. 402.—Examples of phase and vector diagrams.

nating e.m.f. of 5 volts be represented by *OC*, 5 units long, and another e.m.f. of 10 volts, of the same frequency but which lags 60 degrees behind the first, be represented by *OA*, 60 degrees away from *OC* and 10 units long,

then OB (which scales 13 units long) will represent their resultant or sum. In other words, the length of the line OB to scale is equal to the sum of the e.m.fs. of 5 volts and 10 volts which differ in phase by 60 degrees. The line OB is obtained by completing a parallelogram, having OC and OA for two of its sides then the diagonal OB is the sum (vector sum) of OC and OA. The resultant e.m.f., 13 volts, will lag D degrees behind OC and will lead OA by E degrees.

The vector diagram, Fig. 401,*II*, which is equivalent to the phase diagram of *I*, is obtained by drawing lines equal and parallel to those representing the same values in the phase diagram. Note that in the vector diagram, the phase relations (angles) between $O'B'$ and $D'A'$ and between $O'B'$ and $O'C'$ are correct and correspond to those in *I*. But to obtain the true angle (60 *degrees*) between $O'C'$ and $O'A'$, the angle F (in *II*) must be taken. The

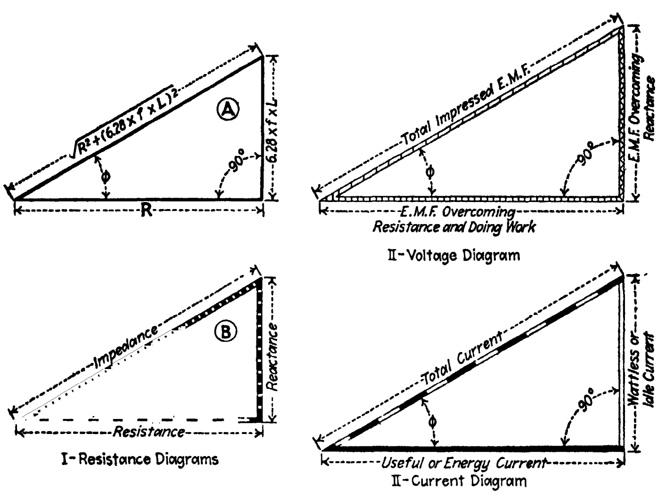


FIG. 403.—Typical vector diagrams.

lines OC, $O'C'$, OA and $O'A'$ are called *components; OB* and $O'B'$ are respectively their *resultants.*

In Fig. 402 are shown the phase and vector diagrams of the resistance and voltage conditions obtaining in the circuit and with the frequency indicated.* These diagrams and their significance will be further explained in following matter. Some other typical vector diagrams, which will also be further treated subsequently, are shown in Fig. 403. The symbol ϕ is a Greek letter pronounced phi; it is often used as a designation or "shorthand" expression to indicate a lag angle. While the diagrams of Figs. 401, 402 and 403 show voltage values, similar diagrams may be plotted to represent current values.

723. The Resolution of an Alternating E.m.f. into Components is a process which is the inverse of combining two alternating

* JACKSON'S ALTERNATING CURRENTS.

e.m.fs. to ascertain their resultant e.m.f. or their sum as just described in the example under Art. 722. Any alternating e.m.f. of a given frequency may be "resolved" into two e.m.fs. of the same frequency which two e.m.fs. will differ in phase from one another by a selected phase angle. That is, the sum of the two e.m.fs. thus obtained will equal the resultant—or their combined —e.m.f. The process involved is similar to that used in mechanics for resolving a force into two components and may be best explained by a specific example:

EXAMPLE.—Resolve an e.m.f. of 280 volts into two component e.m.fs. which differ in phase by 60 degrees so that one of these component e.m.fs. will be 180 volts. It is understood that the component and resultant e.m.fs. are of the same frequency. SOLUTION.—Draw to scale (Fig. 404) a line, OM, 180 units long at an angle of 60 degrees from a base line OP of indefinite length. This line OM represents the component e.m.f. of 180 volts. Draw a line MT, of indefinite length, parallel to OP. With a compass, with O as a center and a radius 280 units long strike a part of a circle QR. From the point, V, where the circle cuts MT, draw VN parallel to MO. Then NO, which scales about 150 units—or 150 volts long—is the other component. Hence, an e.m.f. of 280 volts may be resolved into two component e.m.fs., differing in phase by 60 degrees, of 180 volts and 150 volts respectively. That is, the resultant sum of two e.m.fs. of 180 volts and 150 volts which differ in phase by 60 degrees is 280 volts. While the diagram of Fig. 404 shows voltage values, similar diagrams may be plotted to represent current values.

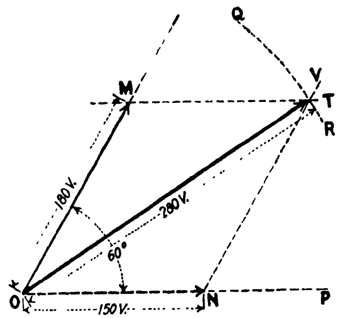


FIG. 404.—Illustrating the resolution of an alternating e.m.f., OV, into two components.

724. An Alternating E.m.f. May Be Resolved Into an Infinite Number of Pairs of Components because it is obvious that for every different value of lag angle between components, the components will be of different values. Furthermore, for every change in value of one of the components there will be a corresponding change in value of the other component. This situation is illustrated in the following example, wherein e.m.f. values are resolved into components; current values may by similar methods, be resolved into components.

EXAMPLE.—Consider Fig. 405 wherein the e.m.f. OE of 150 volts, shown at I, is resolved into component e.m.fs. of different values and differing in

phase by different angles. In the phase diagram of *II* the e.m.f. of 150 volts, *OE*, is resolved into two components, differing in phase by 90 degrees, of 130 volts and 75 volts respectively; at *III* is shown the corresponding vector diagram. In *IV* the e.m f. of 150 volts is resolved into two component e.m.fs. of 100 volts and 110 volts which also differ in phase by 90 degrees; the equivalent vector diagram is plotted at *V*.

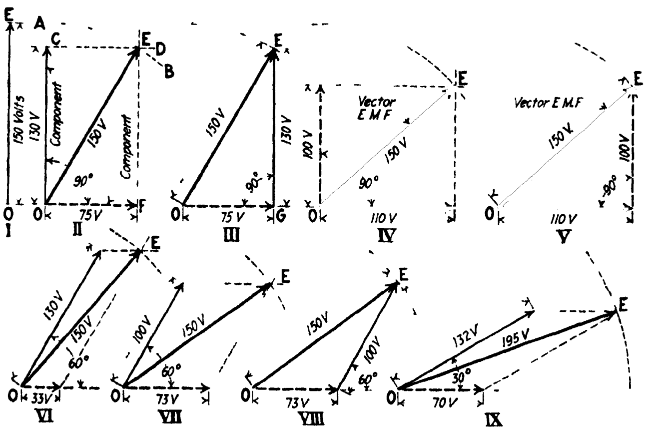


FIG. 405.—Examples of the resolution of an e m.f., *OE*, into components.

In *VI*, the 150-volt e.m f. has been resolved into two components of 130 volts and 33 volts, which differ in phase by 60 degrees, and in *VII* the same 150-volt pressure is again resolved into two e.m.fs. differing in phase by 60 degrees, but in this example they are 100 volts and 73 volts; the equivalent

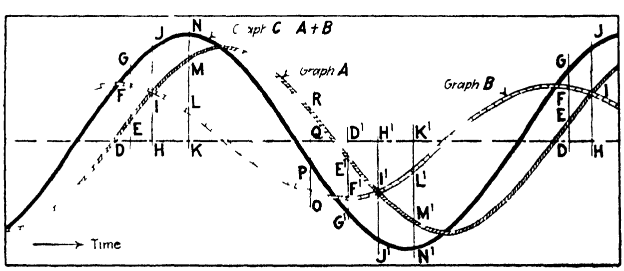


FIG. 406.—Illustrating the addition of sine graphs.

vector diagram is shown in *VIII*. In *IX* an e.m.f. of 195 volts has been resolved into two components of 132 volts and 70 volts, respectively, which differ in phase by 30 degrees.

725. Sine Graphs Can Be Added and Subtracted.—See Fig. 406. If two sine graphs (*A* and *B*, Fig. 406) each of the same

frequency, but differing in phase are added together, a third sine graph (C) of the same frequency can be plotted which will represent their sum.

EXAMPLE.—To obtain graph C (Fig. 406) the heights of corresponding points in A and B are added. Thus, $DG = DE + DF$, $HJ = HI + HI$, $KN = KL + KM$ and $PQ = QO - RQ$.

Similarly, sine graphs of the same frequency can be subtracted from one another, by subtracting heights of points in one curve from the corresponding heights in another.

EXAMPLE.—Curve $C = A + B$, $B = C - A$ and $A = C - B$ all from Fig. 406.

SECTION 45

THE ADDITION AND SUBTRACTION OF ALTERNATING-CURRENT VALUES

726. The Addition and Subtraction of Alternating E.m.f. and Current Values can, where the values are represented by vectors, be readily effected by utilizing the principles of the composition (Art. 722) or resolution (Art. 723) which were briefly described hereinbefore. Where the e.m.f. or current values are represented by sine curves the addition or subtraction of these values can be made by utilizing the principles suggested for the addition and subtraction of sine curves previously outlined in Art. 725. Where the e.m.fs.—or currents—are to be added to or subtracted from other e.m.fs.—or currents—with which they are in phase (Art. 685) the operation is, as will be shown, very simple in that it involves merely arithmetical addition or subtraction. But if the e.m.fs.—or currents— to be added are not in phase, then the process is somewhat more complicated. Where the values are not in phase, the addition or subtraction can most readily be made graphically by plotting suitable phase or vector diagrams. Also, it is frequently desirable, for the purposes of explanation, to plot sine curves in conjunction with the vector or phase diagrams, which curves may represent the same values as those presented by the vectors.

727. Four Different Cases in the Addition and Subtraction of E.m.f. Values Will Be Considered in some detail in Arts. 728 to 731. While these cases which will be recited refer specifically to e.m.f. values precisely the same principles and process would be employed in adding or subtracting alternating-current values. The sources of e.m.f. in each of these four examples will (Fig. 407 and following illustrations) be represented by generators which are supposed to be rotating uniformly in a counterclockwise direction and developing the effective e.m.fs. indicated. All of the generators are of the same frequency. While generators are shown as sources in these examples, any other sources of alternating current e.m.fs. could be used instead. Where the

generators are in phase, Figs. 407 and 413, they are maintained in phase by the link-belt connection. Where the generators are not in phase, Figs. 410 and 416, the phase difference between them is maintained by the link-belt-drive connection.

728. The Method of Adding Two E.m.fs. Which Are in Phase can be best demonstrated by the consideration of a specific example. In this and in the examples which follow it is assumed

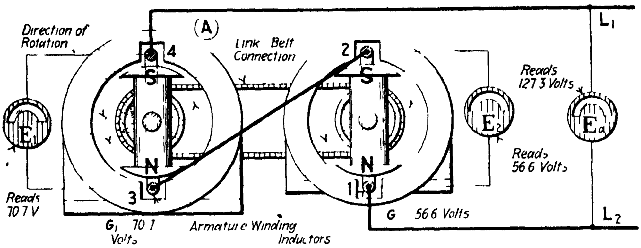


FIG. 407.—Two single-phase, alternating-current generators of the same frequency which are in phase connected in series so that their e.m.fs. augment or add up.

that the conditions outlined in the preceding paragraph are satisfied:

EXAMPLE.—What (Fig. 407) is the sum of an e m.f. of 70 7 volts and one of 56.6 volts which are in phase? In other words, what e m f. will the two generators of Fig. 407, which are in phase and connected in series so that their e.m fs augment, impress upon the line at L_1 and L_2? SOLUTION.—Since these two e.m.fs. are in phase they may be added directly to obtain their sum, or resultant. That is: 70.7 *volts* + 56 6 *volts* = 127 3 *volts*, which is the e.m.f. impressed across L_1 and L_2 as shown in Fig. 407. A graphic statement of this addition, wherein vectors are used to represent the e.m.f. values is shown in Fig. 408. From this it is evident that the length of the vector E_1, which is equivalent to 70.7 volts + the length of the vector E_2, which is equivalent to 56.6 volts = the length of the vector E_a or 127.3 volts. Hence, E_a is the vector representing the magnitude, 127.3 volts, of the e.m.f. impressed across L_1 and L_2 in Fig. 407.

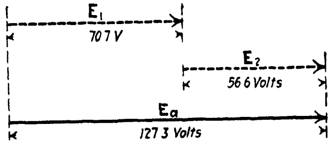


FIG. 408—Diagram showing the vector addition of two alternating e.m.fs. of 80 and 100 volts of the same frequency and in phase.

Now refer to Fig. 409. At *I* the vector diagrams are repeated but in this illustration the lengths of the vectors are, as always must be the case with rotating vectors, proportional to the maximum values of the e.m.fs. (Maximum values may, Art. 697, be obtained by multiplying the corresponding effective values by 1.41.) Since all three of these e.m.fs., E_{1m}, E_{2m} and E_{3m} are in phase they lie directly over one another in the vector

diagram. The graphs of *II* were plotted following the method hereinbefore described in Art. 720. It is evident that the sine curve E_{am} is equal to the

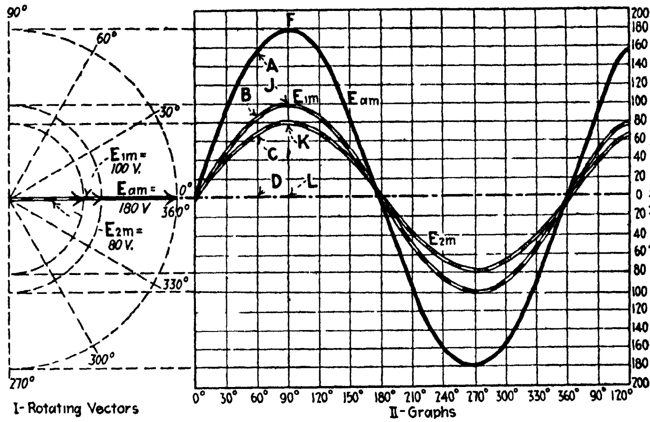


FIG. 409.—Rotating vectors and graphs showing addition of e.m.fs. of 80 and100 volts of the same frequency and in phase.

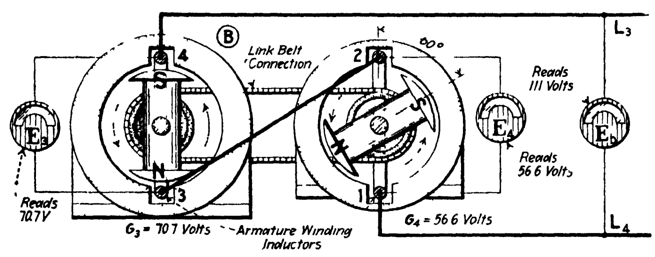


FIG. 410.—Two single-phase, alternating-current generators which are not "in phase" connected in series so that their e.m.fs. augment or "add up."

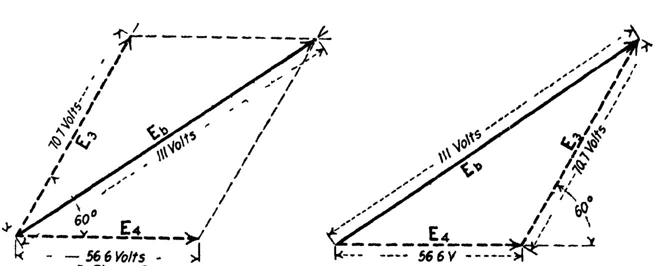


FIG. 411.—Diagrams showing the vector addition of two alternating e.m.fs. of 80 and 100 volts of the same frequency but differing in phase by 60 degrees.

sum (Art. 725) of the sine curves E_{1m} and E_{2m}. For example the distances $AD = DB + CD$ and $FL = JL + KL$.

729. The Method of Adding Two E.m.fs. Which Are Not in Phase will be explained by considering Figs. 410, 411 and 412 and by means of a specific example.

EXAMPLE.—What is the sum of an e.m.f. of 70.7 volts and one of 56.6 volts which differ in phase by 60 degrees? In other words, what e.m.f. will

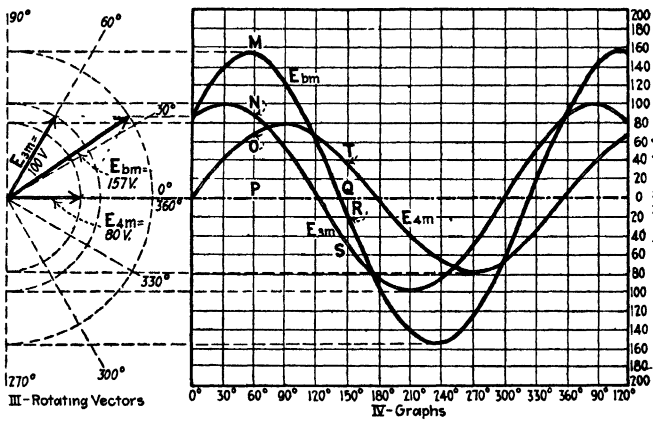


FIG. 412.—Rotating vectors and graphs showing addition of e.m.fs. of 80 and 100 volts of the same frequency but differing in phase by 60 degrees.

the generators G_3 and G_4 of Fig. 410, which are connected in series so that their e.m.fs. augment, impress across the line L_3 and L_4? SOLUTION.—The phase diagram of Fig. 411,*I* shows a graphic solution of this problem, the vector sum of E_3 and E_4 being E_b which is equivalent to scale to 111 volts

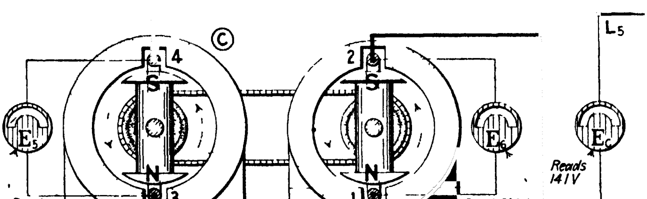


FIG. 413.—Two single-phase, alternating-current generators of the same frequency, which are in phase, connected in series so that their e.m.fs. oppose or buck.

(effective). At *II* in Fig. 411 is shown the equivalent vector diagram which, of course, gives the same results as does the phase diagram. In Fig. 412, at *III* is shown the rotating phase diagram, the vectors being proportional in length to the maximum values and the corresponding quantities indicated on the phase diagram of Fig. 411,*I*. By using the method hereinbefore described (Art. 722) the graph or sine-curve representation of *IV* (Fig. 412), corresponding to the vector representation of *III* (Fig. 412), can be plotted.

By inspection it will be evident that the sine curve E_{5m}. is equal to the sum (Art. 725) of the sine curves E_{3m}. and E_{4m}. For example, $MP = NP + OP$ and $QR = QS - QT$.

730. The Method of Subtracting an E.m.f. from Another E.m.f. With Which It Is in Phase is illustrated in Figs. 413, 414 and 415 and is demonstrated in the following example:

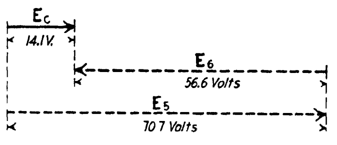


FIG. 414.—Diagram showing the vector subtraction of two alternating e.m.fs. of 56.6 and 70.7 volts of the same frequency and in phase.

EXAMPLE.—The two generators of Fig. 413, which are developing, respectively, pressures of 70.7 and 56.6 volts, are connected in series, but, as shown, the connections are such that the e.m.fs. oppose or "buck" one another. What is the resultant e.m.f.? That is, what pressure is impressed across L_5 and L_6 on the line? SOLUTION.—The vector solution of the problem is shown in Fig. 414, from which it is obvious that if the vector E_6, which is proportional in length to 56.6 volts, is subtracted from the vector E_5, which is proportional to 70.7 volts, the difference is represented by the vector E_c, proportional in length to 14.1. (These vectors may thus be subtracted from one another because the e.m.fs. which they represent are all in phase.) Therefore, the e.m.f. across L_5 and L_6 is, as suggested in

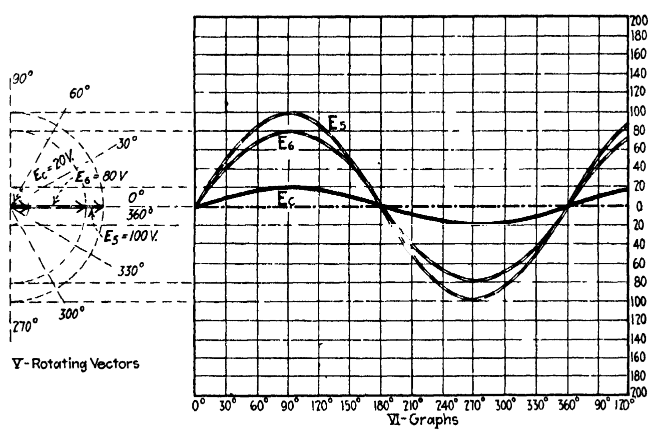


FIG. 415.—Rotating vectors and graphs showing subtraction of e.m.f. of 80 from 100 volts, both e.m.fs. are of the same frequency and are in phase.

Fig. 413, 14.1 volts. In Fig. 415, *V* the rotating vectors (which are, of course, proportional in length to the maximum value of the e.m.fs. involved) are shown. From this rotating vector diagram the sine curves shown in Fig. 415, *VI* were plotted. A consideration of these curves will indicate that the curve E_c, which has a maximum value of 20 volts, is equal to the difference between the curves, E_5, maximum value 100 volts and E_6, maximum value 80 volts.

731. The Method of Obtaining the Difference of Two E.m.fs. Which Are Not in Phase is outlined in Figs. 416, 417 and 418. The conditions specified in Art. 727 also applies for this problem which is solved below.

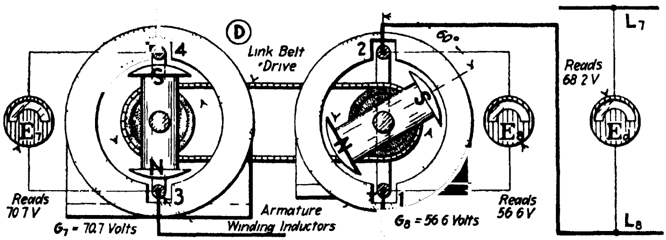


FIG 416 —Two single-phase, alternating-current generators of the same frequency, which are not in phase connected in series so that their e m fs oppose each other

EXAMPLE —What will be the e m f impressed across the line L_7 and L_8 by the two generators of Fig 416 The generators are in series but they are so connected, as shown, that the e m fs which they are developing oppose one another and furthermore, differ in phase by 60 degrees In other words, what is the resultant e m f produced by generators G_7 and G_8 com-

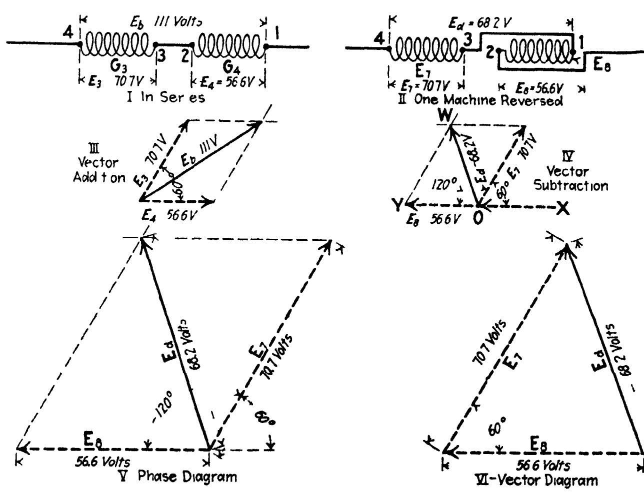


FIG 417 —Illustrating vector addition and subtraction.

bined, under the conditions outlined? SOLUTION.—First consider the vector solution of the problem. When two e m fs are in series so that they "boost" one another (Fig 410 and 417,*I*), they can be added vectorially as shown in the diagram at Fig 417,*III* But if one of the e m fs is reversed, for example E_8 in *II*, then the e m f values must be subtracted from one another,

instead of added, to obtain their resultant. How this subtraction is effected vectorially is diagrammed in *IV*. Since the e m f. E_8 is reversed in direction in *II* (Fig 417) the vector respresenting it in the phase diagram must also be reversed in direction, as shown at *OX*. In other words, to perform the vector subtraction it must be plotted in the position *OY*. Then the vector *OW* is proportional in length to the difference between the e.m fs E_7 and E_8, reversed which differ in phase by 60 degrees. That is, the actual difference in phase between E_7 and E_8 is 120 degrees, as shown in *IV**. *V* (Fig 417) is merely an enlarged phase diagram of *IV* and *VI* shows the corresponding vector diagram. Hence, it is apparent from these diagrams, if the length of E_d is measured, that the effective e m f, E_d across L_7 and L_8, is 68 2 volts.

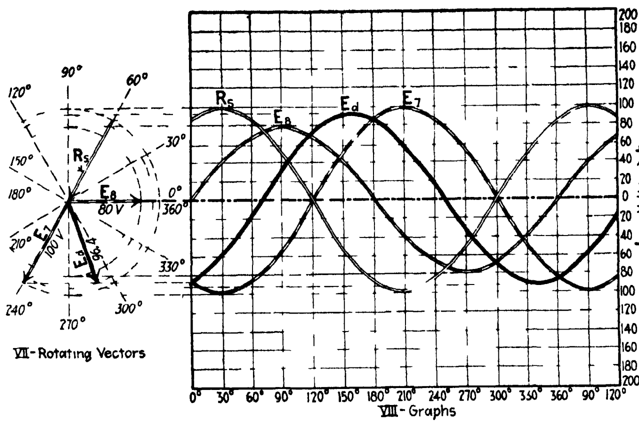


FIG 418 —Rotating vectors and graphs showing subtraction of e m fs of 80 from 100 volts which differ in phase by 60 deg Both e m fs are of the same frequency.

In Fig. 418 are shown the rotating vectors—maximum values—corresponding to the effective-value vectors of Fig. 417,*V*. In the graph of *VIII* is shown the sine curves which may be plotted from these rotating vectors. Curve R_s represents the variation of the e.m.f. *OX* (Fig. 417,*IV*). It will be noted that *curve* E_d = *curve* $E>$ + *curve* E_8 and it is also evident that *curve* E_d = *curve* E_8 − *curve* R_s. This demonstrates the truth of the fact that the vector R_s, leading E_8 by 60 degrees, is equivalent to the vector E_7 which lags behind E_8 by 120 degrees, as diagrammed in Fig. 417,*IV* and in Fig. 418,*VII*.

*If the e m fs in the two coils differ in phase by a certain number of degrees and one of these coils is reversed, then, after the reversal the difference in phase between the two e m fs will be 180 degrees minus the former phase difference in degrees That is, in Fig 417,*I* the difference in phase between the e m fs of the two coils is, because of the conditions of the problem, 60 degrees Then, if one of the coils is reversed, as shown at *II*, the difference in phase between the e m fs of the two coils is 180 degrees − 60 degrees = 120 degrees

SECTION 46

EFFECTS OF RESISTANCE AND INDUCTANCE IN ALTERNATING-CURRENT CIRCUITS

732. The Effect of Resistance in Alternating-current Circuits which contain resistance only, that is, which contain no inductance (Art. 471) or permittance (capacitance, Art. 753), is the same as its effect in direct-current circuits. Resistance in alternating-current circuits limits the current therein and causes a loss or drop in voltage just as it does in direct-current circuits (Art. 124). With a given alternating e.m.f. impressed on a circuit, the current in it will vary inversely as its resistance. In an alternating-current circuit containing resistance only, the current is in phase with the impressed e.m.f. It follows that Ohm's law (Art. 134) is, without qualification, true for alternating-current circuits containing resistance only. Thus:

(175) $$I_E = \frac{E_E}{R} \qquad \text{(amp.)}$$

and

(176) $$E_E = I_E \times R \qquad \text{(volts)}$$

or

(177) $$R = \frac{E_E}{I_E} \qquad \text{(ohms)}$$

Wherein I_E = effective current in the circuit, in amperes. E_E = the effective e.m.f. or voltage impressed across the circuit, in volts. R = the resistance of the circuit, in ohms.

Example.—What current will flow in the alternating-current circuit of Fig. 419 which contains resistance only. The impressed pressure (effective voltage) as indicated by the voltmeter is 110 volts, the resistance of each lead is 0.70 ohm and the resistance of the incandescent lamp is 200 ohms. The frequency of the current may be anything. Solution.—Substitute in the Ohm's law formula (175): $I_E = E_E \div R = 110 \div (0.7 + 0.7 + 200) = 110 \div 201.4 = 0.55$ *amp.*

Example.—What voltage must the alternating-current generator (it may develop any frequency) of Fig. 420,*I* develop to impel a current of 2 amp. (effective) in the circuit shown which, it is assumed, has a total resistance of 50 ohms.? Solution.—Substitute in Equation (176) above.

Thus: $E_E = I_E \times R = 2 \times 50 = 100$ *volts*. Hence to circulate the current of 2 amp. (effective) the generator should produce 100 volts (effective) across its terminals A and B.

733. There Is a Loss of Power When an Alternating Current Flows Through Resistance just as there is when a direct current flows through resistance. The power loss, in watts, in any conductor, conveying either an alternating or a direct current, always equals *the square of the current in amperes (effective current*

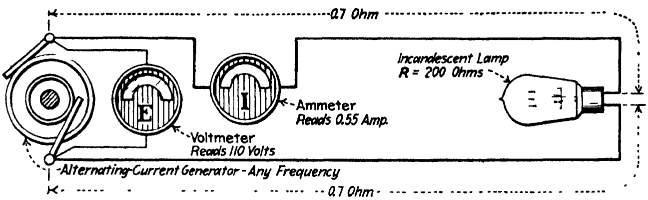


FIG. 419.—Illustrating the effect of resistance in an alternating-current circuit.

in alternating-current circuits) multiplied by the resistance of the conductor in ohms. This rule is perfectly general and applies to alternating currents of all ordinary voltages and frequencies even

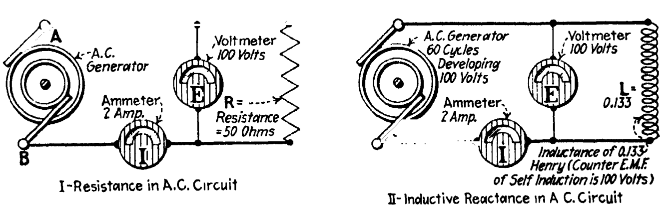


FIG. 420.—Examples of resistance and inductive reactance in alternating-current circuits.

if they contain inductance (Art. 471) or permittance (capacitance, Art. 753). There is no power loss in inductance or permittance. The electrical power thus lost reappears as heat and raises the temperature of the conductor. Expressed as a formula:

$$P = I^2{}_E \times R \qquad \text{(watts)} \tag{178}$$

or

$$I_E = \sqrt{\frac{P}{R}} \qquad \text{(amp.)} \tag{179}$$

or

$$R = \frac{P}{I^2{}_E} \qquad \text{(ohms)} \tag{180}$$

Wherein P = power lost in the conductor, in watts. I_E = effective current, in amperes, in the conductor. R = resistance of the conductor, in ohms. Note that the above formulas are identical with those of Art. 167 for direct current, except that in this alternating-current formula, the current, I_E, is the effective current.

EXAMPLE.—What is the power loss in the incandescent lamp in Fig. 419? SOLUTION.—The lamp, as shown in the picture, has a resistance of 200 ohms and the current through it is 0.55 amp. Then, substituting in equation (178): $P = I^2_E \times R = (0.55 \times 0.55) \times 200 = 60.5$ watts. In other words, the lamp is a 60½-watt lamp.

EXAMPLE.—What is the power loss in the inductive winding of Fig. 421 when an alternating current of 5 amp. flows in it? The winding has a resistance of 10 ohms. SOLUTION.—Substitute in equation (179): $P = I^2_E R = (5 \times 5) \times 10 = 250$ watts. With a current of 5 amp., the power loss in the winding would be 250 watts regardless of the frequency and regardless of the fact that it is an "inductive" winding.

734. In Alternating-current Circuits, Containing Resistance Only, the Current Is in Phase with the Impressed Voltage as shown in Fig. 422,*III*. That is, in such circuits, the current attains its crest or maximum values at the same instants as those at which the impressed e.m.f. (which circulates the current in the circuit) reaches its crest or maximum values; see definition of "In Phase" in Art. 685.

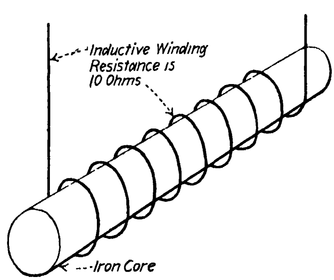


FIG. 421.—Illustrating power loss in an inductive winding.

EXAMPLE.—Fig. 422,*I* shows the diagram of an alternating-current circuit containing resistance only. The alternating e.m.f. (effective) impressed on the circuit, is as indicated by the voltmeter, 100 volts. The resistance of the circuit is 2 ohms. Then, since the circuit contains resistance only, by Ohm's law the effective alternating current in the circuit is: $I_E = E_E \div R = 100 \div 2 = 50$ *amp.* The phase diagram given at *II* shows the phase relation of this e.m.f. and the current which it propagates. They are in phase with each other.

The light line CB, the impressed - e.m.f. vector, is made 141 units long to represent a maximum of 141 volts, which is the maximum corresponding to an effective e.m.f. of 100 volts (Art. 697). The heavy black line CA is made 70.5 units long to represent 70.5 amp. which is the maximum current corresponding to an effective current of 50 amp. (Art. 697). At *III* the graphs of the e.m.f. and current are plotted and they indicate that the e.m.f. and current are in phase as defined in Art. 685.

735. The Effects of Inductance in Alternating-current Circuits are, in general, much more noticeable than those which it produces in direct-current circuits. Why this is the case will be explained. In 471 and following articles (which should be carefully reviewed) the phenomenon of inductance was discussed. It was there shown that whenever the current, in a circuit having inductance, changes in intensity, a *counter e.m.f. of self-induction* (Art. 479) is induced in the circuit. With direct currents, the effects of inductance are most noticeable: (*a*) just after a circuit is closed and a current is forced to circulate in it;

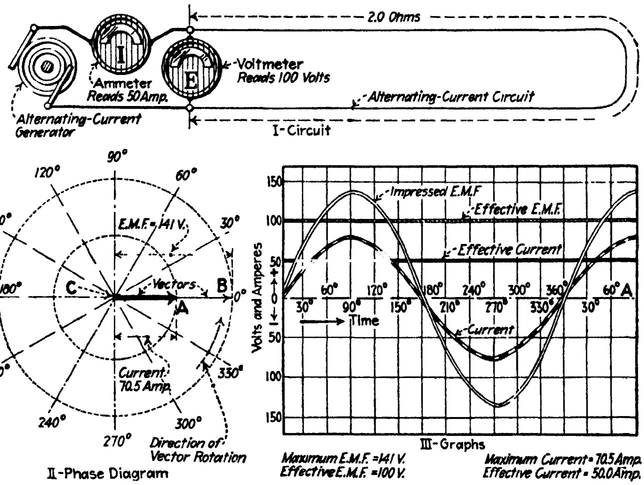


FIG. 422.—Example of an alternating-current circuit containing resistance (2 ohms) only.

(*b*) when a circuit having a current flowing in it is opened; or (*c*) when, for any other reason, there is a change in the intensity of the current in the circuit. Inductive effects are, then, due to *changes* in current intensity.

NOTE.—Now alternating currents reverse in direction at regular intervals (Art. 113) and are constantly changing in intensity, see Fig. 422. Therefore, since in these circuits the current is constantly changing in intensity, the result is that a counter e.m.f. of self-induction is, if they contain inductance, being *generated constantly*—all the time—in them when current is flowing. The counter e.m.f. of self-induction is always in such a direction as to oppose any change in the intensity of the then existing current (Art. 435). This counter e.m.f. of self-induction varies in intensity just as does any alternating e.m.f. but it is not in phase with the e.m.f. or current

which produces it, as will be shown. Its variation with the time follows a sine-wave form—if the current which induces it has a sine-wave form.

In order to predetermine the intensity of the current—the number of amperes—which will flow in an actual alternating-current circuit containing inductance, it is first necessary to—either directly or indirectly—compute the value of the counter e.m.f. of self-induction of the circuit and then subtract it (as will be outlined in Art. 746) from the impressed e.m.f. which it opposes. The difference, the e.m.f. which really circulates the current in the circuit, may be called the *available* or *energy e.m.f.* (The terms "net," "active," "impelling" or "actuating" e.m.f. are all used synonymously with available or energy e.m.f. However, in the text which follows, *available* or *energy e.m.f.* will be used to designate the difference between the impressed e.m.f. and the counter e.m.f. of self-induction.) In practice this subtraction is most readily effected by introducing a new quantity which is called *reactance* which will be treated in following Art. 747. Note that in ideal circuits which contain only inductance or permittance or both but which contain no resistance, there is no available or energy e.m.f. But, for reasons which will be discussed elsewhere, an applied alternating e.m.f. would, nevertheless, propagate an alternating current in such a circuit—if such a circuit could exist. However, it is obvious that there can to be an electric circuit which does not have some resistance.

736. The Counter E.m.f. of Self-induction Always Lags 90 Degrees Behind the Current, that is, the effect of inductance is 90 degrees behind, or at right angles to, the current. This is stated here merely as a fact; why it is true will (Art. 740) be shown later.

737. To Compute the Counter E.m.f. of Self-induction in an Alternating-current Circuit—or in any circuit for that matter—the inductance of the circuit, L, is multiplied by the *rate of change* of current in the circuit, Art. 490. If the inductance, L, is expressed in henrys and multiplied by the *average rate of change* in current, the change of current per second in the circuit, the result will be the *average counter e.m.f.*, in volts, induced in the circuit during that second; this follows from the definition of the henry, Art. 472. Now an alternating current is continually changing in intensity; hence it is continually—if there be inductance in the circuit—inducing a counter e.m.f. However, in dealing with alternating-current circuits, it is not the *average* counter e.m.f. which is of interest but it is the *effective* (Art. 691) counter e.m.f. It can be shown (see proof in the following note) that the *effective rate of change of a sine-wave-form alternating current*—that is the rate of change in the effective current per second equals: $6.28 \times f \times I$. Hence:

$$E_c = 6.28 \times f \times L \times I \qquad \text{(volts)} \tag{181}$$

and

(182) $$f = \frac{E_c}{6.28 \times L \times I} \quad \text{(cycles per sec.)}$$

and

(183) $$L = \frac{E_c}{6.28 \times f \times I} \quad \text{(henry)}$$

and

(184) $$I = \frac{E_c}{6.28 \times f \times L} \quad \text{(amp.)}$$

Wherein E_c = the counter e.m.f. (effective) of self-induction, in volts. f = frequency of the current, in cycles per second. L = inductance of the circuit or conductor, in henrys. I = current (effective) in the circuit or conductor, in amperes.

NOTE.—The symbol π (a Greek letter pronounced "pie") has been universally adopted to stand for the number "3 1416." Now, 2 × 3.1416 =

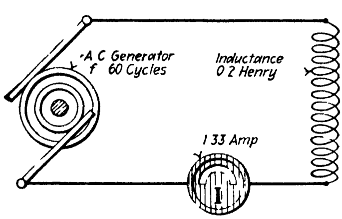


FIG. 423.—Inductive winding in an alternating-current circuit.

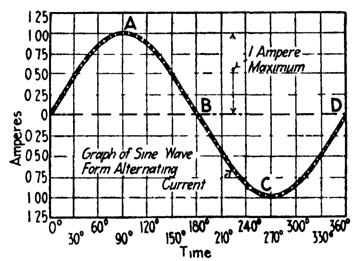


FIG. 424—Illustrating "rate of change" of an alternating current.

6.28. Hence the above formula may also be written: $E_c = 2 \times \pi \times f \times L \times I$, which is the form very frequently adopted in text books.

EXAMPLE.—What counter e.m.f. is generated in the coil of Fig. 423, which has an inductance of 0.2 henry, when an effective alternating current of 1.33 amp. flows in it, at a frequency of 60 cycles per sec.? SOLUTION.—Substitute in the above formula (181): $E_c = 6.28 \times f \times L \times I = 6\ 28 \times 60 \times 0.2 \times 1.33 = 100$ *volts* (almost). This means that the generator must impress 100 volts (effective) across the terminals of the coil to circulate a current of 1.33 amp. through the coil. It also means that when a current of 1.33 amp. at 60 cycles flows in the coil the counter e.m.f. of self-induction developed in the coil is 100 volts.

NOTE.—PROOF THAT 6.28 × f × I = EFFECTIVE RATE OF CHANGE OF A SINE-WAVE-FORM ALTERNATING CURRENT.—Consider the rate of change of current in an alternating-current circuit. Refer, for example, to Fig. 424 which shows the graph of a sine-wave-form alternating current having a crest or maximum value of 1 amp. Note that during 1 cycle ($OABCD$) the current: (1) *increases, OA, from* 0 *amp.* to 1 *amp.*, in the positive direction; (2) *decreases, AB, from* 1 *amp. to* 0 *amp.*, in the positive direction;

(3) *increases, BC, from* 0 *amp. to* 1 *amp.*, in the negative direction; and (4) *decreases, CD, from* 1 *amp. to* 0 *amp.*, in the negative direction. Note then that the alternating current graphed in Fig. 424 changes 1 amp. *four times per cycle twice from* 0 *amp. to* 1 *amp.* and twice from 1 amp. to 0 amp.

It follows that any alternating current changes four times per cycle between zero intensity and maximum intensity; twice from maximum to zero and twice from zero to maximum. Let the symbol I_M stand for maximum current. Then the change of current per cycle $= 4 \times I_M$.

Now, *average rate of change of current = change in amperes ÷ seconds.* With an alternating current: frequency or f = *cycles per second.* Hence the time in seconds, required for the completion of 1 cycle $= 1 \div f = \frac{1}{f}$. (For example, with a 60-cycle current $\frac{1}{60}$ sec. is required for the completion of 1 cycle, Fig. 70.) The time, in seconds, required for each of the form changes from zero intensity to maximum intensity in a cycle is, obviously, one-fourth the time required for the completion of an entire cycle. Therefore, the time required for one of the changes is: $\frac{1}{4} \times \frac{1}{f} = \frac{1}{4 \times f}$ sec. That is, the current changes from 0 amp. to I_M amp. in $4 \times f$ sec.

The average rate of change of current as defined in the preceding paragraph—the change of current per second—must be: *the change in current in amperes divided by the time in seconds elapsing while the change transpires;* that is: (184*C*) *average rate of change of current* $= I_M \div \left(\frac{1}{4 \times f}\right) = 4 \times f \times I_M$.

Now in practical work we deal with effective currents (Art. 691) rather than with maximum currents and hence I_M should be converted into its equivalent effective value. From equation (162): $I_M = 1.41 \times I_E$, or to be more exact:

(185) $$I_M = 1.4142 \times I_E$$

Then substituting (2) in (1):

(186) *Average rate of change of current* $= 4 \times f \times (1.4142 \times I_E)$ (187) Or *average rate of change of current* $= 5.6568 \times f \times I_E$.

If then this expression: $5.6568 \times f \times I_E$, which represents the *average* rate of change of current per second, were multiplied by the inductance, L, in henrys, of an alternating-current circuit, the product (Art. 490) would be the *average* (Art. 690) counter e.m.f. in volts, induced in the circuit. But we are usually interested in and deal with *effective* (Art. 691) counter e.m.fs. not average e.m.fs. Hence, the formula (187) will now be so modified that it will give the *effective* rate of change of current. In equation (171) it was shown that: *Average values* $= 0.90 \times$ *effective values.* Hence:

(188) *Average rate of change of current* $= 0.90 \times$ *effective rate of change of current.*

Now substituting the expression of (188) in (187):

(189) $$0.90 \times \textit{effective rate of change of current} = 5.6568 \times f \times I_E$$

Then simplifying:

(190) $$\textit{Effective rate of change of current} = \frac{5.6568 \times f \times I_E}{0.90}$$

(191) or *Effective rate of change of current* $= 6.28 \times f \times I_E$

NOTE.—Power is not, as will be shown (Art. 784), expended in overcoming a counter e.m.f. of self induction. If there is resistance in the circuit—and there always is some, though in many cases very little—power is expended in forcing the current to circulate through the resistance.

738. The Method of Computing the Counter E.m.f. of Self-induction of Any Coil in an Alternating-current Circuit follows from the principles just outlined. From Art. 492, formula (111), the inductance of any coil, $L_1 = \phi_a \times N \div 10^8$, wherein ϕ_a = the flux produced by the coil *per ampere* of current flowing in it and N = number of turns in the coil. Then if this quantity, which represents the inductance of the coil, be multiplied by the *average rate of change* of current—the change in current per second—the product will be the average counter e.m.f. of self-induction induced (Art. 737). It has been shown (equation 184*A*) that the *average rate of change* of a sine-wave-form alternating current $= 4 \times f \times I_M$. Hence if E_{CA} be taken to represent the average counter e.m.f. induced by the coil and performing the multiplication just suggested:

$$(192)\qquad E_{CA} = 4 \times f \times I_M \times \left(\frac{\phi_a \times N}{10^8}\right) = \frac{4 \times f \times I_M \times \phi_a \times N}{100{,}000{,}000}$$

But, obviously, *maximum amp.* $\times$ *flux per amp.* = *maximum flux*, that is:

$$(193)\qquad I_M \times \phi_a = \phi_M$$

Then substituting ϕ_M for "$I_M \times \phi_a$" in equation (192):

$$(194)\qquad E_{CA} = \frac{4 \times f \times \phi_M \times N}{100{,}000{,}000}$$

But equation (171) *Average Value* = 0.90 $\times$ *Effective Value*, hence:

$$(195)\qquad E_{CA} = 0.90 \times E_{CE}$$

Then substituting (195) in (194):

$$(196)\qquad 0.90 \times E_{CE} = \frac{4 \times f \times \phi_M \times N}{100{,}000{,}000}$$

$$(197)\qquad E_{CE} = \frac{4 \times f \times \phi_M \times N}{0.90 \times 100{,}000{,}000}$$

$$(198)\qquad E_{CE} = \frac{4.44 \times f \times \phi_M \times N}{100{,}000{,}000}$$

Wherein E_{CE} = volts, effective, counter e.m.f. induced in any

coil by a sine-wave-form alternating current. f = frequency of the circuit in cycles per second. ϕ_M = maximum total flux or lines of force produced by the coil. N = number of turns in the coil.

EXAMPLE.—A certain coil has 50 turns. With a 60-cycle alternating current of such value flowing through it that a flux of 300,000 lines maximum is developed, what is the counter e.m.f. then induced in the coil? SOLUTION.—Substitute the above formula (198):

$$E_{CE} = \frac{4.44 \times f \times \phi_M \times N}{100{,}000{,}000} = \frac{4.44 \times 60 \times 300{,}000 \times 50}{100{,}000{,}000} = 39.9 \ \textit{volts}.$$

Hence the induced counter e.m.f. is 39.9 volts (effective).

The Above Equation is Very Important, that is equation (198), inasmuch as it is utilized repeatedly in the design of alternating-current generators, transformers and motors.

739. To Compute the Counter E.m.f. of a Coil in an Alternating-current Circuit on the Basis of the Area of the Core of the Coil and the Flux Density, it is merely necessary to modify equation (198) accordingly, as will be shown. In an alternating-current magnetic circuit it is, obviously, the flux density at the instants when the current is a maximum, that is $\mathbf{B}_M$, which should determine the area of the magnetic circuit. If the magnetic circuit has sufficient area to effectively carry the flux due to the maximum instantaneous current it will, plainly, carry the flux due to currents less than the maximum. Now (Art. 246): $\phi = A \times B$, hence it is obvious that $\phi_M = A \times \mathbf{B}_M$. Then substituting this value for ϕ_M in equation (198), the following working formula is obtained:

$$(199) \qquad E_{CE} = \frac{4.44 \times f \times A \times \mathbf{B}_M \times N}{100{,}000{,}000} \qquad \text{(volts)}$$

Wherein all symbols have the same meanings as under equation (198) except that (Art. 246): A = cross-sectional area of core, in square inches = area—where an iron core is used—of cross-section of iron in the core on which the coil is wound. $\mathbf{B}_M$ = maximum flux density, in lines per square inch, that is, the flux density at the instants when the current in the coil is at its maximum instantaneous values. In design, $\mathbf{B}_M$ is usually taken at some value below the saturation point, Art. 259.

EXAMPLE.—If an iron core similar to that of Fig. 425, except having an area of 1⅝ sq. in., is wound with 700 turns of wire, what will be the effective counter e.m.f. which it develops in a 40-cycle circuit, assuming that the

current through it is such that B_M = 70,000 lines per sq. in.? SOLUTION.—From formula (199):

$$E_{CE} = \frac{4.44 \times f \times A \times B_M \times N}{100,000,000} = \frac{4.44 \times 40 \times 1.625 \times 70,000 \times 700}{100,000,000}$$
$$= 141\ volts.$$

Therefore, the counter e.m.f. developed under the above conditions (and assuming that there is no magnetic leakage) is 141 volts.

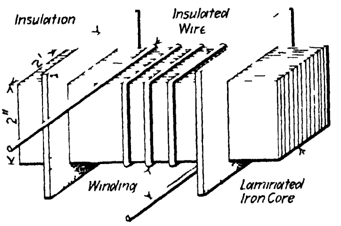


FIG. 425.—Laminated iron core with insulated-wire winding.

EXAMPLE.—It is desired to wind a coil on the laminated iron core of Fig. 425 which will develop a counter e.m.f. of 100 volts when connected in multiple on a 60-cycle circuit. Permissible maximum flux density is taken as 30,000 lines per sq. in. How many turns (assuming no magnetic leakage) will be required on the coil to produce this result? SOLUTION.—Core is 2 in. × 2 in., hence area = 4 sq. in. Solving equation (199) for N:

$$N = \frac{100,000,000 \times E_{CE}}{4.44 \times f \times A \times B_M} = \frac{100,000,000 \times 100}{4.44 \times 60 \times 4 \times 30,000} = 313\ turns$$

Hence 313 or, say, 310 turns are necessary.

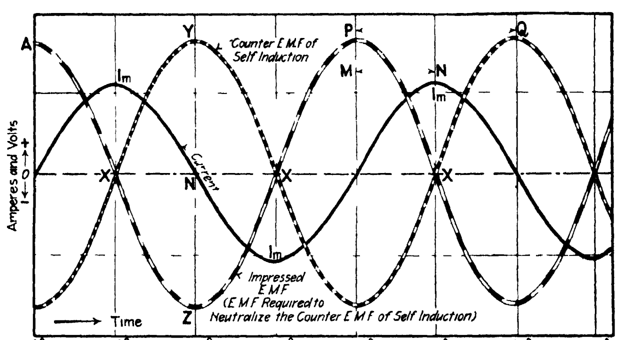


FIG. 426.—Graphs of current and impressed and counter e.m.fs. in an alternating-current circuit containing inductance only. (Actually there cannot be a circuit which does not have some resistance.)

740. Phase Relations of Current and the Counter E.m.f. Due to Inductance are shown in Fig. 426. The counter e.m.f. of self-induction in an alternating-current circuit is never in phase with the current which produces it. Nor is it in phase with the e.m.f. impressed on the circuit by the generator or other source. As has been previously stated (Art. 736) and as now will be shown, the

counter e.m.f. of self-induction always lags 90 degrees behind the current. The reason for this 90-degree lag—which represents a certain definite interval of time—is given in the following explanation.

Explanation.—The intensity of an e.m.f.—or a counter e.m.f. —induced in any conductor is determined by the *rate* at which the conductor cuts or is cut by flux (Art. 438). The greater the rate of cutting, the greater the induced voltage. The slower the rate, the lower the induced voltage. The counter e.m.f. of self-induction in any alternating-current circuit is induced by virtue of the movement of concentric lines of force (Art. 465) which emanate from the center of the conductor. These circular flux lines in expanding outward, as the current increases, or in contracting inward, as the current decreases (as the alternating current varies in intensity or changes in direction), cut the conductor (see Figs. 227 to 230) or the turns of the coil into which the conductor may be formed.

Obviously, the speed of travel and speed of cutting of these lines is proportional to the rate of change of current. It is evident from the values shown in Fig. 427 that with an alternating current the *rate of change of current* is constantly changing. During the time an alternating current is passing through its zero value its rate of change is greatest. For example, during the 10-degree period *AB*, the current graphed in Fig. 427 has changed (increased) in intensity by 17.4 amp. During the 10-degree period *CD*, the rate of change is slower, viz. an increase of only 14.3 amp. At about the time the current is passing through its maximum value, the rate of change is very slow indeed; during the 10-degree period *EF* the decrease shown in the illustration is only 0.004 amp.

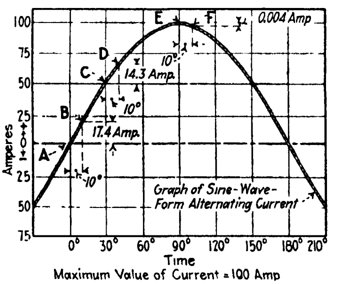


Fig. 427.—Showing how the rate of change of current varies at different times in a cycle.

It can be shown that there is an instant, when the current is just at its maximum value, *E* (Fig. 427), during which the rate of change of current intensity is zero. Therefore, since the intensity of a counter e.m.f. at any instant is determined by the rate of change of current at that instant, the counter e.m.f. of self-induction induced in an alternating-current circuit will be *zero* when the *current* is passing through its maximum value. The graphs of Fig. 426 illustrate this condition. When the current is a maximum at *Im*, the counter e.m.f. is zero at *X*.

The rate of change of an alternating current is greatest when the current is passing through its zero value. Therefore, the counter e.m.f. of self-induction is a maximum when the current to which it is due is a minimum. In Fig. 426 at the instant when the current is zero, at *N*, the counter e.m.f. of self-induction is a maximum, at *Y*.

Since any induced counter e.m.f. is always in such a direction as to

oppose any change in the current producing it (Art. 435), the curve of the counter e.m.f. must be in the relation to the curve of current that is shown in Fig. 426. At the instants when the current is increasing the counter e.m.f. is decreasing. At the instants when the current is decreasing the counter e.m.f. is increasing.

The counter e.m.f. of self-induction will lag behind the current by 90 degrees, which means that the counter e.m.f. will reach its maximum intensities in each cycle, a certain interval of time (represented by 90 degrees; see example under Art. 528) later than the current reaches its maximum intensity.

741. The Current in an Alternating-current Circuit Containing Only Inductance and Resistance, Always Lags Behind the E.m.f. Impressed by the Generator.—The amount of lag is proportional to the amount of inductance (or *inductive reactance*, Art. 747) in the circuit. If there is no inductance in the circuit, then there is no lag and the current will be in phase with the impressed e.m.f. as shown in Fig. 422. In a circuit consisting wholly of inductive reactance—such a circuit is, however, a physical impossibility—the current would lag exactly 90 degrees behind the impressed e.m.f. as shown in Fig. 426. With varying proportions of inductive reactance and resistance, the current—as will be shown later—will lag by some amount between 90 and 0 degrees, behind the impressed e.m.f. With little inductance in the circuit there will be but little lag; with much inductance the lag may be almost 90 degrees.

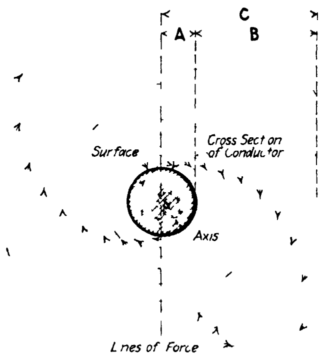


FIG. 428—A graphic explanation of skin effect.

742. Skin Effect is the name of that effect, in alternating-current conductors, whereby the current density (Art. 123*a*) at their surfaces is greater than that along their axes. It amounts to a virtual increase in their resistances.

EXPLANATION.—It has been shown that (see Fig. 227) current in a conductor sets up a field of circular lines of force about itself. This field is represented in Fig. 428. With an alternating current, at the instant when the current is zero there is no field and, as the current increases during an alternation, the field of concentric circular lines forms and expands outwardly, like smoke rings from a locomotive stack, from the center of the conductor. When the current decreases to zero, during an alternation, the lines return to the center of the conductor and vanish. These circular

lines of force, in expanding and returning, cut the conductor and produce self-induction and skin effect. More lines cut the metal at the axis of the conductor than cut the metal near its surface.

In Fig. 428, wherein a field is shown at its maximum value, all of the lines, represented by *C*, have cut the axis while only those represented by *B* have cut the surface. The lines represented by *A* do not, and can not, cut the surface. The result is that there is a greater counter e.m.f. of self-induction at the center of the conductor than at its surface. This tends to force most of the current toward the surface. With large conductors and high frequencies there is practically no current at the axis of the conductor. Most of the current is carried near the surface. A central core of some non-conducting material is often used in large stranded conductors so that the copper in the conductor will be worked at a good efficiency.

While the self-induction of a conductor, which causes voltage drop, and the skin effect in the conductor both originate from the same magnetic field, they are not otherwise related. They are two distinct effects and should always be considered separately. With small conductors, at commercial frequencies, the skin effect is negligible. Inasmuch as, because of skin effect, all of the sectional area of a large alternating-current conductor can not be effectively utilized, the result of skin effect amounts to an increase in the resistance of a conductor—since it increases the voltage drop and energy loss. Skin effect is considered as an increase in resistance. The following tables give constants by which the actual resistances of conductors must be multiplied to obtain their virtual resistances to alternating currents. The counter e.m.f. of self-induction of a conductor requires no energy to overcome it (see 784) but energy is required to "overcome" the virtual increase of resistance due to skin effect.

743. Skin Effect Factors at 20 Deg. C. for Straight Cylindrical Conductors.—Values by means of which the skin effect, of round conductors of copper, aluminum, iron or steel, can be obtained for any frequency (STANDARD HANDBOOK).

Product of circular mils times cycles per second	Skin-effect factor		Product of circular mils times cycles per second	Skin-effect factor	
	Copper	Aluminium		Iron	Steel
5,000,000	1.000	1.000	500,000	1.000	1.000
10,000,000	1.003	1.001	1,000,000	1.000	1.000
20,000,000	1.011	1.004	2,000,000	1.057	1.000
30,000,000	1.025	1.009	3,000,000	1.215	1.000
40,000,000	1.044	1.017	4,000,000	1.485	1.001
50,000,000	1.068	1.027	5,000,000	1.520	1.0015
60,000,000	1.095	1.038	6,000,000	1.640	1.002
70,000,000	1.125	1.052	7,000,000	1.750	1.003
80,000,000	1.156	1.067	8,000,000	1.850	1.004
90,000,000	1.188	1.077	9,000,000	1.950	1.005
100,000,000	1.222	1.101	10,000,000	2.040	1.006
125,000,000	1.340	1.148	12,500,000	2.250	1.009
150,000,000	1.440	1.200	15,000,000	2.440	1.013
175,000,000	1.530	1 260	17,500,000	2.620	1.018
200,000,000	1.620	1.330	20,000,000	2.780	1.023
250,000,000	1.780	1.460	25,000,000	3.080	1.036
300,000,000	1.920	1.570	30,000,000	3.340	1.051
350,000,000	2.060	1.680	35,000,000	3.600	1.069
400,000,000	2.180	1.770	40,000,000	3.820	1.088
450,000,000	2.300	1.870	45,000,000	4.040	1.107
500,000,000	2.410	1.950	50,000,000	4.240	1.131
550,000,000	2.520	2.040	55,000,000	4.440	1.155
600,000,000	2.620	2.110	60,000,000	4.620	1.177

EXAMPLE.—What is the skin-effect factor with a frequency of 60 cycles for a round iron conductor of 500,000 cir. mil area? SOLUTION.—(*circular mils*) × (*cycles per second*) = 500,000 × 60 = 30,000,000. The factor corresponding to 30,000,000 for an iron conductor in the above table is 3.34. The conductor would have an apparent resistance for a 60-cycle current 3.34 times as great as for a direct-current.

SECTION 47

WHAT REACTANCE AND IMPEDANCE REALLY MEAN

744. Ohm's Law Really Applies to All Alternating-current Circuits as well as to all actual direct-current circuits. But the method of its application to some alternating-current circuits involves a somewhat tedious, though not complicated process. First it will be shown, in a general way, why this is true; then a numerical example illustrating the truth will be recited:

With a direct-current circuit (Art. 134): $I = E \div R$, where I = current in amperes, E = e.m.f., in volts, which impels I in the circuit and R = resistance, in ohms, of the circuit. Note particularly that E = *e.m.f. in volts which impels I;* this E is not necessarily the e.m.f. or voltage impressed on the direct-current circuit. If there is a source of counter e.m.f. in the circuit, the voltage E which impels the current will be: *Impressed Voltage minus Counter Voltage.* That is, where there is a counter e.m.f. in the circuit, the available, net or active voltage is something less than the impressed voltage. The available voltage which actually forces current through a direct-current motor, armature which is turning, is the voltage impressed on the motor minus the counter e.m.f. induced in the motor armature due to its rotation. In many—in fact probably in a majority—of direct-current circuits there is no source of counter voltage; hence for a majority of (but not for all) direct-current circuits, *the current equals the e.m.f. impressed on the circuit divided by the resistance of the circuit* because, in these cases where there is no counter e.m.f., the available or energy e.m.f. must necessarily be the impressed e.m.f.

Precisely the same general situation obtains in regard to the application of Ohm's law with alternating- as with direct-current circuits. The current in an alternating-current circuit equals the available (active or energy) e.m.f. divided by the resistance of the circuit. And to determine the available e.m.f. in any circuit, any counter e.m.f. developed in the circuit must be subtracted from the e.m.f. impressed on the circuit—just as with direct-current circuits.

It was shown in Art. 732 that the current in an alternating-current circuit containing resistance only is equal to the e.m.f. impressed on the circuit divided by the resistance. The reason why this rule is a true one is that no counter e.m.f. is developed in an alternating-current circuit containing only resistance. Hence, in such a circuit the impressed e.m.f. is the available or energy e.m.f. But if there is permittance (capacitance) or inductance in an alternating-current circuit, each when current flows, exerts a counter e.m.f. (Arts. 735 and 753), which opposes the impressed e.m.f. Hence, to obtain the value of the *available or energy e.m.f.* (the e.m.f. which is actually serviceable in impelling current) in an alternating-current circuit the counter e.m.fs., if there are such, developed due to inductance or to permittance or to both must be subtracted from the impressed e.m.f.

In direct-current circuits the counter e.m.fs.—if there are such—are in phase (Art. 685) with their impressed e.m.fs.; hence to obtain the net e.m.f. in a direct-current circuit the counter e.m.f. is subtracted arithmetically from the impressed e.m.f. (Art. 730). But in alternating-current circuits, the counter e.m.fs. of self-induction and of permittance are not in phase with the impressed e.m.f. (Arts. 736 and 763). That is, the counter e.m.fs. of inductance and permittance do not attain their maximum values at the same instants as those at which the impressed e.m.f. attains its maximum values. It follows that, to obtain the net —available—e.m.f. in an alternating-current circuit, the subtraction of the counter from the impressed e.m.f. can not be made arithmetically. It must be made vectorially and the phase relations between the e.m.fs. concerned must be given due consideration. Just how this may be done and a verification of the fact that Ohm's law holds for alternating-current circuits, are shown in the illustrative example which follows. In the circuit of this example there are only resistance and inductance. Circuits containing permittance are treated in another section of the book.

EXAMPLE.—In a certain coil in a 60-cycle, alternating-current circuit (Fig. 429), the current is 11 amp. with an impressed e.m.f. of 110 volts. The resistance of the coil is 5 ohms and its inductance is 0.023 henry. Analyze this situation and show that the *current* —11 *amp.—in the coil is equal to the available or energy e.m.f. across the coil divided by its resistance*—5 *ohms.* Also show that the available or energy e.m.f. is equal to the impressed e.m.f. minus the counter e.m.f. of self-induction developed in the coil.

SOLUTION.—Now the resistance of the coil *AB* (Fig. 429) is 5 ohms.

Hence, by Ohm's law (Art. 134) the available, or energy e.m.f. which actually impels the 11-amp. current in it must be: $E_e = R \times I = 5 \times 11 =$ 55 *volts*.

The impressed e.m.f., E_i, is 110 volts. Since the coil has an inductance of 0.023 henry and the current in it is 11 amp., the counter e.m.f. of self-induction developed in the coil must be (Art. 737): $E_c = 6.28 \times f \times L \times I = 6.28 \times 60 \times 0.023 \times 11 = 95.3$ *volts*.

Now proceed with the solution of the problem to show that the energy e.m.f. —55 volts—is equal to the impressed e.m.f.—110 volts—minus the counter e.m.f. of self-induction developed in the coil—95.3 volts. That is, show that 55 volts is the difference between an impressed e.m.f. of 110 volts and a counter e.m.f. of self-induction of 95.3 volts.

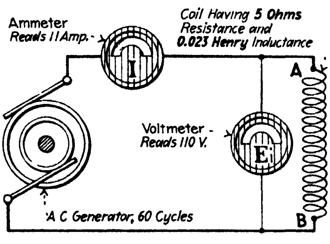


FIG. 429.—Current and voltage relations in a circuit having 5 ohms resistance and 0.023 henry inductance.

First draw reference lines (Art. 721) at right angles to one another as shown at *I* in Fig. 430. Now plot a vector, OE_r, as shown in *II*, to represent in length to scale, the energy e.m.f. of 55 volts. This vector is plotted horizontally because the e.m.f. it represents is in phase with the current and (as suggested in Art. 721) it is usually most convenient to plot horizontally, vectors representing values which are in phase with the current.

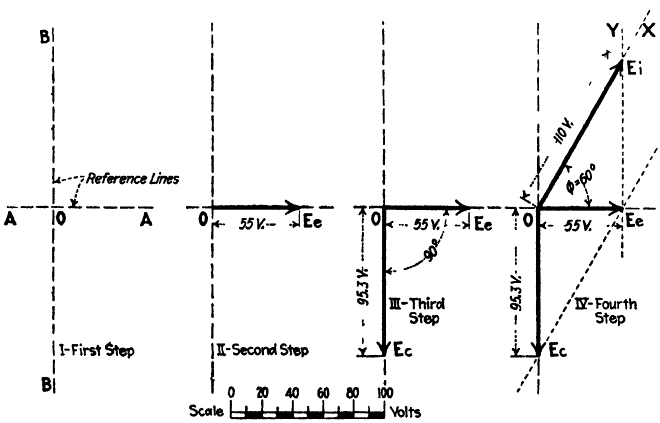


FIG. 430.—Illustrating the process of a vector solution of a problem.

Now plot the vector representing the counter e.m.f. of self-induction of 95.3 volts. Since (Art. 736) the counter e.m.f. of self-induction always lags 90 degrees behind the current—and consequently 90 degrees behind the energy e.m.f.—this vector plotted to scale 95.3 volts long will fall in the position OE_c, as shown at *III*.

There have now been plotted, in *III*, two vectors OE_e and OE_c representing, in length and phase relation, the energy e.m.f. of 55 volts and the counter

e.m.f. of self-induction of 95.3 volts. It now remains to plot another vector which will be called O_1E_i, which will represent in length and phase relation the impressed e.m.f. of 110 volts. If the vector OE_e (in *III*) represents the difference between two e.m.fs. and OE_c represents one of the two, it follows from the suggestions of Arts. 724 and 731 that OE_i (in *IV*) must represent the other e.m.f. of the two. The vector OE_i is plotted thus: Draw the construction line E_cE_e, then draw OX through O parallel to E_cE_e. Now draw construction line E_eY parallel to OE_c. Then the point, E_i, of intersection of the OX and E_eY will determine the length of the vector OE_i which is plotted along OX.

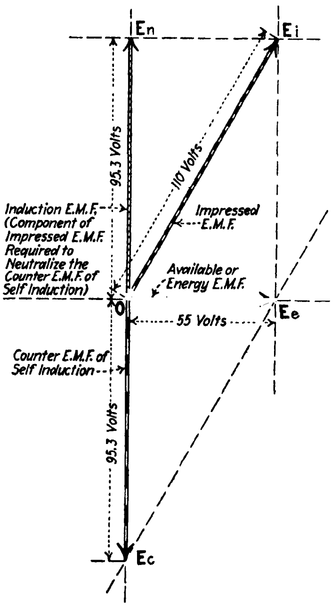


FIG. 431.—Phase diagram illustrating induction component of impressed e.m.f.

Now by scaling it will be found that the length of OE_i equals 110 volts, which as stated in the specifications of the above example is the value of the impressed e.m.f. Thus it has been shown that the available or energy e.m.f. of 55 volts which impels the 11-amp. current in the coil of Fig. 429 is equal to the difference between the impressed e.m.f. of 110 volts and the counter e.m.f. of self-induction of 95.3 volts.

By scaling with a protractor it is found that the angle ϕ, in Fig. 430, *IV*, is 60 degrees. Hence, it follows that, in this particular circuit, the current—and the energy e.m.f.—lag 60 degrees behind the applied e.m.f.

745. The Portion or Component of Impressed E.m.f. Which Is Necessary to Neutralize the Counter E.m.f. of Self-induction must be just equal in intensity and opposite in phase to the counter e.m.f. of self-induction. That this is true is evident almost without explanation. Any impressed e.m.f. may be considered as comprising two components (Art. 724) parts or portions: an energy e.m.f. and an induction e.m.f. Thus, consider Fig. 431 which has been plotted to the values of the example just above recited. The vector OE_i (Fig. 431) represents the impressed e.m.f. It may be considered as comprising two components OE_n and OE_e differing in phase by 90 degrees. The component OE_e represents the energy part of the impressed e.m.f. which is available for actually circulating the current in the circuit. The component OE_n represents the "Induction"

component or part of the impressed e.m.f. which is "used up" in neutralizing the counter e.m.f. of self-induction.

If the induction component of the impressed e.m.f. is to neutralize the counter e.m.f. of self-induction, the induction e.m.f. must be a maximum in the negative direction (Z, Fig. 426) at the instant (Y) the counter e.m.f. of self-induction is a maximum in the positive direction. That is, these two e.m.fs. should always differ in phase by 180 degrees as shown in the phase diagram of Fig. 431 and the graphs of Fig. 426.

EXAMPLE.—If a current of 2 amp. is forced through the inductance of 0.133 henry (no resistance or permittance in this imaginary circuit) of Fig. 420, *II*, the generator must impress across the inductance a pressure just equal to the counter e.m.f. induced by the 2-amp. current. As shown in the example relating to this Fig. 420, which is given under Art. 747, the counter e.m.f. developed by the 2-amp. current through the inductance of 0.133 henry is 100 volts. It follows, therefore, that the impressed e.m.f. necessary to impel this 2-amp. current must also be just 100 volts—since it is assumed that in this imaginary circuit there is no resistance. Furthermore, since it is assumed that there is no resistance in the circuit, there is no energy e.m.f. The current which circulates in this resistanceless circuit does no work and represents no energy (see "Power-factor," Art. 782). The counter e.m.f. and the impressed e.m.f. in this imaginary circuit must differ in phase (see Fig. 426) by 180 degrees.

746. It Is Not Usual in Solving Problems in Alternating Currents, to First Determine the Energy E.m.f. and then compute the current by dividing this energy or available e.m.f. value by the resistance, as was done in the solution of the problem relating to Fig. 429. It is, as will be shown, possible to treat the counter e.m.f. effects of inductance (and of permittance, Art. 753) as if these effects were a kind of resistance—which is called *reactance*. They—reactance effects—can be combined with the actual "ohmic" resistance of the circuit to determine the total opposition offered to flow of current in the circuit. Then, if the impressed-voltage value be divided by this composite quantity called "impedance" (Art. 748) (which is expressed in ohms) which represents the combined opposition due to actual resistance and inductance (and permittance if there is appreciable permittançe in the circuit) the value of current, in amperes, in the circuit will result. The situation is discussed and illustrative examples are given in following articles.

747. Inductive Reactance is the name which has been given to the opposition offered to the flow of alternating currents due to the counter e.m.f. of self-induction (Art. 479). It is numer-

ically equal but opposite to the counter e.m.f. of self-induction in an alternating-current circuit. Reactance is expressed in ohms just as is resistance. Just as an e.m.f. is necessary to impel current through resistance, an e m.f. is also necessary to "overcome" or neutralize the counter e.m.f. induced in an alternating-current circuit which contains inductance—that is, inductive reactance. The formula whereby the inductive reactance in any alternating-current circuit may be computed is:

(200) $$X_s = 6.28 \times f \times L$$ (ohms)

(201) $$f = \frac{X_s}{6.28 \times L}$$ (cycles per sec.)

(202) $$L = \frac{X_s}{6.28 \times f}$$ (henry)

Wherein X_s = inductive reactance of the circuit in ohms. f = frequency of e.m.f. impressed on circuit, in cycles per second. L = inductance in the circuit, in henrys.

NOTE (Art. 737) that the counter e.m.f. of self-induction, $E_c = 6.28 \times f \times L \times I$ and that the portion "$6.28 \times f \times L$" of this quantity is called inductive reactance. When the reactance (x_s, in ohms) is multiplied by the current (I, in amperes), the portion or component of the impressed e.m.f. in volts which is necessary to neutralize (Art. 745) the counter e.m.f. of self-induction (so that there may be available an energy e.m.f. to force a current, I, to flow) is the result. The similarity between resistance and reactance is shown by the following examples:

EXAMPLE.—In the circuit of Fig. 420,*I*, which contains only resistance, the e.m.f. necessary to produce a current of 2 amp. is, substituting in the Ohm's law formula (Art. 134): $E = R \times I = 50 \times 2 = 100$ *volts.* With an inductance of 0.133 henry in circuit, as in Fig. 420,*II*, the 60-cycle, alternating e.m.f. necessary to force a current of 2 amp. through the reactance having an inductance of 0.133 henry is similarly obtained. Thus:

$$E = X_s \times I$$
$$E = (6.28 \times f \times L) \times I$$
$$E = (6.28 \times 60 \times 0.133) \times 2 = 50 \times 2$$
$$E = 100 \text{ volts.}$$

The inductive reactance in this example is therefore 50 ohms.

For direct-current circuits and for alternating-current circuits having no inductance:

(203) $$Current = \frac{e.m.f.}{resistance} \quad \text{or} \quad I = \frac{E}{R}.$$

Similarly: For alternating-current circuits containing inductive reactance only:

(204) $$Current = \frac{e.m.f.}{reactance} \quad \text{or} \quad I = \frac{E}{X_s}$$

and

$$E = I \times X_S \quad \text{(volts)} \tag{205}$$

or

$$X_S = E \div I \quad \text{(ohms)} \tag{205A}$$

Wherein E = the e.m.f., in volts (effective e.m.f. in an alternating-current circuit) impressed on the circuit. I = current in amperes in the circuit. X_S = reactance, in ohms, of the circuit.

NOTE.—Reactance can not be added directly to resistance although their effects are similar in that they both tend to limit the current intensity in a circuit. How resistance and reactance can be added is shown in Art. 749. Reactance may also be defined as that quantity which, when multiplied by the current, gives that *component* or portion of the impressed e.m.f. which is at right angles to the current.

748. Impedance is the name given to that quantity which represents the combined resisting effects of: (1) *Actual* (*ohmic*) *resistance* and (2) *the apparent resistance* (*reactance*) *or the opposition due to counter e.m.fs. of self-induction and permittance.* Impedance is expressed in ohms. If the *impedance* (ohms) of a circuit be multiplied by the current (amperes) in the circuit the resulting value will be the alternating e.m.f. in volts impressed on the circuit. It follows then that, for alternating-current circuits:

$$I = \frac{E}{Z} \quad \text{(amp.)} \tag{206}$$

hence

$$Z = \frac{E}{I} \quad \text{(ohms)} \tag{207}$$

and

$$E = I \times Z \quad \text{(volts)} \tag{208}$$

Wherein I = current (effective) in the circuit, in amperes. E = the e.m.f., effective, in volts, impressed on the circuit. Z = the impedance of the circuit in ohms.

749. To Obtain the Value of Impedance, resistance must be combined with, that is, added to, reactance (Art. 747). However, since the effects of resistance and reactance differ in phase by 90 degrees they can not be added arithmetically. It can be shown that (Art. 750 and Fig. 434) the counter or opposing effect of inductive reactance (the $I \times X_S$ drop) leads the opposing effect of resistance (the $I \times R$ drop) by 90 degrees. They are at right angles to each other and can be represented by phase or vector diagrams as in Fig. 432.

EXAMPLE.—Fig. 432,*I* shows a circuit the reactance (8 ohms) of which was computed from equation (200) of Art. 747. The impedance (the sum of the resistance and reactance) was obtained by drawing diagonals in the parallelograms of the phase diagrams and by drawing a hypotenuse in the vector diagram. The resulting value is the same in each case.

In a right-angled triangle:

$$(209) \qquad Hypotenuse = \sqrt{(one\ side)^2 + (other\ side)^2}$$

It follows that:

$$(210) \qquad Impedance = \sqrt{(resistance)^2 + (reactance)^2}$$

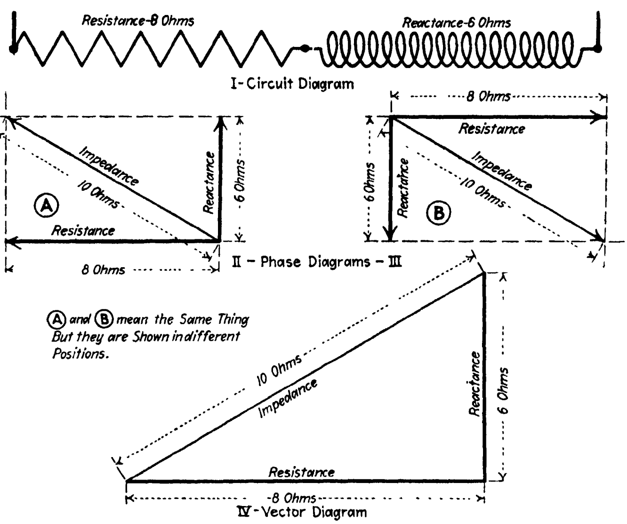


FIG. 432:—Impedance diagrams.

Now it was shown (Art. 747) that inductive reactance = $6.28 \times f \times L$. Therefore (for a circuit which contains only inductance and resistance—no permittance):

$$(211) \qquad Z = \sqrt{R^2 + (6.28 \times f \times L)^2} \qquad \text{(ohms)}$$

Wherein Z = impedance in ohms; R = resistance in ohms.

EXAMPLE.—What is the impedance of the coil of Fig. 433,*I*? It has an inductance of 0.03 henry and a resistance of 30 ohms. The frequency is 60 cycles. What will be the current with an impressed e.m.f. of 100 volts?

SOLUTION.—Substituting in the impedance formula:

$$Z = \sqrt{R^2 + (6.28 \times fL)^2} = \sqrt{(30 \times 30) + (6.28 \times 60 \times 0.03)(6.28 \times 60 \times 0.03)}$$
$$= \sqrt{(30 \times 30) + (11.3 \times 11.3)} = \sqrt{900 + 127.7} = \sqrt{1{,}027.7} = 32.1 \textit{ ohms}.$$

The problem could be solved graphically, as suggested at *II*, by laying

out the sides of a right-angled triangle to scale to represent the resistance and reactance and then scaling the hypotenuse to obtain the impedance. The current that would flow through the coil AB with a 60-cycle e.m.f. of 100 volts impressed across it would be (Formula 206):

$$I = \frac{E}{Z} = \frac{100}{32.1} = 3.12\ amp.$$

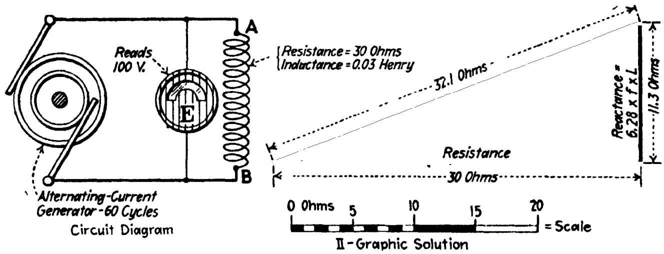


FIG. 433.—Example in computing impedance.

750. The Relations of the Different E.m.fs. and the Components Thereof in Alternating-current Circuits containing resistance only, and containing resistance and inductance, are shown respectively, in Fig. 434, at I and II. In a circuit containing

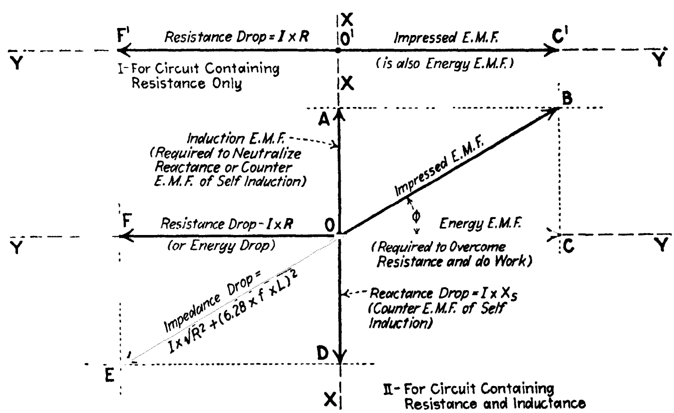


FIG. 434.—Graphic statement of e.m.f. relations in alternating-current circuit.

resistance only, the e.m.f. $O'C'$ (as shown at I), impressed by the generator or other source of alternating e.m.f., is spent wholly in "overcoming" or neutralizing the resistance drop $O'F'$. The current (I) in the circuit, (for which no vector is shown) is in phase with the impressed e.m.f. $O'C'$. In a circuit (II) containing resistance and inductance, there are two counter forces which the impressed e.m.f. must overcome: (1) *The resistance drop OF*.

(2) *The reactance drop OD*—which is the counter e.m.f. of self-induction (Art. 747.) The resultant or vector sum of *OF* and *OD* is the impedance drop *OE*. The e.m.f. *OB* impressed on the circuit under consideration must be equal and opposite to the impedance drop *OE*. This impressed e.m.f. *OB* may, as before outlined in Art. 745, be considered as being composed of two components: (1) *An induction component, OA*, which neutralizes the counter e.m.f. of self-induction and which represents no real energy. (2) *An energy component, OC*, which actually impels the current which does work through the resistance and which repre-

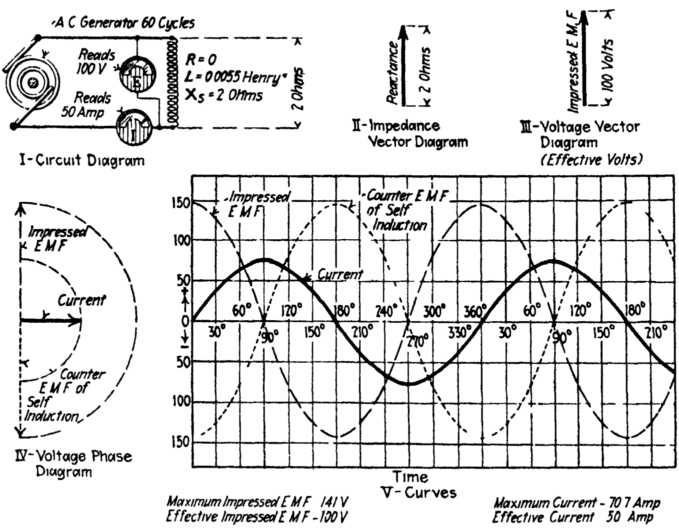


FIG. 435—Relations in an alternating-current circuit of 2 ohms reactance, no resistance.

sents real energy. Component *OA* is equal and opposite to *OD*. The current will always be in phase with *OC* and will lag behind the impressed e.m.f. *OB* by the angle ϕ. The cosine of the angle ϕ is the power-factor (Art. 783) of the circuit.

751. Further Discussion of Impedance, as Relating to Circuits Containing Permittance, and the Solution of Problems Involving Impedances in Series and in Multiple will be discussed in a following section of this book (Art. 772) after the phenomenon of permittance in alternating-current circuits has been treated.

752. Examples of Voltage and Current Relations in Circuits Having Different Proportions of Resistance and Inductance

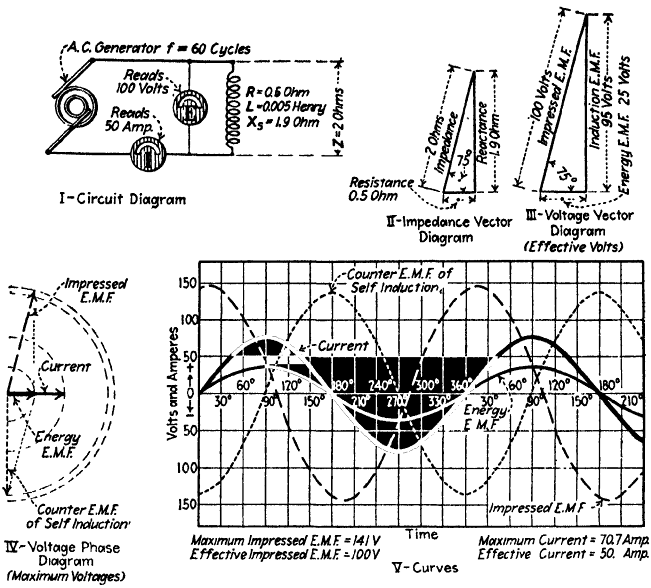


FIG. 436.—Relations in alternating-current circuit of 1.9 ohm reactance and 0.5 ohm resistance.

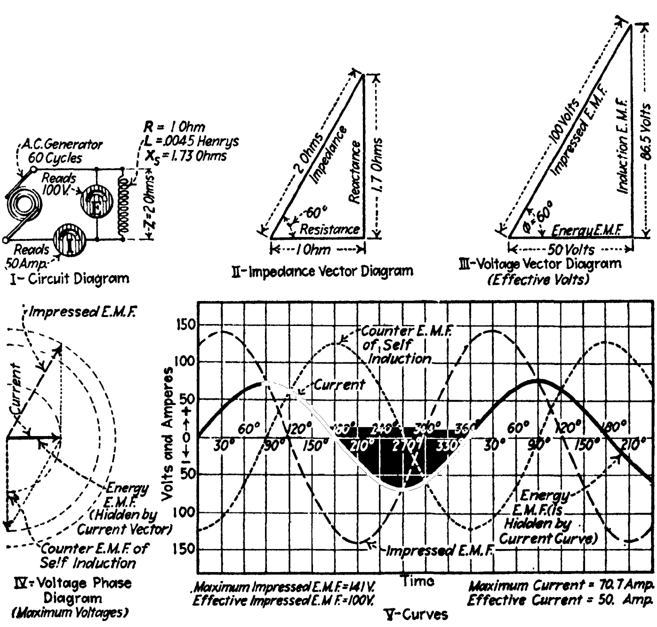


FIG. 437.—Relations in an alternating-current circuit of 1.7 ohm reactance and 1 ohm resistance.

are shown in Figs. 435, 436, 437 and 438. In all cases the current will be in phase with the energy or available e.m.f. (note under Art. 735) but will, since there is inductance in these circuits, lag behind the impressed e.m.f. The greater the relative amount of inductance the greater will be this lag.

NOTE.—In each example the reactance was first computed as directed in Art. 747 and then the impedance and voltage triangles were drawn in accordance with the preceding suggestions of Art. 744. The angle, Φ, between

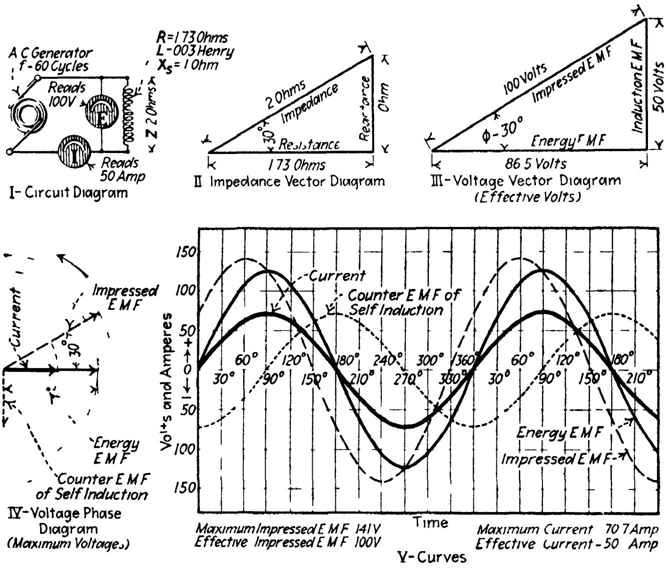


FIG. 438.—Relations in an alternating-current circuit of 1.73 ohm resistance and 1.0 ohm reactance.

the hypotenuse and the base of each of the triangles is the angle by which the current lags behind the impressed e.m.f. Note that the lag angles in the triangles are the same as in the corresponding phase diagrams and graphs. If a circuit having no resistance but some reactance could exist, its voltage and current relations would be as shown in Fig. 435; the current would lag exactly 90 degrees behind the impressed e.m.f. The condition shown in Fig. 435 can never be realized in practice because all conductors have some resistance—however, the condition suggested may be approximated. The relations in a circuit containing only resistance are shown in Fig. 422.

SECTION 48

PERMITTANCE OR CAPACITY IN ALTERNATING-CURRENT CIRCUITS

753. The Action of Inductance and Permittance (Capacitance) When the Voltage of or the Current in a Circuit Changes is this: The reaction of a permittor or condenser instead of tending—as does inductive counter e.m.f—to prevent any *change* in current, tends to prevent any change in voltage. That is, it tends to keep the voltage constant.

EXAMPLE.—If an e.m.f. is impressed across a "dead" circuit having inductance (Fig. 243), the inductance (Art. 471) tends to prevent (Fig. 246) a current from "building up" in the circuit. Now, with the current flowing, if the e.m.f. is discontinued and the circuit short-circuited (Fig. 243), the inductance will tend to prevent (Fig. 247) the "decay" of the current.

But if an e.m.f. be impressed across a "dead" circuit containing permittance, the permittance does not oppose the "building up" but instead "permits" the displacement current to flow until the counter e.m.f. of the permittor due to its stressed dielectric, is equal to the impressed e.m.f. Then, with the dielectric "charged," if the e.m.f. is discontinued and the circuit short-circuited, the permittance instead of retarding the "decay" of the current will accelerate it by virtue of the counter e.m.f. which it imposes. Obviously, then, inductance tends to prevent a current from "building up" in a dead circuit while permittance tends to *permit* it to "build up." Furthermore, inductance tends to prevent the "decay" of current in a live circuit when the e.m.f. is discontinued while permittance tends to *promote* such decay.

NOTE.—THE TERMS "CAPACITY," "CAPACITANCE" AND "PERMITTANCE" ARE SYNONYMOUS when used as relating to electrostatics. However, the term "capacity" may have *two* distinct meanings in electrical parlance. Hence, to eliminate misunderstandings, "capacity" should be used only in referring to "power capacity," "current-carrying capacity" or in similar senses. In referring to "electrostatic capacity," either "permittance" or "capacitance" should be used. "Capacitance" is the word recommended in the A. I. E. E. STANDARDIZATION RULES, but "permittance" (which was suggested by O. Heaviside) is preferred by the author because it is the more expressive of the actual property of the dielectric.

754. Inductance or Permittance (Capacitance) Effects Are Produced Respectively by Changes of Current or of Voltage.—The effects due to inductance occur in a circuit only when the

current in the circuit changes. The effects due to permittance (or capacitance) in a circuit, occur only when the voltage of the circuit changes.

EXAMPLE.—If it were possible to change the current in a given circuit without altering the voltage conditions therein, then, in that case, only inductance effects would be observed—no capacity effects. If it were possible to change the voltage impressed on a circuit without changing the current intensity (amperes) in any part of the circuit, then only permittance effects would be observed. In actual circuits it is practically impossible to change the impressed voltage without changing the current, and *vice versa*. However, in practice, except where high voltages are involved or where the circuit is provided artificially with an unusual amount of permittance (with a permittor or condenser), the effects of permittance are of little consequence. The effects of inductance, since they are determined solely by the inductance of and the current in the circuit, may be and are frequently, very noticeable in low-voltage circuits—particularly in alternating-current circuits.

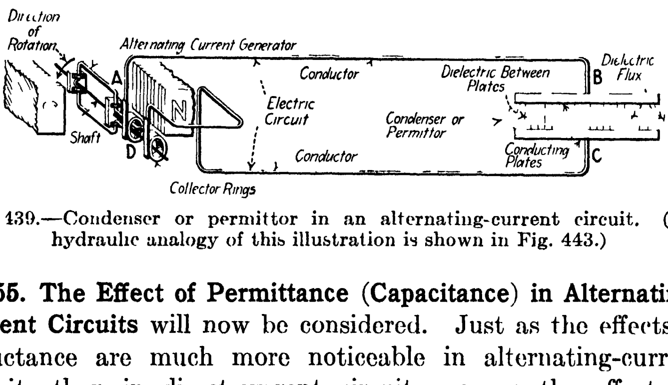


FIG. 439.—Condenser or permittor in an alternating-current circuit. (The hydraulic analogy of this illustration is shown in Fig. 443.)

755. The Effect of Permittance (Capacitance) in Alternating-current Circuits will now be considered. Just as the effects of inductance are much more noticeable in alternating-current circuits than in direct-current circuits, so are the effects of permittance much more pronounced in alternating than in direct-current circuits. Whenever there is a *change* in the voltage impressed across a permittor (condenser), electricity—that is electrons—is displaced in the dielectric of the permittor and a displacement current flows.

In an alternating-current circuit the e.m.f. reverses in direction periodically and is *constantly changing* in value. From this it would be inferred that a displacement (charging) current must flow constantly in an alternating-current circuit containing permittance—and such is the case. Now note what occurs when an alternating e.m.f. is impressed across a permittor.

EXAMPLE.—Figs. 439 and 440 show an alternating-current generator connected across a permittor (condenser). If the alternator is rotated at a

uniform speed, in the direction shown, a sine-wave-form e.m.f. will be induced. During the first 90 degrees (in Fig. 440) the e.m.f. will be in the

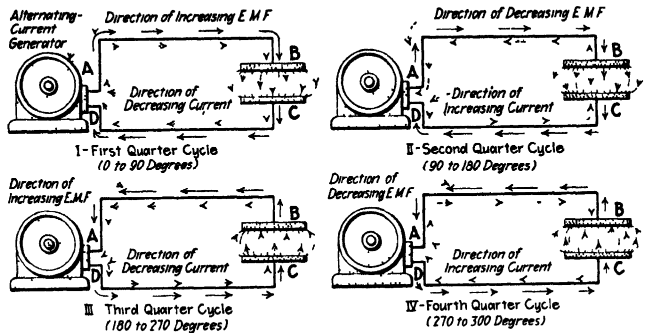


FIG. 440.—Showing how a permittor (condenser) "charges" as the impressed voltage increases and how it "discharges" when the voltage decreases.

direction $ABCD$. The portion TR of the e.m.f. sine curve of Fig. 441 shows how this induced voltage increases during the first 90 degrees. During this

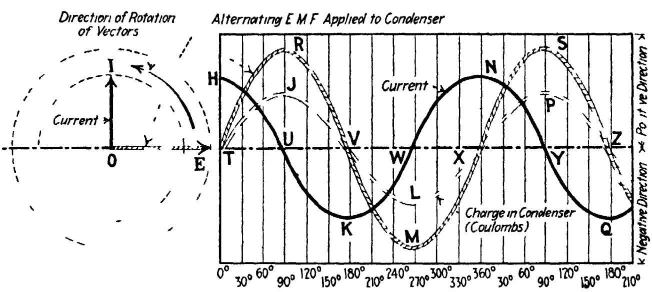


FIG. 441.—Relations in alternating-current circuit containing capacity only. The current leads the applied e.m.f. by 90 degrees.

period the permittor would be "charged" in the direction BC (Fig. 440) as indicated by TJ in the graph, Fig. 441. Also, during this period the dielectric would be stressed in the direction BC (Fig. 439 and 440) and a displacement current would flow around the circuit in the direction $ABCD$.

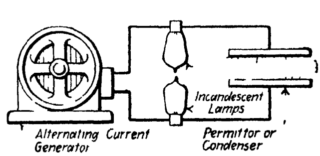


FIG. 442.—Incandescent lamps in a circuit containing permittance.

During the next quarter cycle (Fig. 440, *II*), the voltage RV (Fig. 441) is in the same direction as before but it decreases to zero. During this 90-degree period the "elasticity" of the dielectric asserts itself and the permittor "discharges" as the e.m.f. decreases—and a displacement current is thereby forced around the circuit in the direction $DCBA$

During the next quarter cycle the e.m.f. increases from zero to a maximum (*VM*) but it is now in the opposite direction (*DCBA*), as shown in Fig. 440,*III*, from that obtaining in the preceding half cycle *TRV*. Now it again "charges" the permittor but in the opposite direction. The displacement current continues to flow in the direction *DCBA*.

During the next quarter cycle (Fig. 440,*IV*) the e.m.f. decreases from *M* to *X* and the permittor "discharges" in the direction *ABCD*. Thus, an alternating displacement or charging current (of the frequency of the applied e.m.f.) flows so long as the alternating e.m.f. is impressed across the permittor in spite of the fact that the circuit is "open" in the usual sense of the word. If incandescent lamps were connected, as shown in Fig. 442, in the circuit, they would be lighted by the alternating displacement current —provided the permitter had sufficient permittance (capacitance) to allow a current to flow of sufficient intensity to heat the filaments.

756. Permittance (Electrostatic Capacity) Effects Are of Little Consequence in Low-voltage Circuits.—This follows from the fact that, permittance effects occur only when there is a

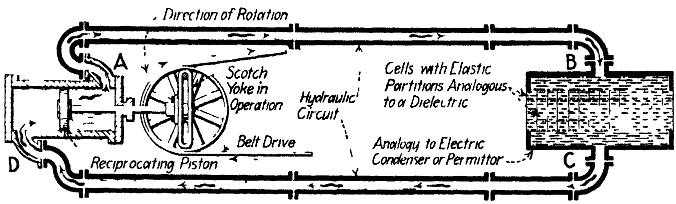


FIG. 443.—Hydraulic analogy of an alternating-current circuit containing an alternating-current generator and a condenser or permittor. (At this instant the permittor dielectric is unstressed. The pressure or voltage is a minimum but the current is a maximum in the positive direction.)

change in voltage. The greater the changes in voltage, the greater will be the permittance effects. Obviously, in low-voltage circuits the *changes* in voltage must be relatively small. The magnitude of the permittance effects—if the dielectric is not "stretched" beyond the rupturing point—is proportional to the applied voltage. On circuits operating at voltages lower than 6,000, it is seldom that the permittance effects—that is, the charging or displacement current effects—are particularly noticeable; they are, though possibly inconsequential, present nevertheless.

757. A Hydraulic Analogue of the Action of Permittance in an Alternating-current Circuit is outlined in Figs. 443 to 450. Fig. 439 illustrates an electric circuit analogous to the fluid circuit of Fig. 443. Fig. 441 is a graphic record of the phenomena occurring in the circuit as the pump is operated. The

reciprocating pump, driven at a uniform speed, is analogous to an alternating-current generator. Its piston, by virtue of the Scotch-yoke drive, has a harmonic longitudinal motion, the speed of which, as it varies with the time, could be represented by a sine curve. When the pump is driven, it impels in the fluid circuit an alternating current of the non-compressible, weightless fluid shown which is analogous to electricity.

The contrivance at the right of Fig. 443 (which is shown enlarged in the other illustrations) is analogous to a permittor (condenser). It consists of two chambers (condenser plates) with a cellular structure between them. The cellular structure has partitions of an imaginary semi-porous elastic material and is analogous to a dielectric (Art. 92). The pipes and the cellular structure are all filled with the weightless, non-compressible fluid just as all matter is permeated with electricity. The pipes are frictionless (the circuit has no electrical resistance).

As the pump is driven, its piston displaces the fluid (electricity) in the circuit—first in one direction as the piston travels to the right, then in the other as it travels to the left. The fluid which resides in the condenser when it is in its normal unstressed condition (Fig. 443) has been rendered in darker lines—merely to illustrate how the displacement of the fluid in the circuit occurs. The force exerted by the piston on the fluid is analogous to an applied e.m.f. The current (rate of flow) of fluid is analogous to an electric displacement current. The quantity of fluid (gallons) displaced is analogous to coulombs. The reactive pressure exerted by the elastic walls of the "condenser" when they are stressed is analogous to the counter e.m.f. due to permittance—which will be further discussed in Art. 758.

EXPLANATION.—Assume that the pump starts from the position of Fig. 443 in which the piston is at the center of the cylinder. Under these normal conditions the "dielectric" in the permittor is unstressed as shown in the enlarged diagram of Fig. 444. As the piston is forced from this "neutral" position, it displaces fluid around the circuit and stretches the elastic-walled cells (stresses the dielectric). Follow the cycle of events referring at the same time to the graphs of Fig. 441. Fig. 445 pictures the situation after the crank has been rotated through 30 degrees.

At the instant at which the piston is at the left end of its stroke (Fig. 446,*IV*), the force which it is exerting must be a maximum because then the "dielectric" is stretched as much as is possible with the arrangement shown. Consequently, the dielectric is exerting its maximum elastic reaction (counter e.m.f.). During the next 90 degrees (Figs. 447,*V* to 448,*VII*), the pressure exerted by the piston is decreasing and the permittor is discharg-

ing—the dielectric is "unstretching" back to the unstressed condition. During the next 90-degree period (Figs. 448,*VIII* to 449,*X*) the permittor is being "charged" in the opposite direction. During the following 90-de-

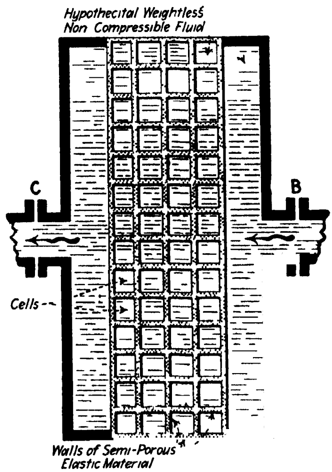


FIG. 444.—Analogy to the electric condenser or permittor. (This shows the dielectric unstressed, the condition obtaining when there is no difference of potential across the condenser between *B* and *C*.)

gree period (Figs. 450,*XI* to 450,*XII*) it discharges—in the opposite direction from the former discharge.

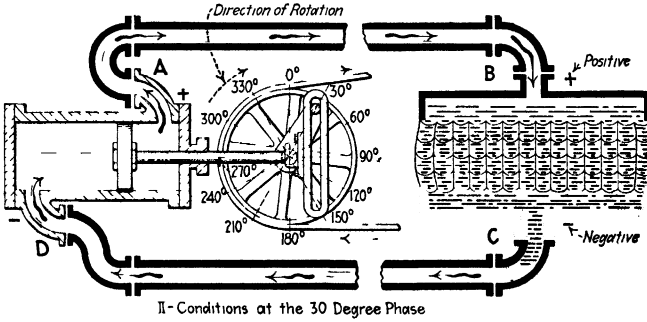
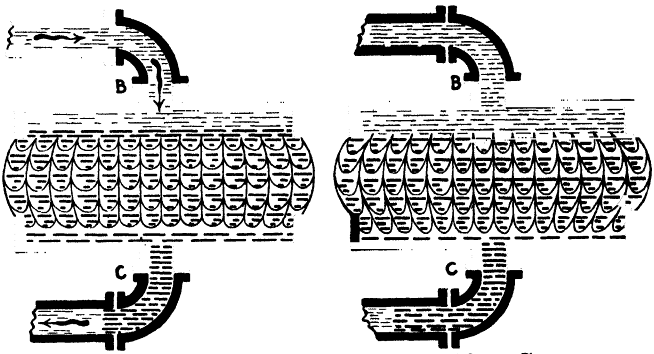


FIG. 445.—Pressure (voltage) impressed by the pump is in the positive direction and is increasing and the "dielectric" is becoming "stretched." The current (rate of flow) is also in the positive direction and is decreasing.

Note that the phase relations in this fluid circuit are, as will be shown, the same as those in an electric circuit containing permittance only (Art. 763) on which an alternating e.m.f. is impressed. At the end of each stroke the force exerted by the piston is a maximum (Figs. 446,*IV* and 449,*X*)

but at that instant the piston is reversing in direction and the fluid current —rate of flow— is zero. When the piston is at the center of each stroke, the force impressed by it is zero but the current is a maximum—it is flowing

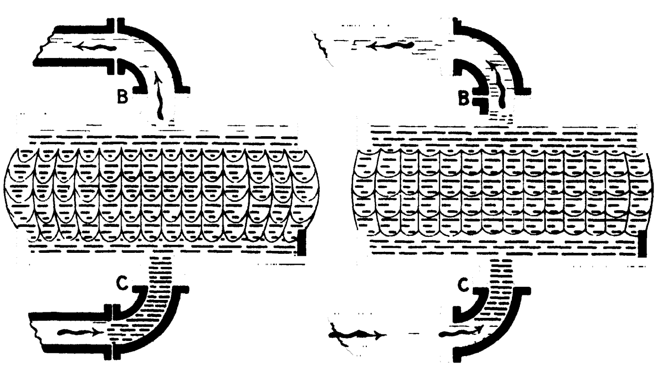


III-60 Degree Phase
(Impressed Pressure is in the Positive Direction and is Increasing, Current in Positive Direction but Decreasing)

IV-90 Degree Phase
(Impressed Pressure is in the Positive Direction and is a Maximum, Current is Zero)

FIG. 446.—Showing conditions in "Permittor" at the 60- and 90-degree instants.

at the maximum rate because the piston is then moving at the maximum rate. Thus, the fluid current in this circuit leads the impelling force impressed by the piston by 90 degrees; Fig. 441 states the situation graphically.



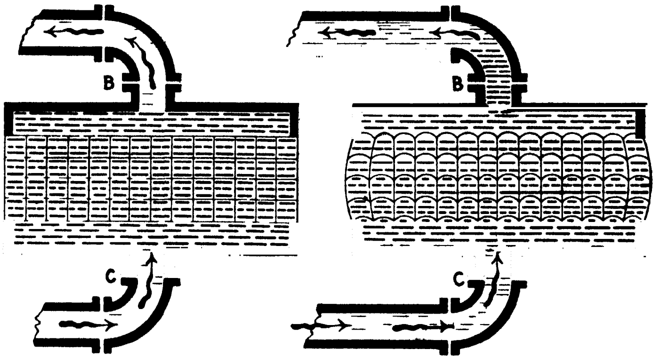
V-120 Degree Phase
(Impressed Pressure in the Positive Direction and Decreasing, Current in Negative Direction and Increasing)

VI-150 Degree Phase
(Impressed Pressure in Positive Direction and Decreasing; Current in the Negative Direction and Increasing)

FIG. 447.—Showing conditions in "Permittor" at the 120- and 150-degree instants.

758. A Counter E.m.f. Is Exerted by Permittance in an Alternating-current Circuit in a somewhat similar manner—but not in the same manner—as an inductance produces a counter e.m.f. in an alternating-current circuit. The dielectric of the permittor

when it is being stressed during the charging period resists the stretching by virtue of its elastic properties. It therefore, in effect, exerts a counter e.m.f. However, the permittance counter

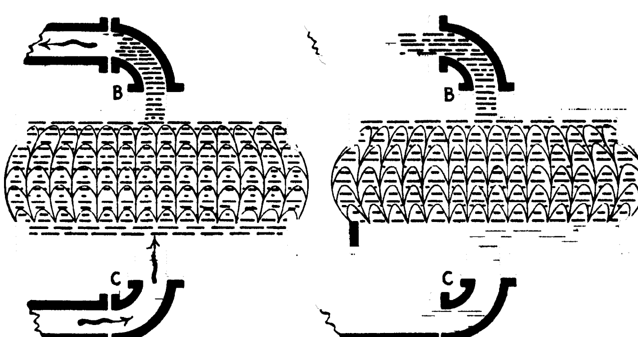


VII-180 Degree Phase
(Impressed Pressure is Zero, Current in the Negative Direction and a Maximum)

VIII-210 Degree Phase
(Impressed Pressure in the Negative Direction and Increasing Current in Negative Direction and Decreasing)

FIG. 448—Showing conditions in "Permittor" at the 180- and 210-degree instants.

e.m.f. attains its maximum value, in each cycle, in the positive direction at the instant at which inductance counter e.m.f. attains its maximum value in the negative direction. Also,



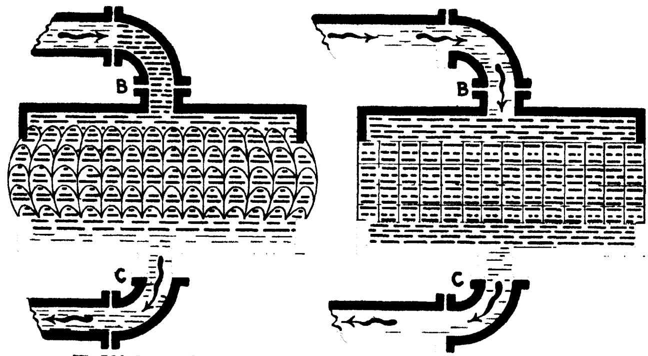
IX 240 Degree Phase
(Impressed Pressure in the Negative Direction and Increasing, Current in the Negative Direction and Decreasing)

X-270 Degree Phase
(Impressed Pressure in the Negative Direction and is a Maximum, Current is Zero)

FIG. 449.—Showing conditions in "Permittor" at the 240- and 270-degree instants.

the converse is true. A consideration of the hydraulic analogy of Figs. 443 to 450 in connection with the graphs of Fig. 441 will verify the above statements.

759. The Counter E.m.f. Due to Permittance in an Alternating-current Circuit Is Always Directly Opposite in Phase and Equal in Value to the Component of the Impressed E.m.f. Required to Neutralize It, as shown in Fig. 451. That this must



XI-300 Degree Phase
(Impressed Pressure in Negative Direction and Decreasing, Current in Positive Direction and Increasing)

XII-360 Degree Phase
(Impressed Pressure is Zero, Current in the Positive Direction and a Maximum)

FIG. 450.—Showing condition in "Permittor" at the 300- and 360-degree instants

be true will be evident from a consideration of the foregoing statements of Arts. 755 and 757. The phase diagram of Fig. 452 indicates vectorially these relations. Hence there is always a difference in phase of 180 degrees between the counter e.m.f.

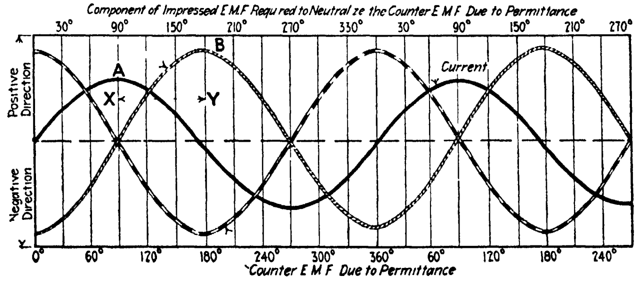


FIG. 451.—Showing how the counter e.m.f. due to permittance directly opposes the impressed e.m.f.

of permittance and the component of the impressed e.m.f. required to neutralize it, the counter e.m.f. leading the component of the impressed e.m.f. In a circuit containing only permittance —if such a circuit could exist—the counter e.m.f. of permittance

e.m.f. would be directly opposite to—would lead by 180 degrees —and would be of the same value as the impressed e.m.f.

760. To Compute the Counter E.m.f. Due to Permittance in an Alternating-current Circuit, the e.m.f. impressed on it being of sine wave form, the value of the current in the circuit is multiplied by the quantity: $1 \div 6.28 \times f \times C$. Hence it follows that:

$$E_C = I \times \left(\frac{1}{6.28 \times f \times c}\right) \qquad \text{(volts)} \tag{212}$$

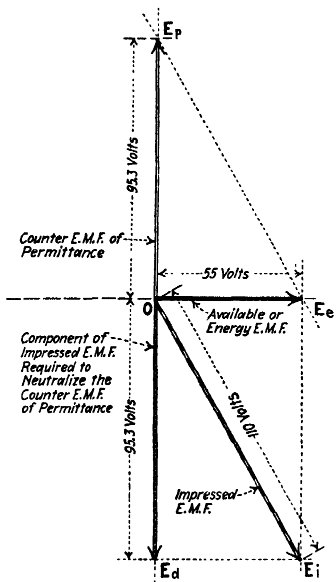


FIG. 452.—Showing relations of e.m.fs. in a circuit containing resistance and permittance.

or, since $2 \times \pi = 6.28$, as it is frequently written:

$$E_C = \frac{I}{2 \times \pi \times f \times c} = \frac{I}{6.28 \times f \times c} \qquad \text{(volts)} \tag{213}$$

Wherein E_C = the counter e.m.f., in volts, exerted by the permittor. I = the effective current, in amperes, in the circuit. f = frequency of the current, in cycles per second. C = permittance or electrostatic capacity of the circuit, in farads. The proof of the above equation follows. Compare it with that of Art. 737 for the counter e.m.f. of inductance.

PROOF.—The "charge" or displacement in a permittor on which an alternating e.m.f. is impressed changes from zero to a maximum during the time

of ¼ cycle (see Fig. 441). That is, the charge changes $Q_m = C \times E_m$ (coulombs) in $1 \div 4 \times f$ sec. Now:

(214) $$Charge = average\ current \times time$$

that is,

(215) $$Q_m = C \times E_m = I_{av} \times \frac{1}{4 \times f} \qquad \text{(coulombs)}$$

or,

(216) $$C \times E_m = \frac{I_{av.}}{4 \times f}$$

Then from the above:

(217) $$I_{av.} = 4 \times f \times C \times E_m \qquad \text{(amp.)}$$

That is, the average charging current equals four times the product of the frequency, permittance and maximum e.m.f.

But (Art. 697):

(218) $$I_m = I_{av.} \times 1.57$$

Then

(219) $$I_m = 4 \times f \times C \times E_m \times 1.57$$

substituting

(220) $$I_m = 6.28 \times f \times C \times E_m \qquad \text{(amp. max.)}$$

Then

(221) $$I = 6.28 \times f \times C \times E \qquad \text{(amp. effective)}$$

and

(222) $$E_c = \frac{I}{6.28 \times f \times C} \qquad \text{(volts effective}$$

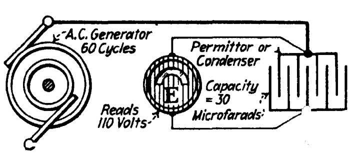


FIG. 453.—Illustrating permittive reactance.

Since the counter e.m.f. exerted by a permittor must always equal the e.m.f. impressed it follows that:

(223) $$E_c = \frac{I}{6.28 \times f \times C} \qquad \text{(volts)}$$

761. Permittive Reactance is the opposition offered by permittance to the flow of alternating current. It is similar to but not the same as inductive reactance (Art. 747). It is measured in ohms. The equation for permittive reactance is:

(224) $$X_c = \frac{1}{6.28 \times f \times C} \qquad \text{(ohms)}$$

Wherein the symbols have the same significance as in the preceding article, except that X_c = permittive reactance, in ohms.

EXAMPLE.—What is the reactance, in ohms, of the permittor shown in Fig. 453? What current will flow in the circuit (assuming that it has no resistance or inductance) there shown, when an effective alternating voltage of 110 is impressed across the permittor? SOLUTION.—Substituting in the above formula for reactance:

$$X_c = \frac{1}{6.28 \times f \times C} = \frac{1}{6.28 \times 60 \times 0.00003} = \frac{1}{0.011304} = 88 \ ohms$$

Then the current, from the Ohm's law formula, would be:

$$I = \frac{E}{X} = \frac{110}{88} = 1.25 \ amp.$$

762. Permittance and Inductance Produce Precisely Opposite Effects in Alternating-current Circuits.—That is, their effects are 180 degrees apart in phase. Hence, one may partially or wholly "neutralize" the other. That this should be true follows from the statements of Art. 754. When inductance and permittance do neutralize in a circuit, then if the e.m.f. in volts, impressed on the circuit, be divided by the resistance, in ohms, of the circuit, the current, in amperes, which will flow in the circuit will be the result. When inductance and permittance neutralize, the resulting condition is called "Resonance," Art. 769. If the permittive effect in a circuit is greater than the inductive effect, then the current in the circuit will lead (Art. 717) the applied e.m.f. But if the inductive effect is greater than the permittive, then the current will lag behind the applied e.m.f.

763. The Current in a Circuit Containing Permittance Only, Leads the Impressed E.m.f. by 90 Degrees as shown in Fig. 441 and 451. When the impressed e.m.f. is increasing in one direction the alternating displacement current is in the same direction—but it is decreasing in intensity. When the impressed e.m.f. is decreasing in one direction, the displacement current is increasing but is in the opposite direction. The displacement or charging current "through" permittance is greatest when the *rate of change* of the impressed e.m.f. is greatest—that is, at the instants at which the impressed e.m.f. is zero. At the instants at which the impressed e.m.f. is a maximum (when its rate of change is zero) the displacement current is zero. In any cycle the displacement current, in a circuit containing only permittance, attains its maximum intensity 90 degrees before the impressed e.m.f. reaches its maximum intensity, thus the current

leads the impressed voltage by 90 degrees. The alternating displacement current lags 90 degrees behind the counter e.m.f. due to permittance as shown in Fig. 451.

764. The Current in an Alternating-current Circuit Containing Only Permittance (Capacitance) and Resistance Always Leads the E.m.f. Impressed by the Generator.—The amount of lead is proportional to the amount of permittance in the circuit. If there is no permittance—only resistance—in the circuit, then there is no lead and the current will then be in phase with the impressed e.m.f. as shown in Fig. 422. In a circuit consisting wholly of permittance—such a circuit, however, is a physical impossibility—the current would lead the impressed e.m.f. by exactly 90 degrees as shown in Fig. 441. With varying proportions of permittance and resistance, the current will lead the impressed e.m.f. by some amount between 90 and 0 degrees. With little permittance in the circuit there will be little lead; with much permittance in the circuit the lead may be almost 90 degrees.

765. Power Is Not Lost in Impelling an Alternating Current in a Circuit Containing Only Permittance.—The reason is this: All (see note below) of the energy expended in displacing the electricity in one direction and in stressing the dielectric of the permittor, as the voltage increases during the first half alternation of a cycle, is returned to the circuit during the last half alternation (when the voltage is decreasing) as the dielectric exerts its elasticity and pulls back into an unstressed condition. Pressure must be exerted by the pump to stretch the "dielectric" cells and displace "electricity" through the "permittor" from the position of Fig. 443 to that of Fig. 446,*IV*. But as the pump pressure decreases (Figs. 447,*V* to 448,*VII*), the stressed "dielectric" returns to its normal condition forcing the "electricity" back again. However, power *is* required to overcome the frictional resistance (if there is any) that the pipe circuit offers to the current of water as it surges back and forth. Similarly, power (watts) is expended in overcoming the resistance (ohms) that any conductor in a permittor circuit offers to the current displaced in the circuit by virtue of the permittor. The power loss in any conductor in which *any* current flows is always $I^2 \times R$ (Art. 167).

NOTE.—It is not strictly true that *all* of the energy expended in displacing the electricity in charging a permittor is returned to the circuit when the

permittor discharges. There are small energy losses, which occur when an alternating current "flows through" a permittance, called "*Dielectric Hysteresis Losses.*" These may be thought of as representing the energy wasted in a sort of intermolecular friction which occurs when the dielectric material is "stretched" and then "returns" to its original condition. The exact nature of these losses is not clearly understood. In any event, dielectric hysteresis losses are relatively very small and consequently are of little importance in ordinary practical work.

766. Elastance is the name which has been given to the reciprocal of permittance. Just as permittance may be measured in farads, elastance may be measured in darafs. A permittor which has a permittance of 10 farads would have an elastance of: $1 \div 10 = 0.1$ darafs. A permittor having a permittance of 0.5 farads would have an elastance of: $1 \div 0.5 = 2$ darafs. The relation between permittance and elastance is analogous to that between conductance and resistance (Art. 130) and permeance and reluctance (Art. 238).

NOTE that "daraf" is "farad" spelled backwards.

SECTION 49

FIGURING ALTERNATING-CURRENT CIRCUITS

767. Susceptance is the name which has been given to the reciprocal of reactance. Just as the reactance of a circuit is a measure of the difficulty encountered in forcing an alternating current through the circuit, susceptance is a measure of the ease with which the current may be forced through the circuit. Reactance is a quantity similar to resistance (Art. 747). Susceptance is one similar to conductance (Art. 130) and it is likewise measured in mhos. A circuit having a reactance of 10 ohms, has a susceptance of: $1 \div 10 = 0.1$ mho. A circuit having a reactance of 0.5 ohms has a susceptance of $1 \div 0.5 = 2$ mhos.

768. The Net Reactance of Any Circuit is Equal to the Sum of Its Permittive Reactance and Its Inductive Reactance.—

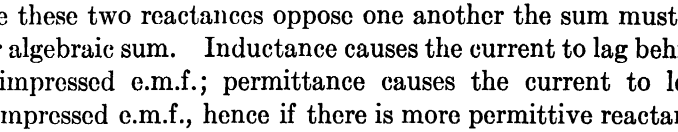


FIG. 454.—Illustrating addition of reactances in series.

Since these two reactances oppose one another the sum must be their algebraic sum. Inductance causes the current to lag behind the impressed e.m.f.; permittance causes the current to lead the impressed e.m.f., hence if there is more permittive reactance in a circuit than inductive reactance the current will lead, and *vice versa*. It follows that:

(225) $$X = X_l - X_c \quad \text{(ohms)}$$

Where X = the total reactance of the circuit, in ohms. X_l = the inductive reactance in the circuit, in ohms. X_c = the permittive reactance in the circuit, in ohms.

NOTE.—Inductive reactance is usually of most importance in circuits except when the voltages involved are very high.

EXAMPLE.—The total reactance of the circuit (which contains only inductance and permittance) of Fig. 454 $= X_l - X_c = 15 - 7 = 8$ ohms.

EXAMPLE.—The reactance of the circuit of Fig. 454,*II* = $X_l - X_c$ = 6 − 23 = − 17 ohms. The minus sign preceding the value 17 indicates that the resultant reactance is permittive, hence will cause the current in the circuit to lead the impressed e.m.f.

769. Resonance is that condition which occurs in an alternating-current circuit when the inductive reactance in the circuit is just equal to, and hence neutralizes, the permittive reactance. Under these conditions the current in the circuit is limited only by the resistance and may therefore attain enormous values. Obviously, resonance occurs when:

(226) $$6.28 \times f \times L = \frac{1}{6.28 \times f \times C}$$

It follows that when resonance occurs:

(227) $$L \times C = \frac{1}{(6.28 \times f)^2}$$

or

(228) $$f = \frac{1}{6.28 \times \sqrt{L \times C}} \text{ (frequency)}$$

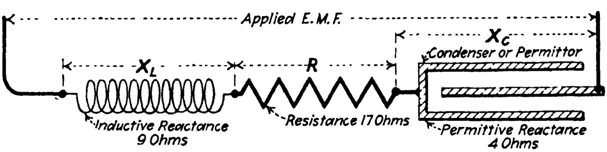


FIG. 455.—Inductive reactance, permittive reactance and resistance in series.

Obviously, resonance can occur in a given circuit only when an e.m.f. of a certain frequency, f, above, is impressed on the circuit.

EXAMPLE.—If an e.m.f. of such a frequency were impressed across the circuit of Fig. 455 so that the permittive reactance of X_c just equalled the inductive reactance of X_l then the current through the circuit would be limited only by the resistance R.

770. Admittance is the name which has been given to the reciprocal of impedance (Art. 748) and it is expressed in mhos. The relation between impedance and admittance is similar to that between resistance and conductance. A circuit which has an impedance of 40 ohms, has an admittance of: 1 ÷ 40 = 1/40 *mho* = 0.025 *mho*. A circuit which has an impedance of 0.2 ohm has an admittance of: 1 ÷ 0.2 = 5 mhos. In the problem of Fig. 456, which will be discussed later, the admittance of the two impedances in parallel is 0 8 mho.

771. The Impedance of a Circuit Containing Only Resistance and Permittive Reactance can be readily computed by drawing a vector diagram as shown at Fig. 457,*II*. Although resistance and permittive reactance may be so associated in a circuit that it is impossible to separate them physically, for the purposes of

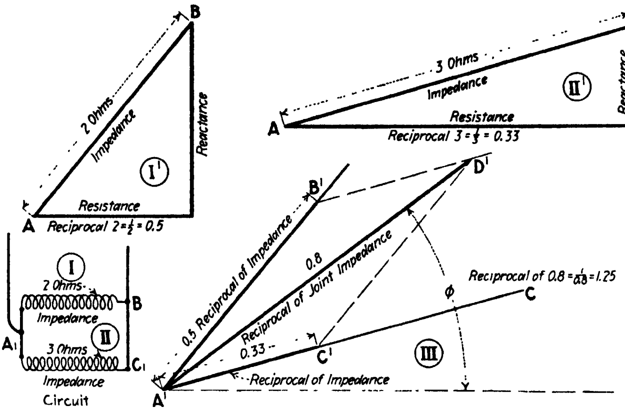


FIG. 456.—Graphic method of determining joint impedance of two impedances in parallel.

computation it may be assumed that they are separate properties, as suggested at *I*. On the basis of the explanation of permittive reactance which has been hereinbefore given, Art. 761, it can be shown that:

$$Z = \sqrt{R^2 + X_c^2} \qquad \text{(ohms)} \tag{229}$$

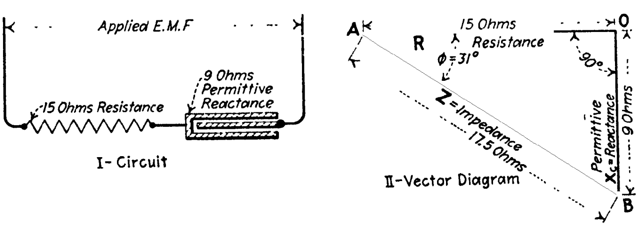


FIG. 457.—Indicating method of computing impedance of resistance and permittive reactance.

EXAMPLE.—To compute the impedance of the circuit of Fig. 457, *I*. The line *OB* is drawn vertically, proportional in length to 9 ohms, to represent the permittive reactance. It is drawn downward to indicate that permittive is the opposite of inductive reactance. Then the line *OA*, to represent the resistance, is laid off at right angles to *OB*. *AB* will then be proportional in length to the impedance of the circuit, which is 17.5 ohms.

The angle ϕ, 31 degrees in this case, is the angle by which the current in the circuit will lead the impressed e.m.f.

772. The Impedance of a Circuit Containing Resistance, Inductance and Permittance in Series, may be readily computed. If one piece of apparatus or a circuit contains resistance, inductance and permittance, it can be assumed that each of these is a separate quantity as shown in Fig. 455. The permittive reactance, X_c, is first subtracted from the inductive reactance X_l which gives the reactance X. Then this reactance is combined with the resistance to obtain impedance, hence:

$$Z = \sqrt{R^2 + (X_l - X_c)^2} \qquad \text{(ohms)} \tag{230}$$

or

$$Z = \sqrt{R^2 + X^2} \qquad \text{(ohms)} \tag{231}$$

EXAMPLE.—What is the impedance of a piece of apparatus or a circuit (Fig. 455) which has a resistance of 17 ohms, an inductive reactance of 9 ohms and a permittive reactance of 4 ohms? SOLUTION.—It may be assumed that the reactances and resistances are separate quantities, as shown in Fig. 455. Then the vector diagram, Fig. 458, is constructed. Draw OR 17 units long to represent the resistance. Draw OL 9 units long to represent the inductive reactance. Draw OC vertically downward 4 units long to represent the permittive reactance. The total reactance is the difference between OL and OC, or OX, which is 5 units long. That is X_l = 5 ohms. Then RX is drawn and it will be proportional in length to the impedance of the circuit, 17.7 ohms in this case.

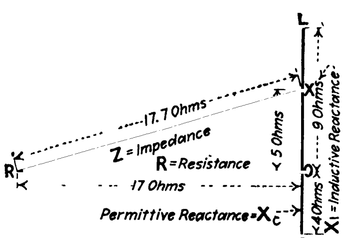


FIG. 458.—Solution of problem of Fig. 455.

NOTE.—For a circuit containing only resistance and permittance—no inductance—it follows, where $1 \div (6.28 \times f \times C)$ = permittive reactance, (Art. 761) that:

$$Z = \sqrt{R^2 + \left(\frac{1}{6.28 \times f \times c}\right)^2} \tag{232}$$

For a circuit containing resistance, inductance and permittance:

$$Z = \sqrt{R^2 + \left[(6.28 \times f \times L) - \left(\frac{1}{6.28 \times f \times c}\right)\right]^2} \tag{233}$$

773. The Joint Impedance of Several Impedances in Series can be computed either graphically or arithmetically as shown in Fig. 459. It is necessary to know the resistance and reactance of each component to obtain their joint impedance because the arithmetical sum of the individual impedances of the com-

ponents is not equal to their joint impedance. The angle ϕ is the angle of lag of the circuit—the angle by which the current will lag behind the impressed e.m.f. The cosine of this angle is, as will be shown later, the power-factor (Art. 783) of the circuit.

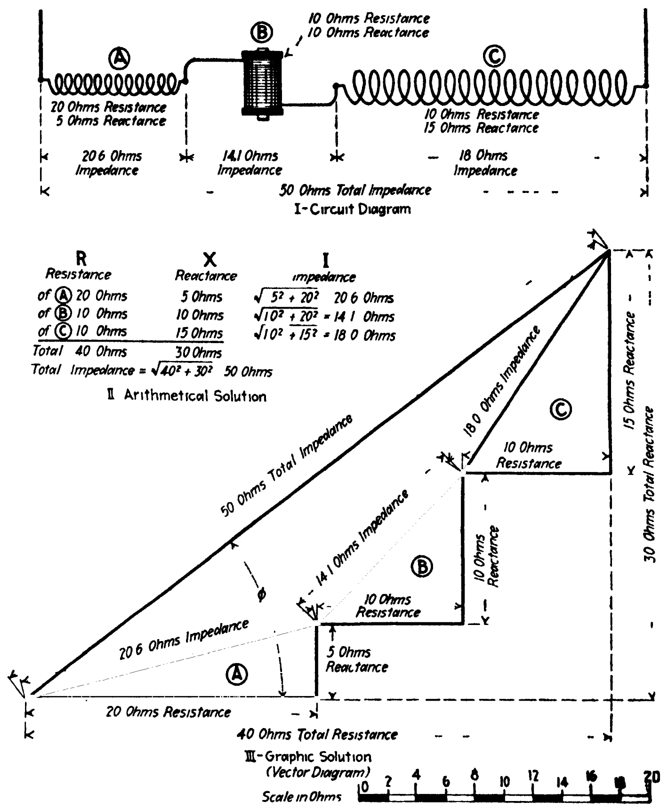


FIG 459.—Method of determining the joint impedance of several impedances in series

774. The Method of Computing the Joint Impedance of a Resistance and an Impedance in Parallel is illustrated in Fig. 460. The resistance, A_1B_1, shown at I is 10 ohms; the impedance, A_1C_1, is 12.8 ohms. The impedance of 12.8 ohms was computed graphically as shown at III. The line $A'B'$ (IV) is drawn horizontally, and in length proportional (to any convenient scale) to the reciprocal—conductance (Art. 130)—of the resistor. Now $A'C'$ is drawn parallel to AC and proportional (to the same

scale as that used for $A'B'$) to the reciprocal of the impedance AC. That is, $A'C'$ represents the admittance (Art. 770) of A_1C_1. The resultant $A'D'$ will be proportional in length to the joint admittance of A_1B_1 and A_1C_1 in parallel. That is, $A'D'$ will be proportional to the reciprocal of the joint impedance.

To obtain the joint impedance, compute the reciprocal of the value represented by $A'D'$, which, in the example illustrated is: $1 \div 0.168 = 5.95$. Hence, the joint impedance of A_1B_1 and A_1C_1 is 5.95 ohms. The angle ϕ in III is the angle by which a current in the circuit L_1L_2 would lag behind the impressed e.m.f.

775. To Compute the Joint Impedance of Two Impedances in Parallel, the method delineated in Fig. 456 may be used. First

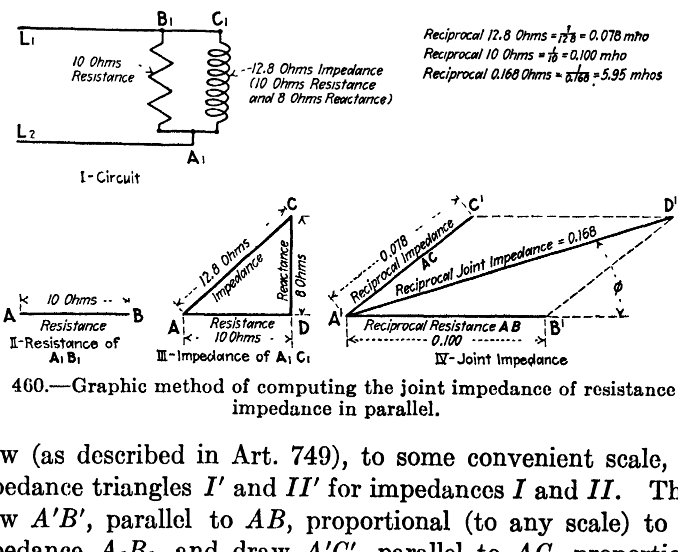


FIG. 460.—Graphic method of computing the joint impedance of resistance and impedance in parallel.

draw (as described in Art. 749), to some convenient scale, the impedance triangles I' and II' for impedances I and II. Then, draw $A'B'$, parallel to AB, proportional (to any scale) to the impedance A_1B_1, and draw $A'C'$, parallel to AC, proportional (to the same scale) to the impedance A_1C_1. The resultant $A'D'$ will be proportional to the *reciprocal* (admittance) of the joint impedance of A_1B_1 and A_1C_1. To obtain the joint impedance, compute the reciprocal of the value represented by $A'D'$, which in the example shown is: $1 \div 0.8 = 1.25$. Therefore, the joint impedance of the two impedances is 1.25 ohms. The angle ϕ is the angle by which the current in leads A_1 and B_1 lags behind the e.m.f. impressed by the generator.

776. Impedance of Motors and Transformers and Energy Resistance are phenomena which should be understood. In a

direct-current circuit which does not contain a motor or other source of counter e.m.f., the current by Ohm's law (Art. 134) always equals the impressed e.m.f. divided by the resistance of the circuit. In alternating-current circuits, as has been shown, this relation seldom exists because the current in such circuits is limited not only by the resistance of the circuit but also by a counter e.m.f. of self-induction (Art. 479). In an alternating-current circuit with which no iron (transformers or motor) is associated, the current with a given impressed e.m.f. is determined by the impedance (Art. 748) of the circuit. That is, the current is determined by the sum of the resistance and reactance.

However, in circuits in which are included transformers, motors or other devices which convert electrical energy, the energy imparted to the circuit is obviously expended in other ways than by merely overcoming resistance. In such circuits the current

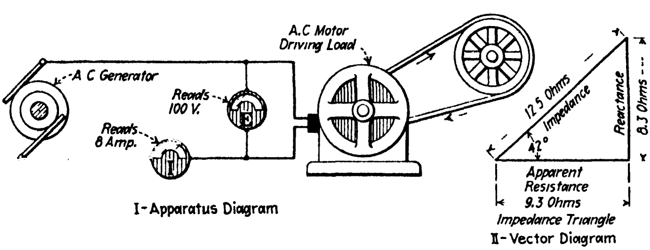


FIG. 461.—Illustrating impedance of motor.

is not equal to the *impressed e.m.f.* ÷ (*resistance* + *reactance*). The e.m.f. impressed on a motor or transformer may be considered as comprising two components: (1) *An induction component* which overcomes the counter e.m.f. of self-induction of the circuit. (2) *An energy component* which does actual work in making the motor or transformer deliver energy and in forcing current through the resistance of their windings.

EXAMPLE.—Consider a motor (Fig. 461) which is driving a load. (Conditions similar to those to be described would obtain for a loaded transformer). If the e.m.f. (100 *volts*) impressed on the motor be divided by the current at some certain load, the result will be the impedance, in ohms, of the motor at that load. Assume 100 volts impressed and a current of 8 amp. Then the impedance of the motor will be: 100 ÷ 8 = 12.5 *ohms*. Assume the angle of lag of the current to be 42 degrees (power-factor = 74 per cent.) and draw the impedance triangle as indicated at *II*. The resistance of the motor from this triangle would appear to be 9.3 ohms. Actually, the resistance of such a motor would be less than 2 ohms. Hence

the balance of this apparent resistance of 9.3 ohms or at least 7.3 ohms would represent not resistance but actual work being done by the motor in pulling the load.

This apparent resistance (9.3 ohms), representing partly real resistance and partly the doing of work, may be called *energy resistance.* Hence, in simple circuits without iron, such as overhead line and interior-mining circuits, the components of the impedance of the circuit consist only of reactance and resistance. But if transformers or other devices containing iron are introduced, then the apparent impedance is composed of reactance and energy resistance.

NOTE.—Impedance of isolated circuits without iron, like overhead lines, is practically the same for all current strengths—but in circuits with iron, impedance may vary with the current. Voltage drop in an electrical energy transmission circuit can not be correctly estimated on the basis of the line impedance alone, since the drop may not correspond in phase with the total impedance of the circuit.

SECTION 50

POWER AND POWER-FACTOR IN ALTERNATING-CURRENT CIRCUITS

777. Power in Alternating-current Circuits is determined by applying the same general laws (Art. 164) which relate to direct-current circuits. The power in any electric circuit (alternating-current or direct-current) is, at any instant, always equal, in watts, to the *current* at that instant multiplied by the *voltage* (at that instant) which impels the current. That is:

(234) $$P_i = I_i \times E_i \qquad \text{(watts)}$$

Wherein P_i = instantaneous power of the circuit, in watts at a given instant, I_i = instantaneous current in the circuit in amperes at the same instant, E_i = instantaneous voltage, in volts, at the same instant, it being understood that this E represents the available or energy voltage which actually impels the current at the given instant. In a direct-current circuit, containing no source of counter e.m.f. the voltage impressed on the circuit (except during the *transient* intervals, Art. 481) is the voltage which impels the current. In an alternating-current circuit, the voltage impressed on the circuit may or may not be the one which impels the current: (1) If the circuit contains no inductance nor capacity, that is, if the current is in phase with the impressed e.m.f. (Art. 732), the impressed voltage is the one which impels the current. (2) If the circuit contains inductance or permittance (capacity) or both, that is, if the current is not in phase (Art. 685) with the impressed e.m.f., some voltage other than the impressed voltage will impel the current.

778. Power Taken by an Alternating-current Circuit in Which the Current Is in Phase with the Impressed E.m.f. (this condition obtains in a circuit containing resistance only) is equal to the product of: *volts* $\times$ *amperes*. That is:

(235) $$P = E_E \times I_E \qquad \text{(watts)}$$

and

(236) $$E_E = \frac{P}{I_E} \qquad \text{(volts)}$$

also

$$I_E = \frac{P}{E_E} \qquad \text{(amp.)} \tag{237}$$

Wherein P = power taken by the circuit, in watts. E_E = e.m.f. effective, impressed on the circuit, in volts. I_E = current, effective, in the circuit, in amperes. Fig. 462 illustrates conditions in such a circuit. If the current at any instant, for example OA, be multiplied by the impressed e.m.f. (OB) at the same instant, then product (OC) will be the power at that instant. By making this computation for a number of different instants in the cycle, a corresponding number of points in the power curve may be obtained. The power curve can then be plotted

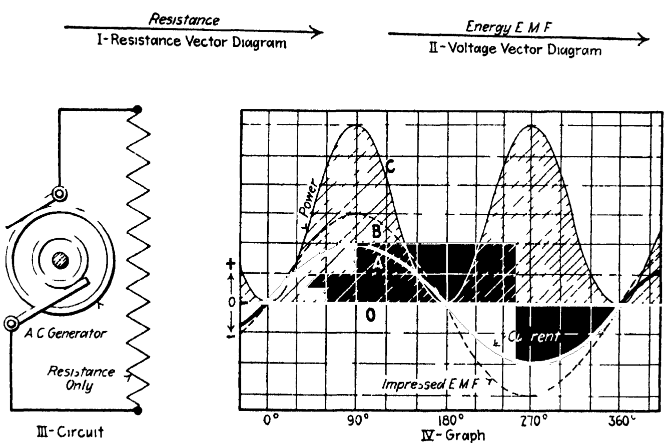


FIG. 462—Illustrating conditions for 100 per cent. power-factor. Circuit composed wholly of resistance and current is in phase with impressed e.m.f.

through them as shown. The total power taken by the circuit during any alternation is proportional to the area (shaded in the illustration) within the power loop of that alternation.

NOTE.—All of each power loop will lie above the zero reference line—that is the power will be positive—because when two + quantities or two − quantities are multiplied together the result will be a + quantity. By *positive power* is meant power which is delivered by the generator (or other source of e.m f.) to the circuit. Sometimes a circuit may, as will be shown, deliver power back to the source of power which feeds it; such is called negative power.

EXAMPLE.—What power is being taken by the incandescent-lamp load of Fig. 463,*I* (all incandescent-lamp loads are practically non-inductive) which is being served by an alternating-current generator? The voltmeter

reads 200 volts and the ammeter reads 24 amp. SOLUTION.—From the above: $P = E \times I$, hence $P = 200 \times 24 = 4{,}800$ *watts* or 4.8 kw. Therefore, the generator is delivering 4.8 kw.

EXAMPLE.—The alternating-current wattmeter of Fig. 463,*II* indicates that the non-inductive iron is taking 660 watts. The current, as shown by the ammeter, is 6 amp. What is the voltage impressed across the iron? SOLUTION.—As outlined above, $E = P \div I$, hence $E = 660 \div 6 = 110$ *volts*, which is the pressure impelling the current in the iron.

EXAMPLE.—The incandescent lamp of Fig. 463,*III* is taking 0.5 amp. at 110 volts. What is its power consumption? SOLUTION —$P = E \times I = 110 \times 0.5 = 55$ *watts.*

779. Power Taken by an Alternating-current Circuit in Which The Current Is Not in Phase with the Impressed E.m.f. will not, as will be shown, be equal to the product of *impressed volts* $\times$ *amperes.* In any alternating-current circuit which contains inductance, and consequently reactance, the current lags (Art.

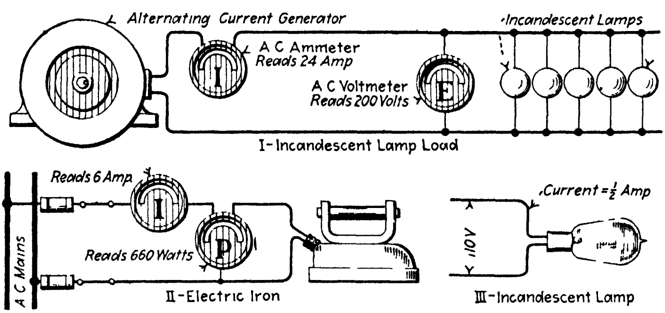


FIG. 463.—Examples in computing alternating-current power and power factor.

716) behind the impressed e.m.f. Consider Fig. 464 illustrating conditions in a circuit which has just enough reactance that the current lags behind the impressed e.m.f. by 30 degrees. If the instantaneous currents at the different instants are multiplied by the instantaneous e.m.fs. at the same instants and a power curve is plotted through points representing these products, it will have the form shown in *IV*. Note that a portion of the power curve N_1 and N_2 lies below the zero line, because when a + quantity is multiplied by a − quantity a − quantity results. The areas of these little loops (N_1 and N_2) represent *negative power* or power which is returned to the generator by the line and hence is not available for doing work in the circuit. Note that no power is required to "overcome" reactance. The vector diagrams *I* and *II* show the voltage and resistance relations, the

angle of lag being the same in these as in the graph. It is evident then from a consideration of Fig. 464 that the power in an inductive alternating-current circuit is not equal to the product of

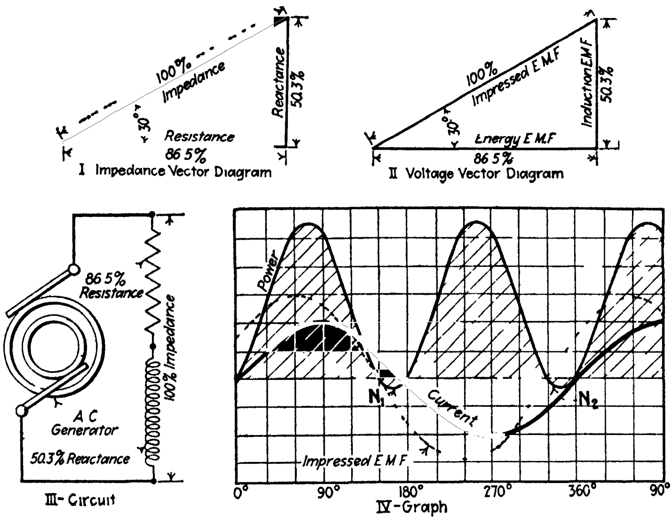


Fig. 464.—Illustrating conditions for 86.5 per cent power factor. Current lags 30 degrees behind impressed e.m.f.

effective volts × *effective amperes* because this product would not take into consideration the negative power (N_1 and N_2, Fig. 464) which is not available for doing work in the circuit.

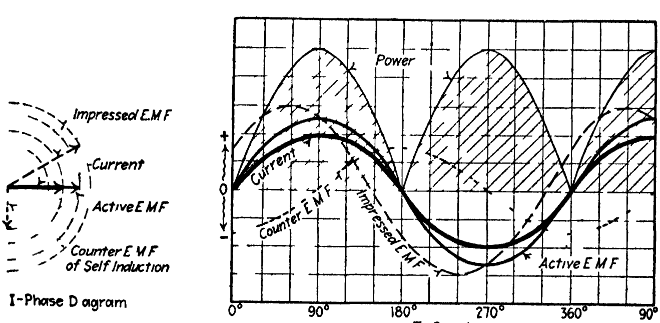


Fig. 465.—Showing how A.C. power is proportional to the product of *available or energy e.m.f. and current.*

780. The Product of the Available or Energy E.m.f. Times the Current Gives Real Power.—Thus, in Fig. 465, if the instan-

taneous energy or active e.m.f. at different instants be multiplied by the instantaneous currents at different instants, the products will be the instantaneous power at those instants. Points can thus be determined through which the power curve can be completed as shown at *II*. The shaded areas within loops represent, and are proportional to, the power taken by the circuit because the current is always in phase with the energy or active e.m.f. The areas of the loops in Fig. 465,*II*, are equal to the shaded areas above the reference line in Fig. 464.

NOTE.—Ordinarily in alternating-current circuits the value of the energy e.m.f. is not known; only the values of the impressed e.m.f. and the current are known. But with these two values, and knowing the angle by which the current lags behind the impressed e.m.f., the power taken by the circuit can be readily computed, as will be shown.

781. The Computation of the Power Taken by an Inductive Circuit may be made by using a vector diagram or a phase diagram, Fig. 466. The e.m.f. impressed on any inductive circuit is "used up" in two ways (Art. 745): *First*, to impel the current in the circuit through its resistance and to force the current to do work; this part of the impressed e.m.f. represents actual power (see also Fig. 431). *Second*, to overcome or neutralize the counter e.m.f. of induction in the circuit; this portion of the impressed e.m.f. does not represent actual power. Thus (Fig. 466) if E be laid out to scale to represent the effective impressed e.m.f., it may be resolved into two components: (1) An induction component E_1, which merely neutralizes the counter e.m.f. of self-induction and hence does no real work and (2) the energy component E_A, which impels current through resistance and also does work. The energy component must be in phase with the current, hence lags behind the impressed e.m.f. E by the angle ϕ. The vectors E_1 and E_A will be at right angles to one another because the current always lags 90 degrees behind the component of the voltage which neutralizes the counter e.m.f. of self-induction. Since E_A is the only component available for doing work, it follows that the average power taken by the circuit is equal to the product of *effective current* $\times$ *the available*

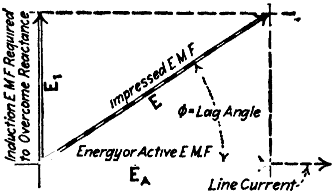


FIG. 466.—Phase diagram showing voltage relations in an inductive circuit.

or energy component of the effective e.m.f. (of the component of the e.m.f. in phase with the current) hence:

(238) $$P = E_A \times I \quad \text{(watts)}$$

but

(239) $$E_A = E \times \cos\phi \quad \text{(volts)}$$

hence

(240) $$P = I \times E \times \cos\phi \quad \text{(watts)}$$

or

(241) $$P = (I \times E) \times \cos\phi \quad \text{(watts)}$$

Wherein P = the power in watts taken by the circuit. I = effective current in the circuit. E = effective e.m.f. impressed on the circuit. $Cos\ \phi$ = cosine of the angle by which the current lags behind the impressed e.m.f.

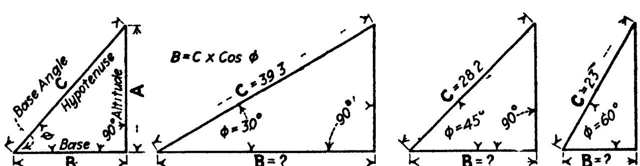


FIG. 467.—The cosine and problems involving its application.

NOTE.—Cos is the abbreviation for cosine. If the length of the hypotenuse of any right-angled triangle be multiplied by the value called the cosine of the angle (base angle) the length of the base will be the result. That is (Fig. 467,*I*) $B = C \times \cos\phi$. Obviously, there is a different cosine value for every different angle. For example, cos 30 degrees = 0.866, cos 45 degrees = 0.707, cos 60 degrees = 0.500, etc. Complete tables of cosine values are given in handbooks and trigonometries. An abbreviated table is given in the author's AMERICAN ELECTRICIANS' HANDBOOK. See for comparison Art. 516 in regard to sines.

EXAMPLE.—What is the length of base of the triangle of Fig. 467,*II*? SOLUTION.—From a table of trigonometric functions it is found that cos 30 degrees = 0.866, hence $B = C \times \cos\phi = 39.3 \times 0.866 = 34$, which is the length of base.

EXAMPLE.—The cos of 45 degrees = 0.707, hence the length of base of *III* = 28.2 × 0.707 = 20.

EXAMPLE.—The cos of 60 degrees = 0.500. Hence, in *IV*, B = C × cos ϕ = 23 × 0.5 = 11.5.

EXAMPLE.—In the circuit shown in Fig. 438 the relation of reactance to resistance is such that the energy e.m.f. as shown at *III*, lags just 30 degrees (as may be determined by constructing the impedance or voltage-vector

diagrams, Art. 749) behind the impressed e.m.f. Now cos 30 degrees = 0.866. Hence $P = E \times I \times cos\ \phi = 100 \times 50 \times 0.866 = 4{,}330$ watts = 4.3 *kw.*, which is the power being taken by the circuit shown.

EXAMPLE.—Referring to Fig. 437 the lag angle ϕ = 60 degrees. Cos 60 degrees = 0.50. Therefore, the power being taken by this circuit = $P = E \times I \times cos\ \phi = 100 \times 50 \times 0.50 = 2{,}500$ watts = 2.5 *kw.*

EXAMPLE.—Similarly, in the circuit of Fig. 436, ϕ = 75 degrees, Cos 75 degrees = 25.9. Hence, in this circuit, $P = E \times I \times cos\ \phi = 100 \times 50 \times 25.9 = 1{,}295$ watts = 1.3 *kw.*

NOTE.—In all of the last three examples the current and the impressed e.m.f. are the same but the power taken by the circuits decreases as their inductances—or reactances—increase.

782. Power-factor is the name given to that quantity by which the product of effective volts and effective amperes in a circuit must be multiplied to obtain the true power in watts taken by the circuit. From equation (240) above it is evident that (for a circuit containing inductance) the power-factor is equal to the cos of the angle by which the current lags behind the impressed e.m.f. That is, *power-factor* = *cos* ϕ. Hence, the greater the amount of inductance in a circuit, the lower will be the power-factor. Power-factor is expressed as a percentage. Refer to the author's AMERICAN ELECTRICIANS' HANDBOOK for a further discussion of the subject of power-factor.

NOTE.—Obviously the nature of the load on a circuit determines its power-factor. In practice the average values of power-factors for circuits with different kinds of loads will be approximately as follows:

Incandescent lighting—no motors,.............	95 per cent.
Lighting and motors,.........................	85 per cent.
Motors only,.................................	80 per cent.

If a circuit contains resistance only, its power-factor will be 100 per cent. and in such a circuit *volts* $\times$ *amperes* = *true power* (Art. 732). Power-factor can not be greater than 100 per cent. See the author's AMERICAN ELECTRICIANS' HANDBOOK for a much more complete table of power-factors of circuits having loads of different characteristics.

783. The Formulas for Power-factor are for single-phase circuits (since cos ϕ = *power-factor* which can be expressed merely as *p.f.*) these:

(242) $$P = I \times E \times p.f. \quad \text{(watts)}$$

(243) $$I = \frac{P}{E \times p.f.} \quad \text{(amp.)}$$

(244) $$E = \frac{P}{I \times p.f.} \quad \text{(volts)}$$

(245) $$p.f. = \frac{P}{I \times E} \quad \text{(power-factor)}$$

Wherein all of the symbols have the meanings given above.

Examples of circuits of 100 per cent. power-factor are given in Fig. 463.

EXAMPLE.—What is the actual power being taken by the multiple-arc-lighting circuit of Fig. 468. The impressed e.m.f. is 110 volts. The cur-

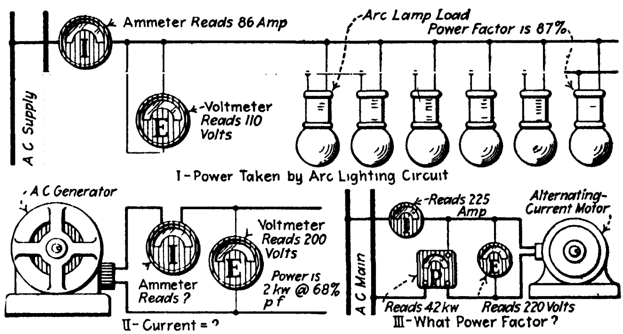


FIG. 468.—Problems involving power factor.

rent is 86 amp. and the power-factor of the load is 87 per cent. SOLUTION.—Substitute in the formula (242): $P = I \times E \times p.f. = 86 \times 110 \times 0.87 =$ 8,230 watts = 8.2 kw.

EXAMPLE.—The alternating e.m.f. impressed on the alternating-current circuit of Fig. 468, *II* is 200 volts. It is known that the circuit is taking

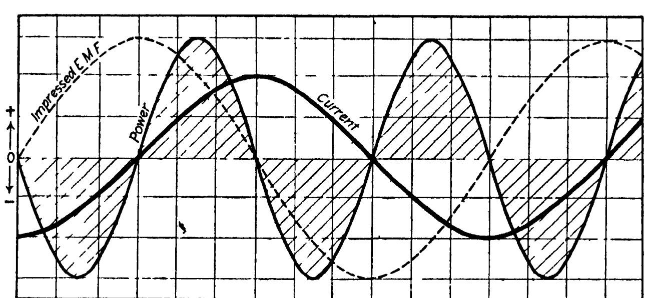


FIG. 469.—Illustrating conditions for zero (0) power factor. Circuit is wholly inductive and current lags 90 degrees behind impressed electromotive force.

2 kw. at 68 per cent. power-factor. What is the current? SOLUTION.—2 kw. = 2,000 watts. Then substituting in the formula (243): $I = P \div E \times p.f. = 2{,}000 \div (200 \times 0.68) = 14.7$ amp.

EXAMPLE.—In an alternating-current circuit feeding a motor (Fig. 468, *III*) the following readings are taken: Power = 42 kw. E.m.f. = 220 volts. Current = 225 amp. What is the power-factor? SOLUTION.—42 kw. =

42,000 watts. Substitute in the formula (245): $p.f. = P \div (I \times E) =$ $42{,}000 \div (225 \times 220) = 0.85 = 85$ per cent. power-factor.

784. The Power-factor of a Wholly Inductive Circuit would, if such a circuit could exist, be zero. Fig. 469 illustrates graphically the conditions in such a circuit. Note that the area of the positive power loops above the reference line is the same as that of the negative power loops below the reference line, hence the net power taken by the circuit is zero.

NOTE.—The graph of Fig. 469 is of theoretical interest only, inasmuch as it is apparent that no actual circuit could exist without some resistance. Some circuits, however, are almost wholly inductive and therefore the conditions in them approximate those delineated in Fig. 469. A circuit feeding the primary of a transformer, the secondary of which is open, is almost wholly inductive and the same is true of a circuit serving an induction motor operating at no load. It follows that the currents in such circuits are largely "wattless" (Art. 785).

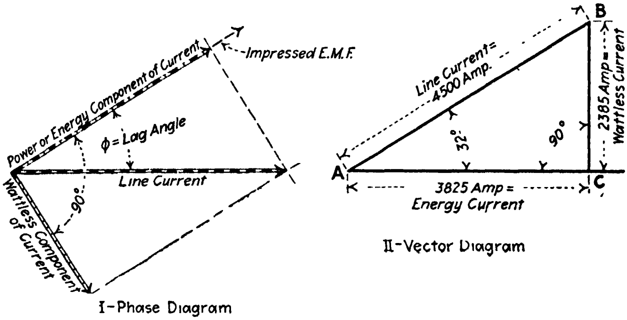


FIG. 470.—Illustrating the meaning of the term "wattless current."

785. Wattless Current is the name given to that part, portion or component of an alternating current which may, when the situation is viewed in one way, be considered as being productive of no actual work. There is only one current—the line current—in a circuit and it is always in phase with the available or energy e.m.f. (Art. 735) but it is sometimes convenient to assume that this current comprises two portions or "components": (1) *the power or energy component*, in amperes, which is of such value that when it is multiplied by the impressed e.m.f. in volts, the product will be the actual watts power taken by the circuit; (2) *the "wattless" component*, in amperes, which, when multiplied by the impressed e.m.f. in volts, will give the apparent power or volt-amperes taken by the circuit. Thus, Fig. 470,*I* shows

how the line current may be resolved graphically into so-called energy and wattless components; which must, because of their definitions, differ in phase by 90 degrees. Refer also to Fig. 471. Furthermore, the energy component, to satisfy its definition as given above, is in phase with the impressed e.m.f. The angle between the energy component and the line current is—because the line current is always in phase with the energy component of e.m.f.—the same as the angle by which the line current lags behind the impressed e.m.f.

EXAMPLE.—Fig. 470,*II* is a vector diagram illustrating the relation between the line current, "wattless current" and "energy current" for the problem of Fig. 472. The line-current vector *AB* was drawn proportional in length to 4,500 amp. *AC* was laid off 32 degrees (this being the lag angle) below *AB* and of indefinite length. *BC* was dropped from *B* perpendicular to *AC*. Then the length *AC*, which scales 3,825 amp. represents the "energy" current and *CB*, scaling 2,385 amp., represents the "wattless" current. *AC* could have been obtained by multiplying *AB* by the cosine (note under Art. 781) of 32 degrees. *BC* could have been obtained by multiplying *AC* by the sine (Art. 516) of 32 degrees.

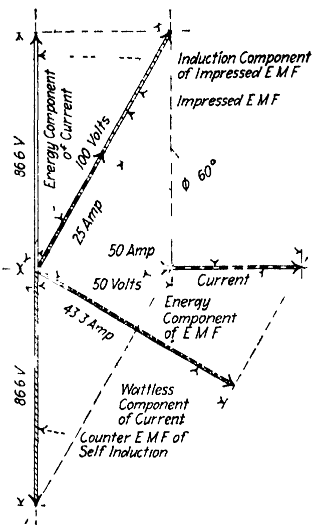


FIG. 471.—Showing vector relations between e.m.f. and current components of the circuit shown in Fig. 429.

786. The Effect of Low Power-factor in a constant-potential circuit is to increase the current necessary for the transmission of a given amount of power over that current which would transmit the same power in a circuit of unity power-factor (a non-inductive circuit). That is, in circuits of low power-factor the "wattless" current is considerable. This excess of current does not, in itself, represent an additional expenditure of energy, that is, it does not require more coal burned under the boilers. It does, however, involve slight additional energy expenditure because it increases the $I^2 \times R$ power loss in the conductors which it traverses. It has the further undesirable feature of decreasing, because of this same I^2R heating effect, the effective capacities of the generators. See the AMERICAN ELECTRICIANS' HANDBOOK for further information.

787. Low Power-factor May Be Corrected by the installation of synchronous motors or synchronous condensers. Low power-factor in practice is frequently due to underloaded induction motors.

788. The Practical Determination of the Power-factor of a Circuit may be made with an ammeter, a voltmeter, and a wattmeter. Alternating-current wattmeters always at any given instant indicate the product of the *average instantaneous current in a circuit* × *the average instantaneous energy e.m.f.* at that instant, hence, they indicate true power. Ammeters and voltmeters indicate effective values and do not show anything in regard to phase relations. With 20 amp. in a circuit, the ammeter will read 20 amp. regardless of whether the current reaches its maximum and intermediate values at the same instant as does the voltage. That is, ammeters and voltmeters take no cognizance of phase relation. Hence, the product of volts and amperes as indicated by these instruments is (in circuits containing inductance or permittance) apparent power taken by the circuit. The true power may be determined from wattmeter readings. Then the power-factor can be readily computed.

EXAMPLE.—Fig. 468,*III* and the accompanying solution gives an example of the foregoing.

EXAMPLE.—Fig. 472 shows an alternating-current circuit serving a mixed motor and lighting load. The power as indicated by the wattmeter, taken by the circuit is 420 kw. The impressed e.m.f. is 110 volts and the current is 4,500 amp. What is the power-factor? SOLUTION.—*Kilovolt-amperes* = (*volts* × *amperes*) ÷ 1,000 = (110 × 4,500) ÷ 1,000 = 495 kva. Now from equation (249): *p.f.* = *kw.* ÷ *kva.* = 420 ÷ 495 = 0.85 = 85 per cent. power-factor.

789. A Vector Diagram Showing the Relation of True Power to Apparent Power may be plotted as suggested in Fig. 473. The general method involved is the same as that used in drawing impedance and voltage vector diagrams.

EXAMPLE.—In the problem of Fig. 472 the apparent power was shown to be 495 kva. and the true power 420 kw. Hence if a right-angled triangle (Fig. 473) be drawn so that the base is porportional in length to the true power, 420 kva., and the hypotenuse so that it is proportional in length to the apparent power, 495 kw., the angle ϕ between them will be the angle by which the current lags behind the impressed e.m.f. The cosine of this angle will then be the power-factor of the circuit. In the example illustrated the angle ϕ is found to be 32 degrees and cos 32 degrees is, by referring to a table, found to be 0.85. Hence the power-factor of the circuit is 85 per cent.

790. True Watts and Apparent Watts and True Power and Apparent Power are terms that are frequently used. The true watts or true power delivered to a circuit is the actual power P (as indicated by a wattmeter) which is being consumed in a circuit. True power may be expressed either in watts or in kilowatts. The apparent watts or power (which is always

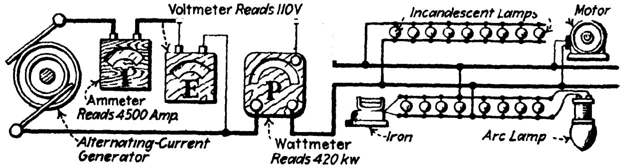


FIG. 472.—Determination of the power factor of an alternating-current circuit.

greater than the actual power in a circuit containing inductance or permittance,) is the product of the *effective voltage impressed* on the circuit times the *effective current* in the circuit. Apparent power may be expressed in volt-amperes or in kilovolt-amperes. Hence it follows from equation (242) since $I \times E =$ *apparent watts,* and $\cos \phi =$ power-factor (the following applies specifically to single-phase circuits):

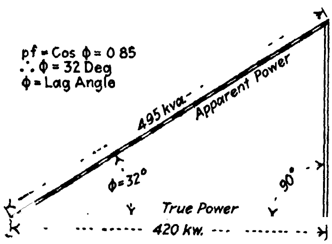


FIG. 473.—Vector triangle indicating relation of true to apparent power for problem of Fig. 472.

(246) $\quad$ *true watts* = *apparent watts* × *power-factor.*

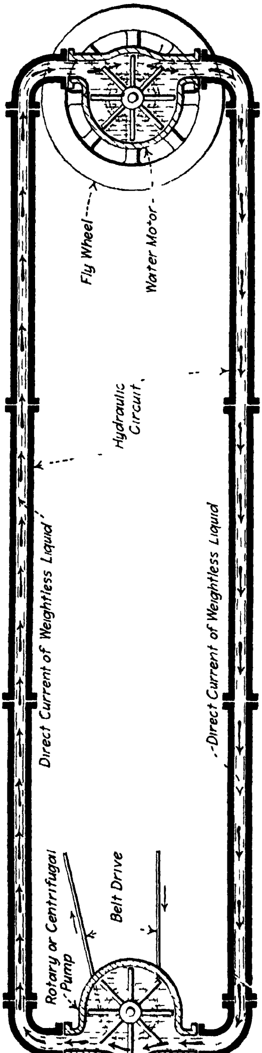
(247) $\quad$ *watts* = *volt-amperes* × *power-factor.*

(248) $\quad$ *kilowatts* = *kilovolt-amperes* × *power-factor.*

Therefore, power-factor equals the ratio of true watts to apparent watts, that is:

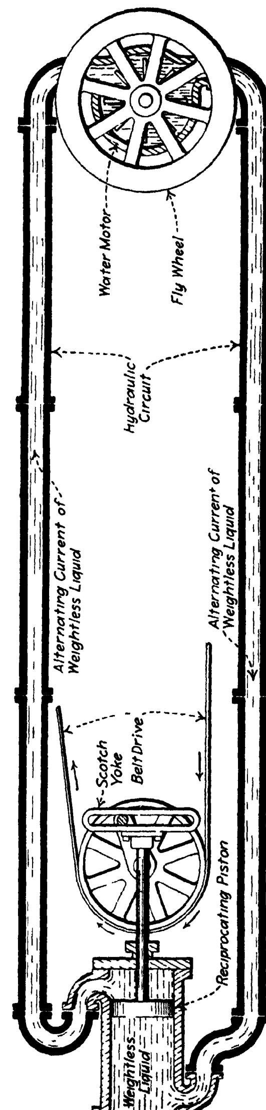
$$(249) \quad \textit{power-factor} = \frac{\textit{true watts}}{\textit{apparent watts}} = \frac{\textit{watts}}{\textit{volt-amperes}} = \frac{\textit{kilowatts}}{\textit{kilovolt-amperes}} = \frac{\textit{kw.}}{\textit{kva.}}$$

NOTE.—The above may be graphically expressed by a right-angled triangle, Fig. 473, in which the hypotenuse represents watts, true watts, kilo-



I-Analogy to Inductance in a Direct Current Circuit

FIG. 474.—Analogy to inductance in a direct-current circuit.



II-Analogy to Inductance in an Alternating-Current Circuit

FIG. 475.—Analogy to inductance in an alternating-current circuit.

watts, or kw., and the base represents, respectively, apparent watts, volt-amperes, kiolvolt-amperes, or kva. Then the base angle, ϕ, between them

will be the lag angle. The cosine of this angle, ϕ which is numerically equal to the ratio between the hypotenuse and the base, is the power-factor.

791. Power-factor Due to Combined Inductance and Permittance in a circuit can be computed on the basis of the method suggested in Fig. 458.

NOTE.—A hydraulic analogy to the effect of inductance in a direct-current circuit is suggested in Fig. 474, the inertia of the fly-wheel being analogous to the inductance of an electric circuit. The flywheel, by virtue of the blades of the water motor to which it is attached, tends to oppose the starting of a flow of current in the circuit. That is, it introduces a counter pressure for the period during which the flow of water is being started. After the flywheel has attained its speed, that is, when the flow of water has attained its normal value, the only counter flow then offered is due to the frictional resistance which is analogous to electrical resistance. Similarly, if the belt were thrown off the centrifugal pump, so that it no longer exerted an impelling force in the circuit, the inertia of the flywheel acting through the blades of the water motor would continue the flow of water in the circuit, after the driving power had been discontinued from the pump. Note that the inertia of this hydraulic circuit is not due to the current of liquid in it, because the liquid is supposed to be weightless.

NOTE.—A hydraulic analogy to inductance in an alternating-current circuit is illustrated in Fig. 475. The inertia of the flywheel acting through the blades of the water motor tends to oppose any change in the rate of flow of the weightless liquid, which is analogous to electricity. If the rate of flow tends to increase, the inertia of the flywheel opposes it. On the other hand, if the current tends to decrease or change in direction, the flywheel acting through the water-motor blades also opposes such changes.

792. Power-factor Due to Permittance or Capacitance is a quantity that applies to permittive circuits in a way similar to that in which power-factor, due to inductance, applies to inductive circuits. However, since the charging current due to permittance is leading, the power-factor due to permittance is a leading power-factor, while that due to inductance is a lagging power-factor.

NOTE.—Leading power-factors of appreciable value are seldom encountered in circuits excepting these operating at relatively high voltages. Leading power-factors can be produced in a circuit by operating an over-excited synchronous motor or "condenser" in the circuit. Hence with a synchronous motor or condenser lagging power-factor in a circuit can be neutralized.

SECTION 51

POLYPHASE CIRCUITS AND SYSTEMS

793. A Polyphase System is one of more than one phase (Art. 685). A polyphase circuit is one for transmitting electrical energy in a polyphase system. There are two important polyphase systems: (1) the two-phase and (2) the three-phase.

NOTE.—Polyphase systems are used for two reasons: (1) because they are more economical than the single-phase (Art. 687), and (2) because induction motors, which are of very simple, rugged and reliable construction —they require no commutators—can be used with them.

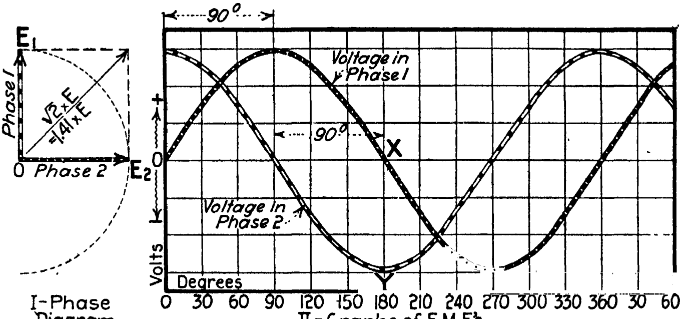


FIG. 476.—Phase diagram and graphs for the e.m.fs. produced by a two-phase alternating-current generator.

794. Two-phase or Quarter-phase is a term (A. I. E. E. STANDARDIZATION RULES) which characterizes the combination of two circuits, each energized by alternating e.m.fs. which differ in phase by a quarter of a cycle, *i.e.*, by 90 degrees (see Fig. 476). Hence, with a two-phase e.m.f. or two-phase current, there are at any instant two "phases" (Art. 685). While two-phase systems are not used as frequently as formerly, an understanding of the principles underlying them is essential.

795. The Production of a Two-phase E.m.f. obviously consists in the generation of two alternating sine-wave e.m.fs., 90 degrees apart, as shown in Fig. 476. Each of these e.m.fs. may, as shown in Fig. 477, be impressed on its own separate cir-

cuit. A single-phase e.m.f. (Art. 687), may be produced by rotating a suitably arranged magnetic field within a properly disposed core on which is wound the alternating-current armature winding. In Fig. 478 this construction is diagrammatically indicated. Two, single-phase alternating-current generators could be made to produce a two-phase e.m.f. if their rotating parts were firmly keyed to the same shaft so that the 90-degree phase relation would be preserved. However, it is more economical to arrange both of the phase windings on one frame, that is, to incorporate them in one machine, rather than to use two frames. With two-phase, as with single-phase, alternating-current generators there are two general types: revolving-armature generators, Fig. 479; and revolving-field generators, Fig. 477.

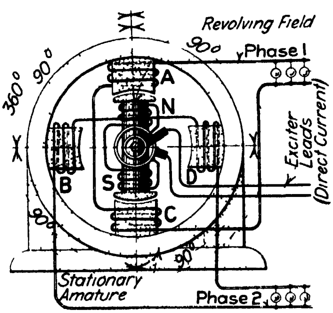


FIG. 477.—Elementary two-pole, two-phase, revolving-field alternator.

796. Really Two Separate Single-phase E.m.fs. Are Produced by a Two-phase Generator.—These may, as above noted, be used to impel two different currents in two distinct circuits. However, it is advisable sometimes to combine or interconnect the two circuits in a manner which will be described later.

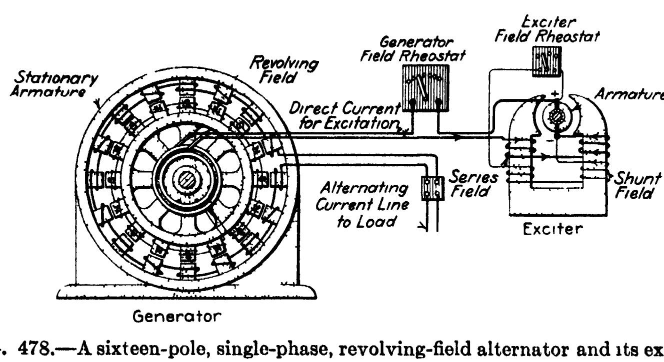


FIG. 478.—A sixteen-pole, single-phase, revolving-field alternator and its exciter.

797. A Revolving-armature, Two-phase Generator is shown diagrammatically in Fig. 479. If the armature winding of any

direct-current generator is tapped at four points 90 electrical degrees apart, as shown in Fig. 479,*I*, and each of these taps is connected to a collector ring, an e.m.f. will be impressed across each of the pairs of rings, which will differ in phase by 90 degrees from the e.m.f. impressed on the other pair of rings. This follows from the principles outlined in Arts. 702 and 703. Hence a bipolar direct-current generator can be converted into a two-phase alternating-current generator by arranging on, but insulating from, its commutator four collector rings. Each of these collector rings is connected to a commutator bar by a screw passing through the ring and into the bar. However, the four bars must be 90 degrees apart in the commutator as shown in Fig. 479,*II*. Revolving-armature, alternating-current generators are seldom used except for machines of very small capacity.

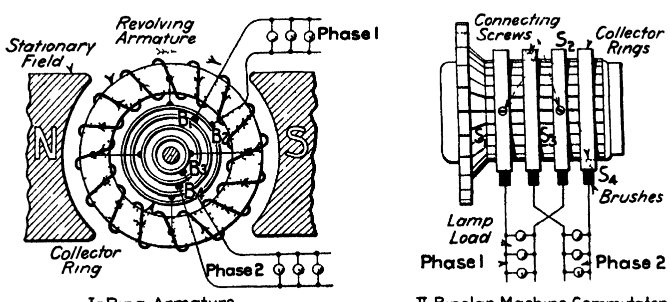


FIG. 479.—Illustrating methods of obtaining two-phase currents from a bipolar, revolving-armature generator

However, the principle involved is frequently utilized for rotary converters.

798. Revolving-field, Two-phase Generators are the most frequently used. Fig. 477 illustrates the principle. It is obvious that if the field core, *NS*, which is excited with direct current, is rotated at a uniform speed, it will induce in the set of coils *AC* an e.m.f. which will differ in phase by 90 degrees from that induced in the set *BD*. Note that to satisfy these conditions the armature coils must be located 90 electrical degrees apart. The arrangement of a practical two-phase, revolving-field generator is shown in Fig. 480. Note that the distance between the centers of similar field poles, for examples *S* and *S*, constitute 360 electrical degrees and that the sides of the armature coils are on this basis located 90 degrees apart. Consideration will show that

the principle of operation of the generator of Fig. 480 is the same as that outlined diagrammatically in Fig. 477 and that the e.m.f. induced in phase 1 (the black set of coils) will differ in phase by 90 degrees from that induced in phase 2 (the cross-hatched set).

NOTE.—A HYDRAULIC ANALOGY OF A TWO-PHASE, FOUR-WIRE ALTERNATING-CURRENT GENERATOR AND CIRCUIT is shown in Fig. 481. The two reciprocating pumps are analogous to two single-phase, alternating-current generators or, with their gear drive, to one two-phase, alternating-current generator, and the two pipe circuits are analogous to the two distinct electrical circuits of a two-phase, four-wire system. Note that the circuits and "generators" are precisely identical except that there is a definitely fixed difference of 90 degrees in the locations of the two cranks, C_1 and C_2, which drive the two pumps. Study Fig. 481 in connection with Fig. 476 and note that at the instant pictured in Fig. 481 when the current of fluid in phase 2 is being impelled at maximum velocity, that is, the current intensity is a maximum (piston head passing through center of stroke) in the direction shown by the arrows the fluid in phase 1 is at rest, that is, at this instant the current is zero in phase 1. This corresponds with the 180 degree

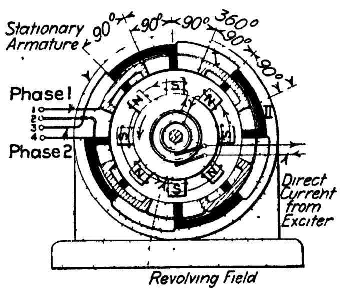


FIG. 480.—Diagrammatic representation of an eight-pole, two-phase revolving-field alternator.

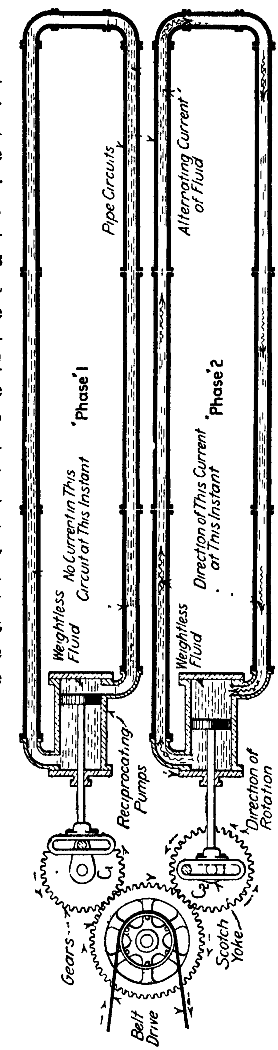


FIG. 481.—Hydraulic analogy to a two-phase, four-wire, alternating-current generator and circuit.

instant in Fig. 476, where the voltage in phase 1 is at zero (X), and the voltage in phase 2 is a maximum (Y), in the negative direction. If the analogy be followed further, it will be found that the fluid currents in the two circuits of Fig. 480 will increase and decrease in intensity and

change in direction precisely as is graphed in Fig. 476,*II*, which, though it shows graphs of the e.m fs., can also be taken as representing graphs of a two-phase current.

799. There are Two Commerical Methods of Connecting the Phase Windings of two-phase generators. In the two-phase, four-wire system of Fig. 482,*II*, each of the two groups of arma-

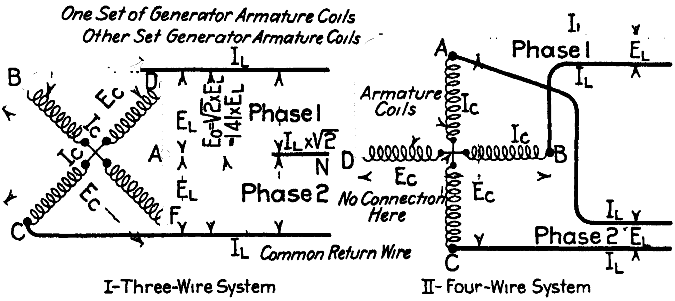


FIG. 482.—Two methods of connecting two-phase-generator armature windings

ture coils is connected to its own distinct external circuit. The hydraulic analogy of this arrangement is shown in Fig. 481. In the two-phase, three-wire system of Fig. 482,*I*, the two groups of armature coils are connected in series within the generator as shown. Connections to the external circuit are made from the junction point *A* of the two groups of coils and from the un-

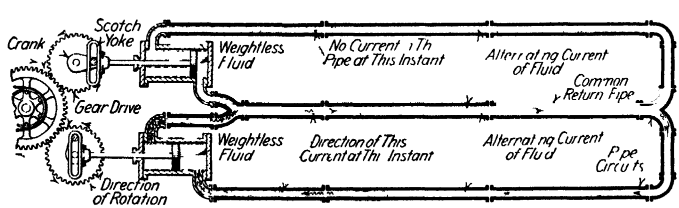


FIG. 483.—Hydraulic analogy to a two-phase, three-wire, alternating-current generator and circuit.

joined ends *B* and *C* of the two groups. Fig. 483 delineates a hydraulic analogy of a two-phase, three-wire circuit.

800. The Voltage and Current Relations in a Two-phase, Four-wire System may be determined by inspection Fig. 482,*II*, by noting that each phase or circuit may be treated as if it were a separate circuit as shown in Fig. 484. Then, assuming that the load is equally balanced on each of the two phases:

(250) $$E_C = E_L \quad \text{(volts)}$$

and

(251) $$I_C = I_L \quad \text{(amp.)}$$

Wherein E_L = the e.m.f. impressed across the line wires. E_C = the e.m.f. induced in each set of armature coils. I_L = the cur-

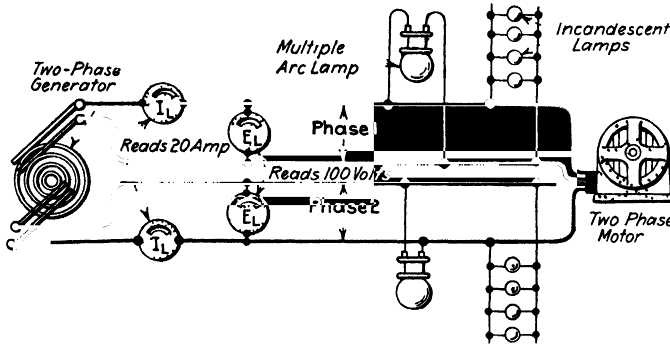


FIG. 484.—Example of balanced two-phase, four-wire circuit.

rent in each line wire. I_C = the current in each set of armature coils.

801. The Voltage and Current Relations in a Two-phase, Three-wire System, Fig. 485, may be determined by an inspection of Fig. 482,*I*. The voltage *Eo* between the two outside

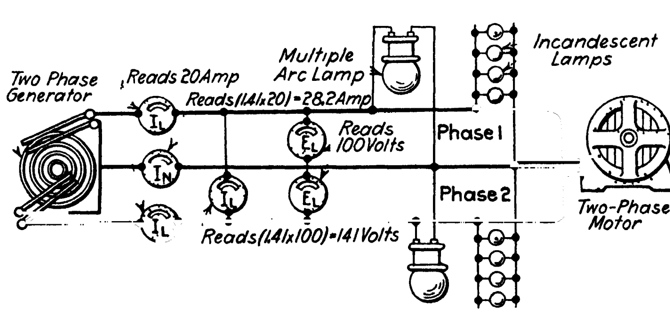


FIG. 485 —Example of balanced two-phase, three-wire circuit

wires will equal the vector sum of the voltages $E_C + E_C$ (or $E_L + E_L$) induced in each set of armature coils. These two voltages (*Ec* and *Ec*) can not be added directly, since they differ in phase by 90 degrees but they can be added vectorially as shown in Fig. 476,*I*. Since the voltages are equal in intensity

and differ in phase by 90 degrees their sum will be proportional to the length of the diagonal of a square. Now the diagonal of a square always equals 1.41 times the length of one of its sides. Hence:

(252) $$E_O = 1.41 \times E_C = 1.41 \times E_L \qquad \text{(volts)}$$

The middle wire AN (Fig. 482,*I*) is similar to the neutral wire of a three-wire system in that it carries the currents of both of the phases. However, these currents differ in phase by 90 degrees and therefore can not be added directly. Obviously, their vector sum equals 1.41 times the current in the outside wires. Then referring to the circuit diagrams of Figs. 482,*I*, and 485:

(253) $$I_L = I_C \qquad \text{(amp.)}$$

and:

(254) $$E_L = E_C \qquad \text{(volts)}$$

(255) $$I_N = 1.41 \times I_L = 1.41 \times I_C \qquad \text{(amp.)}$$

802. The Relations Between Power, Current, Voltage and Power-factor for any balanced, two-phase circuit follow from the fact that a two-phase circuit is merely two single-phase circuits. That is:

(256) $$P = 2 \times I_L \times E_L \times p.f. \qquad \text{(watts)}$$

and:

(257) $$I_L = \frac{0.50 \times P}{E_L \times p.f.} \qquad \text{(amp.)}$$

hence:

(258) $$E_L = \frac{0.50 \times P}{I_L \times p.f.} \qquad \text{(volts)}$$

therefore:

(259) $$p.f. = \frac{P}{2 \times I_L \times E_L} \qquad \text{(power-factor)}$$

Wherein I_L = current, effective, in outside wires, in amperes P = the power transmitted, in watts. E = the pressure between phases, in volts. $p.f.$ = power-factor. Note that the total power is equal to the sum of the power in the two phases, so that in making power measurements where the load is unbalanced either a wattmeter must be used in each phase, Fig. 486,*I*, or a two-phase wattmeter, which automatically adds the power of both phases, can be employed.

Example.—Fig. 487 illustrates the application of the above equations from two problems.

803. Three-phase is a term (A. I. E. E. Standardization Rules) characterizing the combination of three circuits energized by alternating e.m.fs. which differ in phase by one-third of a

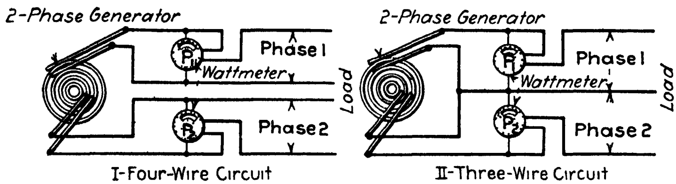


Fig. 486.—Wattmeters for measuring power in two-phase circuits.

cycle, *i.e.*, 120 degrees. That is with a three-phase e.m.f. or current there are at any instant three "phases." The three-phase is the most widely applied of all polyphase systems because of its economy and adaptability (Art. 820).

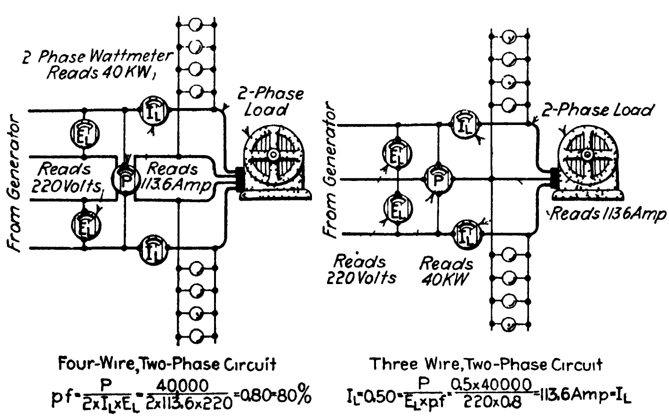


Fig. 487.—Illustrating e.m.f. and current relations in two-phase circuits.

804. To Produce a Three-phase E.m.f., Fig. 488, it obviously follows from the definition given above that it is only necessary to generate three sine-wave-form e.m.fs. which differ in phase by 120 degrees. As with single-phase and two-phase systems, the generators for developing these three-phase e.m.fs. may be either of the revolving-armature, Figs. 484 and 490, or of the revolving-field, Figs. 491 and 492, types.

805. A Three-phase E.m.f. Really Comprises Three Separate E.m.fs.—Likewise, a three-phase current comprises the three separate currents impelled by these three e m.fs. Hence, it is perfectly feasible to develop a three-phase e.m.f. with three single-phase generators, the shafts of which are rigidly coupled

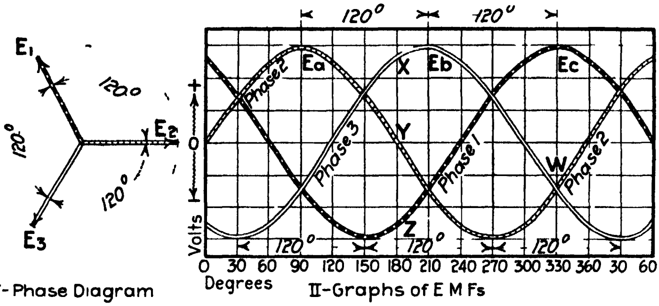


FIG 488 —Phase diagram and graphs for the e m fs produced by a three-phase alternating current generator

together so that the 120-degree phase relation between the e m.fs. developed by each of the three machines would be preserved. However, it is much more economical of material to combine the three sets of armature coils for the development of

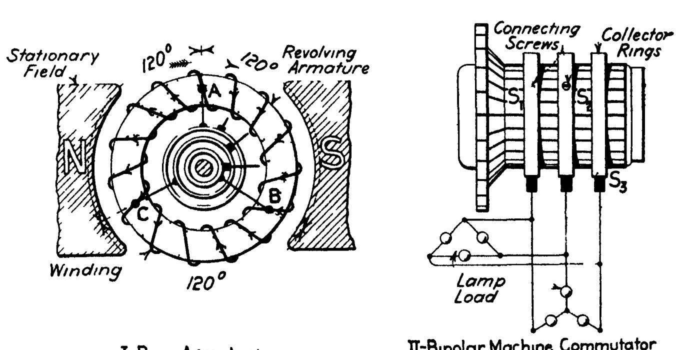


FIG 489 —Showing methods of obtaining three-phase currents from a bipolar revolving-armature generator, delta connection

these three e.m fs. into one machine. Therefore, this is the procedure always followed in practice.

806. The Principle of the Three-phase Generator is suggested in Fig. 491. If three different sets of armature coils be mounted in a frame 120 electrical degrees apart and a magnetic field

arranged as shown (*NS*, Fig. 491) is caused to rotate within them, it is obvious from a consideration of the principles outlined in Art. 708 that three e.m.fs. will be induced, one in each set of coils, which will differ in phase by 120 degrees as shown in Fig.

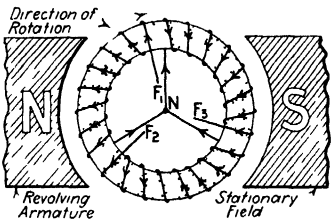


FIG. 490.—Bipolar-machine, ring-wound armature connected in *Y*

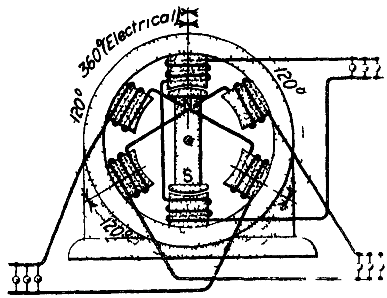


FIG. 491.—Elementary, two-pole, three-phase revolving-field alternator.

488. Furthermore, if three sets of overlapping armature coils be arranged with their sides 60 electrical degrees apart as shown in Fig. 492, three e.m.fs. which differ in phase by 60 degrees, Fig. 493, will be produced by rotating a properly arranged field mag-

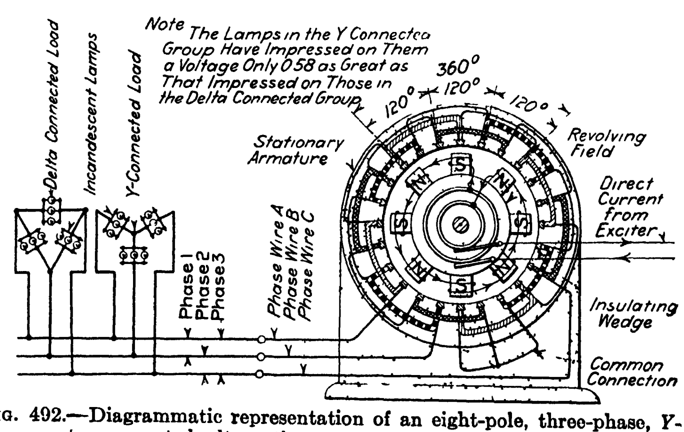


FIG. 492.—Diagrammatic representation of an eight-pole, three-phase, *Y*- or star-connected, alternating-current, revolving-field generator.

net within the structure. If the terminals of the coil group of phase 3 be reversed then three e.m.fs. differing in phase by 120 degrees, Fig. 488, will be impressed on the external circuit.

807. Revolving-armature, Three-phase Generators are now seldom used except for small machines, possibly those of capacities

of 25 kva. or less. It follows from a consideration of the principles of electromagnetic induction (Arts. 702 and 703) that, if a direct-current-generator armature winding be tapped at points 120 electrical degrees apart as shown in Fig. 489 and each of these tap wires is connected to a collector ring, a three-phase e.m.f. will be impressed on these rings. This can be accomplished with a bipolar machine as shown in Fig. 489,*II*, by using three collector rings tapped to three equidistantly spaced commutator bars after the manner hereinbefore described. Fig. 489 shows the delta method of connection which will be described later. The *Y*-method of connection, also described later, is shown in Fig. 490.

808. A Revolving-field, Three-phase Generator is illustrated diagrammatically in Fig. 492. The armature winding is sta-

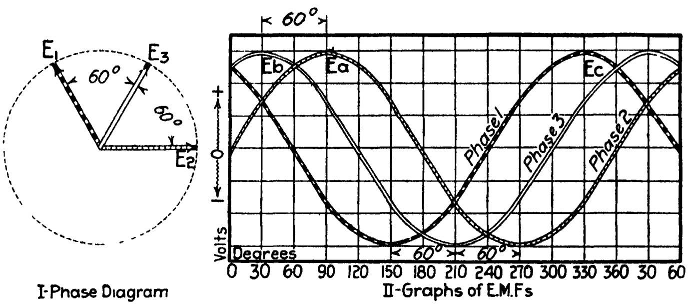


FIG. 493.—Phase diagram and graphs of the e.m.fs. produced by a three-phase, alternating current generator before the winding of phase 3 is reversed.

tionary and the field structure which is excited from a direct-current source—usually a small direct-current generator exciter—is rotated within the armature. Note that the distance between adjacent similar poles, that is from *N* to *N* or *S* to *S*, constitutes 360 electrical degrees. Besides, the armature coils are 60 degrees apart, while the distance between similar sides of the different coils is 120 degrees.

NOTE.—A HYDRAULIC ANALOGY OF A THREE-PHASE GENERATOR AND CIRCUIT is shown in Fig. 494. Each of the three reciprocating pumps is analogous to a single-phase generator and the combination of the three and their gear drive is analogous to a three-phase generator. The three hydraulic circuits are analogous to a three-phase, six-wire circuit. The only difference between each of the three generators and its circuits is that the cranks driving the three pumps are so geared that there is always a "phase" difference of 120 degrees between them. The instant pictured in Fig. 494

corresponds to the 180-degree phase of Fig. 488. That is, at this instant the piston of phase 2 is stationary and there is no current of water in its pipe circuit. This corresponds to point Y in Fig. 488. The fluid current in phase 1 is decreasing in the negative direction, Z, Fig. 488, and the fluid current in phase 3 is increasing, X, Fig. 488, in the positive direction. If the analogy is followed further it will be found that the graphs of Fig. 488,*II*, indicate just how the fluid currents in the three different pipe circuits would increase and decrease in intensity and change in direction individually and in relation to each other as the three pumps are driven by the large gear.

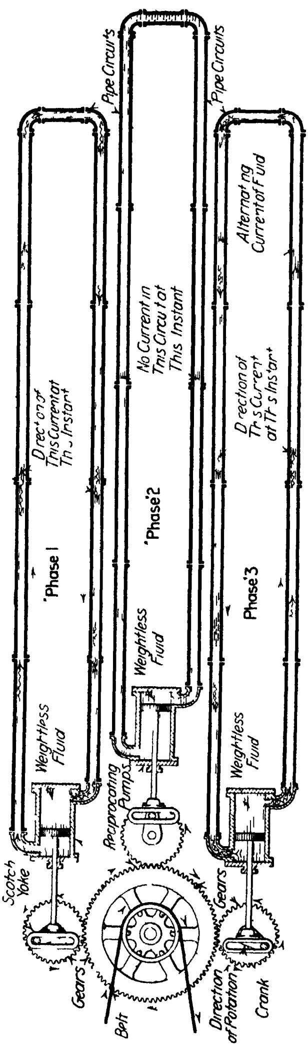


FIG. 494.—Hydraulic analogy to a three-phase, six-wire, alternating-current circuit and generator.

809. There Are Two Methods of Connecting the Windings of Three-phase Generators and, in general, there are two methods of

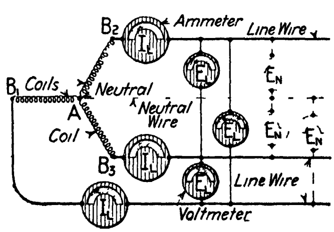


FIG. 495.—Ammeters and voltmeters in a three-phase, Y-connected circuit.

connecting devices of any sort to a three-phase circuit. They are (1) the Y-connection. Fig. 495, and (2) the delta connection, Fig. 496,*II*. Most generators have their coils Y-connected but it is ordinarily impossible to determine from an external inspection of a machine whether it is Y- or delta-connected. Machines connected in either way can be made to provide the same performance, but for a given voltage impressed on the line the armature coils for a Y-connected machine must be different from those for a delta-connected machine, as will be shown.

NOTE.—THREE-PHASE VECTORS CONVENTION.*—A feature which sometimes gives some difficulty is the selection of positive directions. Any selection will do as long as it is consistently adhered to. Thus, it is convenient

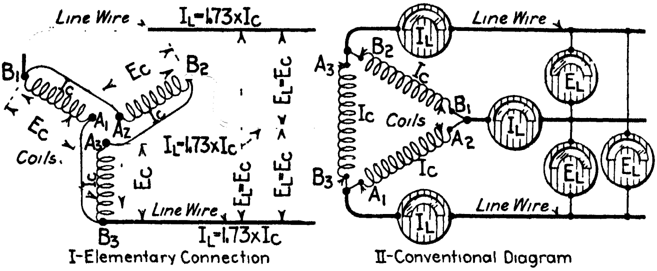


FIG. 496.—Illustrating the delta (Δ) connection of generator or receiver coils.

to consider the counterclockwise direction around a delta, and the direction out from the neutral point of a Y as positive. Confusion can be avoided

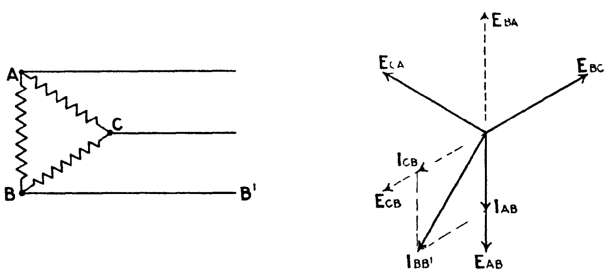


FIG. 497.—Delta connection and corresponding phase diagram.

by following a simple convention. Draw a diagram of the circuit and letter all intersections. It is helpful to arrange these diagrams to correspond to

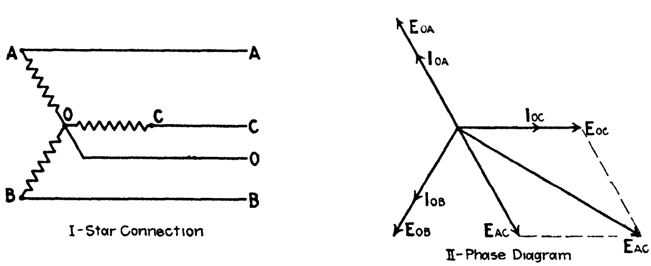


FIG. 498.—Star or Y connection and phase diagram.

the shape of the vector figure. When a vector is drawn and lettered, use two subscripts denoting the points between which the current flows or vol-

* R. H. Willard, in ELECTRIC JOURNAL.

tage exists. Thus in Fig. 497, E_{AB} represents the difference of potential between A and B, A being the higher. E_{BA} represents the difference of potential between B and A which is equal and opposite to E_{AB}. In other words, $E_{AB} = - E_{BA}$. The same is true of currents. If the current in a line is desired, say BB' add I_{AB} and I_{CB} since both these currents feed into B and $I_{BB'}$, feeds out. If the currents are in phase with the voltages, the vector $I_{BB'}$, the sum of I_{AB} and I_{CB}, Fig. 497, will represent the current in BB'.

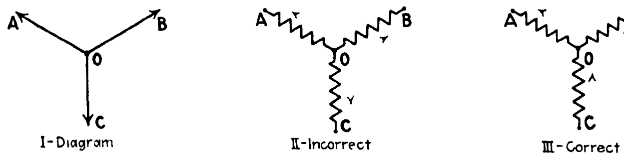


FIG. 499.—Vector diagram of currents and correct and incorrect methods of showing the instantaneous currents in a star connection.

Fig. 498 shows a star generator. If voltage E_{AC} is desired, add E_{AO} and E_{OC}. If the current in the neutral of a balanced star is desired $I_{o'o} = I_{OB} + I_{OC} + I_{OA}$. Using vector addition, this gives zero as the current in the neutral, if the currents in the phases are equal and at the same power-factor.

Care must be taken not to confuse vectors with instantaneous currents or voltages in the windings. In Fig. 499, I represents by vectors the cur-

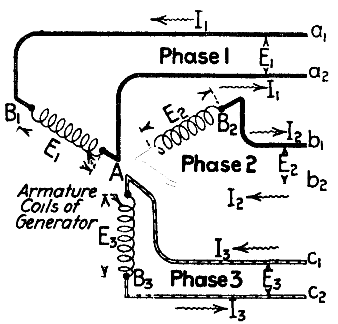


FIG. 500.—Armature coils of a three-phase generator. Coils not interconnected.

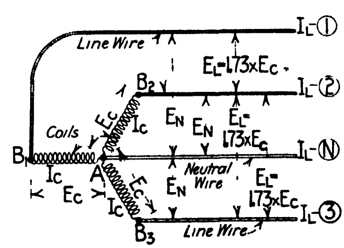


FIG. 501.—Coils of a three-phase generator connected in Y or in star.

rents in a star, but at no instant is the current flowing as in *II* for the current flowing in from C to O at a given instant flows out from O to B and O to A as shown at *III*. That is, for *instantaneous* currents, Kirchoff's law holds, the algebraic sum of all currents flowing to a point is zero.

810. The Y-connection of phase windings or coils is indicated diagrammatically in Fig. 495. A three-phase generator or device may be represented by a diagram like that of Fig. 500 in which each of the three sets of coils AB_1, AB_2 and AB_3 represents

one of the sets of armature windings in which an e.m.f. is induced. However, the number of line wires may be reduced to four as shown in Fig. 501, by using a common return wire AI_L for all three of the circuits, and by connecting it to a junction, A, of the three sets of coils. However, if the three circuits are balanced, that is, equally loaded, as they usually are in practice—approximately at least—then no current would flow in the neutral wire, AI_N; hence it can be omitted as diagrammed in Fig. 495.

811. Why the Neutral Wire of a Y-connected Circuit May be Omitted is suggested in Fig. 502, which indicates the conditions existing in a three-phase circuit at a certain instant. It is assumed that the e.m.f. induced in each of the three sets of armature coils AB_1, AB_2 and AB_3 has a maximum instantaneous value of 100 volts. Each of the three load components, $L_1 - N_1$, $L_2 - N_2$, and $L_3 - N_3$, has a resistance of 10 ohms. At the

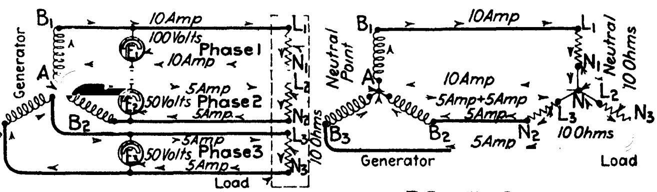


FIG. 502.—Instantaneous currents at a certain instant in three-phase (Y-connected) circuits.

instant shown in Fig. 502, which corresponds to the 330-degree phase of Fig. 488, the e.m.f. impressed on phase 1 is a maximum (E_C) or 100 volts. The e.m.fs. impressed on phases 2 and 3 are then each one-half maximum (W) or 50 volts, but are in opposite direction from that of phase 1. Since the resistance of the load in each of the phases is 10 ohms, the currents of the instant pictured will be 10 amp., 5 amp. and 5 amp., respectively, as shown. Therefore, Fig. 503 indicates a graph of the three-phase current in this circuit, the conditions of Fig. 502,*I*, corresponding to the 90-degree phase (VW) of Fig. 503. Now, if a common return wire, AN, is provided as shown in Fig. 502,*II*, there would be a tendency for a current of 10 amp. to flow in it toward the left, and also for a current of 5 *amp.* + 5 *amp.*, = 10 *amp.*, toward the right. The consequence is that these tendencies neutralize and that there would be no current in the neutral wire, therefore it is unnecessary. Now, while Fig.

502 pictures conditions for only one instant, it is evident from a study of the graph of Fig. 503 that in a three-phase circuit the current in one of the phases is always equal and opposite to the sum of the currents in the other two phases, which indicates why there is no occasion for a return wire. For example, (Fig. 503), at the 120-degree phase, the current represented by the ordinate PQ (8.5 amp.), is equal and opposite to the current represented by the ordinate QR. Also, at the 150-degree phase, $ST + ST$ is equal but opposite to TU.

812. The Voltage and Current Relations in a Star- or Y-connected Three-phase Circuit are indicated in Fig. 501. It is evident that the current in each one of the three coils AB_1,

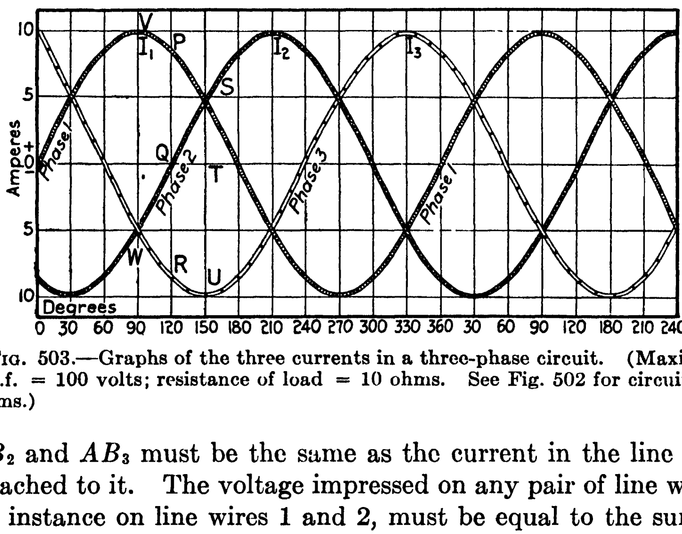


FIG. 503.—Graphs of the three currents in a three-phase circuit. (Maximum e.m.f. = 100 volts; resistance of load = 10 ohms. See Fig. 502 for circuit diagrams.)

AB_2 and AB_3 must be the same as the current in the line wire attached to it. The voltage impressed on any pair of line wires, for instance on line wires 1 and 2, must be equal to the sum of the voltages induced in the two coils which are connected in series between these two wires. For example, the voltage impressed across $B_1 - B_2$, Fig. 501, is equal to the sum of the e.m.f. induced in AB_1 plus that induced in AB_2. However, since these voltages differ in phase by 120 degrees they can not be added arithmetically. They can, however, be added vectorially and then, as is shown in the following paragraph, their resultant = 1.73 times the voltage generated in either coil. That is, for a Y-connected circuit, the following equations must hold:

(260) $$I_C = I_L$$ (amp.)

(261) $$E_L = 1.73 \times E_C$$ (volts)

Therefore

(262) $$E_C = \frac{E_L}{1.73} = 0.58 \times E_L \qquad \text{(volts)}$$

It is evident that the voltage from either one of the three line wires to the neutral wire, if one is provided, must equal the voltage induced in each coil, that is:

(263) $$E_N = E_C \qquad \text{(volts)}$$

Wherein, assuming balanced load and effective values of current and e.m.f., I_L = the current in any one of the three line wires. I_C = the current in any one of the three-phase windings or coils which are connected in Y. E_L = the e.m.f. impressed between any pair of the three line wires. E_C = the e.m.f. induced in or impressed on any one of the three-phase windings or coils.

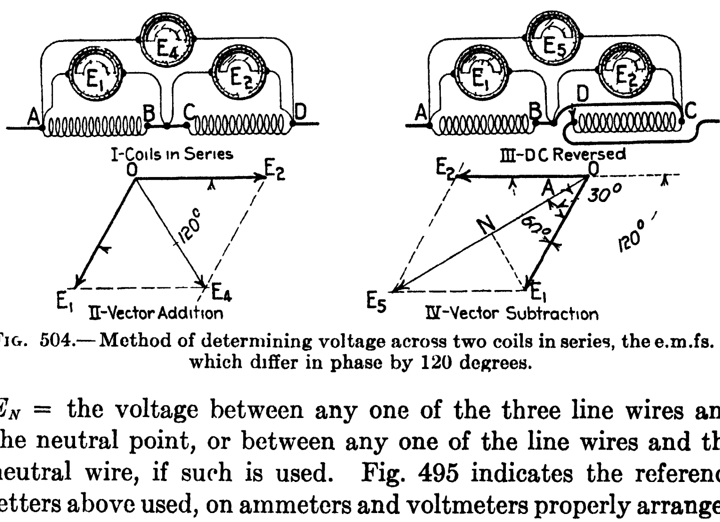


FIG. 504.—Method of determining voltage across two coils in series, the e.m.fs. in which differ in phase by 120 degrees.

E_N = the voltage between any one of the three line wires and the neutral point, or between any one of the line wires and the neutral wire, if such is used. Fig. 495 indicates the reference letters above used, on ammeters and voltmeters properly arranged in a three-phase Y-connected circuit.

813. Why the Line E.m.f. in a Y-connected Circuit Equals 1.73 Times the Coil E.m.f. follows from a consideration of Fig. 504. *AB* and *CD* represent two different coils in each of which are induced equal voltages. However, the voltage, E_1 in *AB* lags 120 degrees behind E_2, in *CD*. If the coils are connected as shown at *I* then the vector sum of the voltages E_1 and E_2 will be, as is evident from the vector diagram of *II*, proportional to the length of the resultant vector OE_4. Now OE_4 (since it is the resultant of two equal vectors 120 degrees

apart) is equal to OE_2 and to OE_1. Hence the sum of the e.m.fs. E_1 and E_2, with the coils connected as shown at *I*, is the same as their e.m.fs. taken singly.

If, however, one of the coils, *DC*, be reversed as at *III*, the e.m.f. in *CD* being equal to that in *AB*, but leading it by 120 degrees, this is equivalent to reversing the e.m.f. (Reversing the coil which generates an e.m.f. changes its phase relation to the other two coils by 180 degrees.) Then E_5 will be the vector difference of E_1 and E_2 rather than their vector sum. The vector diagram for the sum of E_1 and E_2 then becomes that shown in *IV*, where the vector OE_5 is proportional in length to the difference of OE_2 and OE_1. Now, the angle *A* is obviously one-half of 60 degrees or 30 degrees and *ON* equals, then, cos 30 degrees $\times E_1$, and $OE_5 = 2 \times \cos 30$ degrees $\times E_1$. Cos 30 degrees is 0.866; therefore OE_5 or $E_5 = 1.73 \times E_1 = 1.73 \times E_2$. Therefore, since in a *Y*-connected three-phase generator any of the groups of two coils which are in series between the line wires are reversed relative to one another as at *III*, the e.m.f. impressed on the line wires of such a machine equals: 1.73 $\times$ *the e.m.f. induced in each of the sets of coils.* The relations just suggested can be conveniently shown by an equilateral triangle, Fig. 505, the sides of which are proportional in length to the voltage between the line wires.

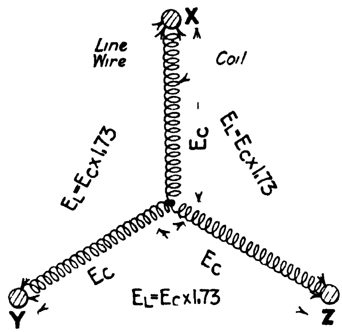


FIG. 505.—Equilateral triangle indicating voltage relations in a three-phase, *Y*-connected circuit.

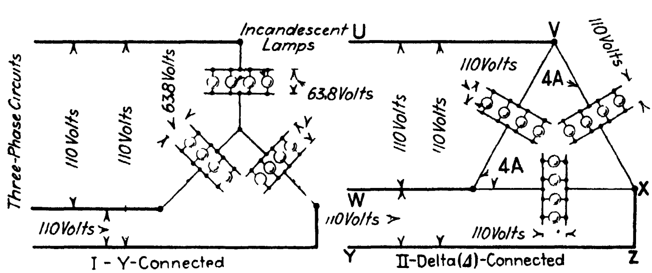


FIG. 506.—Incandescent lamps connected in delta and in *Y*.

EXAMPLE.—With a group of three sets of four incandescent lamps each, connected in *Y* across a 110-volt circuit, as in Fig. 506,*I*, what e.m.f. will be impressed upon each set? SOLUTION.—Substitute in equation (262) $E_C = 0.58 \times E_L \times 0.58 = 110 = 63.8$ *volts.*

814. The Power in a Y-connected Circuit may be determined as follows: It is evident from the power relations in a single-phase alternating-current circuit that the power P_p, developed in any one-phase winding of a three-phase generator is:

$$(264) \qquad P_p = E_C \times I_C \times p.f. \qquad \text{(watts)}$$

But from equation (262): $E_C = E_L \div 1.73$ and from (260): $I_C = I_L$. Substituting these values in (264), the power developed in any one coil is:

$$(265) \qquad P_p = \frac{E_L}{1.73} \times I_L \times p.f. \qquad \text{(watts)}$$

Since, with a balanced load, an equal amount of power is developed in each of the three-phase windings, the total power will be three times that developed in one winding, or:

$$(266) \quad P = 3 \times \left(\frac{E_L \times I_L \times p.f.}{1.73}\right) = 1.73 \times E_L \times I_L \times p.f. \qquad \text{(watts)}$$

815. The Delta Connection of Phase Windings or Coils is indicated at Figure 496,*II*, where *I* shows the elementary diagram and *II* the simplified arrangement. It might be assumed that since the three windings form a closed circuit, current would circle around in it but such is not the case because the resultant sum of the voltages which differ in phase by 120 degrees in the three windings, is at every instant zero as will be evident from a consideration of the graphs of Fig. 488. That is, the voltage in one phase is at any instant always equal and opposite to the sum of the voltages in the other two phases. If, however, three line wires are connected to these phase windings, one to each of the junction points between windings as shown in Fig. 496,*II*, then alternating e.m.fs. will be impressed on the external circuit and a three-phase current will be forced through the circuit if it is closed.

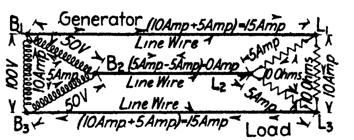


FIG. 507.—Instantaneous currents, at a certain instant, in a delta (Δ)-connected three-phase circuit.

EXAMPLE.—Fig. 507 shows the instantaneous currents in the circuit

shown at the instant at which the e.m.f. in coil B_1B_3 is a maximum. Note that at this particular instant the current in B_1L_1 is 15 amp., but that it is opposite to the current of 15 amp. in B_3L_3. At this instant there is no current in B_2L_2. This example verifies the statement previously made that, in a three-phase circuit, the current in some one of the line wires is at any instant always equal and opposite to that in the other two.

816. Voltage and Current Relations in a Delta-connected Circuit are shown in Fig. 496. It is evident that the voltage impressed between each of the two line wires is equal to the voltage induced in each phase winding because the line wires are connected directly across the phase windings. It is also evident that the current in each one of the line wires is the sum of the currents in two of the phase windings. However, these currents, since they differ in phase by 120 degrees, can not be added directly but must be added vectorially, somewhat as suggested in Fig. 504,*IV*. Hence it follows that for a three-phase, delta-connected circuit:

$$E_C = E_L \qquad \text{(volts)} \tag{267}$$

$$I_L = I_C \times 1.73 \qquad \text{(amp.)} \tag{268}$$

$$I_C = \frac{I_L}{1.73} = 0.58 \times I_L \qquad \text{(amp.)} \tag{269}$$

Wherein, assuming balanced load and effective values of current and e.m.f., all of the symbols have the same meanings as used in the *Y*-connection demonstrations.

EXAMPLE.—In the circuit of Fig. 506,*II*, each of the three groups of 110-volt incandescent lamps, takes 4 amp. What is the current in the line wires *UV*, *WX* and *YZ*? SOLUTION.—Substitute in the formula (268): $I_L = I_C \times 1.73 = 4 \times 1.73 = 6.92$ *amp.*

817. The Power Developed in a Delta-connected Circuit can be determined on the basis of the explanation hereinbefore given for the determination of power in a *Y*-connected circuit. Obviously in a delta-connected circuit the power developed by each phase winding is (Art. 783):

$$P_p = E_C \times I_C \times p.f. \qquad \text{(watts)} \tag{270}$$

But, equation (267): $E_C = E_L$ and equation (268): $I_C = I_L \div 1.73$, hence, substituting these values in (270):

$$P_p = E_L \times \frac{I_L}{1.73} \times p.f. \qquad \text{(watts)} \tag{271}$$

Then the total power, P, for the three windings must be:

(272) $P = 3 \times E_L \times \frac{I_L}{1.73} \times p.f. = 1.73 \times E_L \times I_L \times p.f.$ (watts)

818. The Three-phase, Four-wire System, Fig. 508, is now being used to a considerable extent, principally for the distribution of electrical energy for light and power in cities. Such a system is, as suggested, served by a three-phase, Y-connected

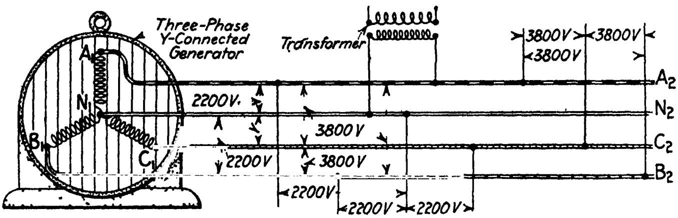


FIG. 508.—Illustrating diagrammatically the three-phase, four-wire system.

generator and is precisely like any other Y-connected system except that the neutral wire $N_1 - N_2$ is carried out from the machine and along the line with the other three wires. The system permits the use of a transmission voltage 1.73 times the voltage impressed on the transformers, a material economy in copper for the same power loss—or a material decrease in energy lost for the same amount of copper—resulting.

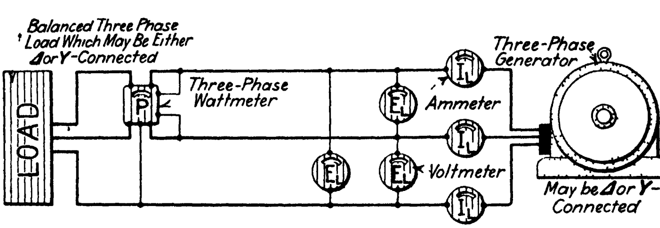


FIG. 509.—Typical three-phase circuit.

EXAMPLE.—In the three-phase, four-wire system of Fig 508, the transformers are connected between the outside wires and the neutral wire, the pressure on their primaries being 2,200 volts. However, the pressure between the phase wires, A, C and B equals $1.73 \times 2{,}200$ volts, is 3,800 volts. Since with a balanced load, and it is usually so—approximately at least—in practice, there is no current in the neutral wire, the energy is really, in the circuit shown in the illustration, transmitted at 3,800 volts.

819. Power and Voltage Relations for Any Three-phase Circuit which may be either Y- or delta-connected are given in the equations below which refer to the symbols of Fig. 509. Note

from the two articles preceding, which indicate how the power may be determined in delta- and in Y-connected circuits, that the final equation is the same for both of these circuits; hence it follows, for any three-phase circuit with a non-inductive load, that:

$$P = 1.73 \times E_L \times I_L \qquad \text{(watts)} \tag{267}$$

For an inductive load, then, the following must be true:

$$P = 1.73 \times E_L \times I_L \times p.f. \qquad \text{(watts)} \tag{268}$$

or

$$E_L = \frac{P}{1.73 \times I_L \times p.f.} = \frac{0.58 \times P}{p.f. \times I_L} \qquad \text{(volts)} \tag{269}$$

$$I_L = \frac{P}{1.73 \times E_L \times p.f.} = \frac{0.58 \times P}{p.f. \times E_L} \qquad \text{(amp.)} \tag{270}$$

$$p.f. = \frac{P}{I_L \times E_L \times 1.73} = \frac{0.58 \times P}{I_L \times E_L} \qquad \text{(power-factor)} \tag{271}$$

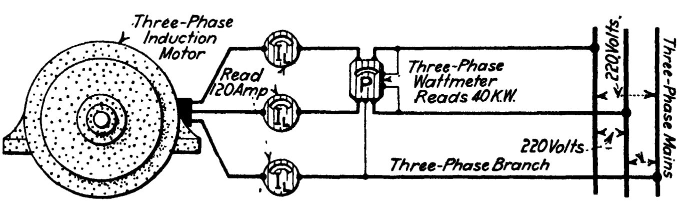


FIG. 510.—Three-phase load feeding from a three-phase main.

Wherein, assuming balanced load and effective values of current and e.m.f., P = the power, in watts, being transmitted in the circuit. E_L = the e.m.f., in volts, between line or phase wires. I_L = the current, in amperes, in each line or phase wire. $p.f.$ = the power-factor of the circuit.

EXAMPLE.—A 220-volt, three-phase circuit feeds an induction motor as shown in Fig. 510. The current in each line wire is 120 amp. and the power taken by the motor is 40 kw., as indicated by a wattmeter. What is the power-factor of the circuit? SOLUTION.—$p.f. = 0.58 \times P \div I_L \times E_L = 0.58 \times 40{,}000 \div 120 \times 220 = 0.878$. That is, the power-factor of the circuit is, say, 88 *per cent.*

EXAMPLE.—Fig. 511 shows a three-phase branch circuit carrying energy from a distributing center to a load. The current in each of the line wires is 65 amp. The line e.m.f. is 220 volts and the power-factor of the load is known to be 70 per cent. What power in kilowatts is being transmitted? What horsepower? SOLUTION.—$P = 1.73 \times E_L \times I_L \times p.f. = 1.73 \times 220 \times 65 \times 0.70 = 17{,}300$ *watts* = 17.3 kw. 17.3 kw. $\div$ 0.746 = 23.4 *h.p.*

EXAMPLE.—Fig. 512 shows the different values of current and voltage in a certain group of receivers connected in Y and delta on a three-phase circuit. A study of this illustration will be of great assistance in fixing in mind the voltage and current relations of three-phase circuits. The generator

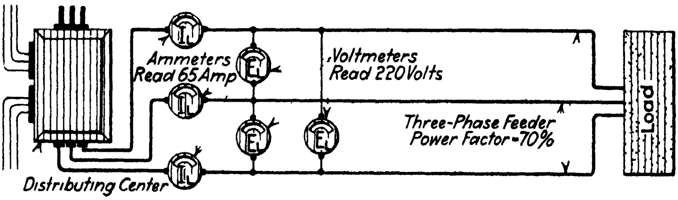


FIG. 511—Three-phase load feeding from a distribution center

shown may be either Y- or delta-connected without affecting the external results.

820. The Three-phase System Is Very Widely Applied and is now usually utilized in preference to any other system for the

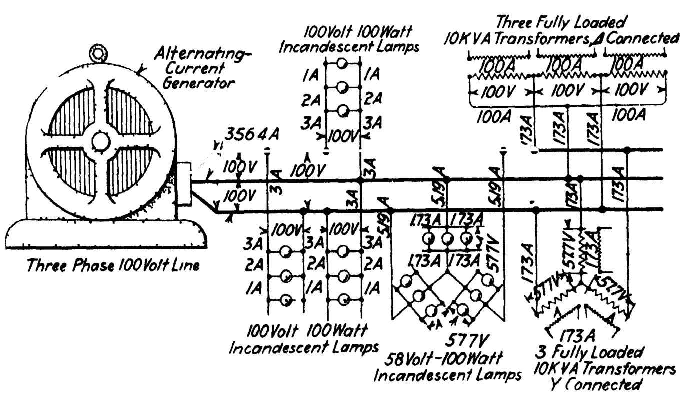


FIG. 512.—A 100-volt, three-phase line showing receivers (lamps and transformers) connected in delta and in Y.

transmission of electrical energy over a considerable distance. It is simpler than the two-phase system, is more economical of copper and its operating performance in practice is quite as satisfactory.

SECTION 52

TRANSFORMERS, THEIR PRINCIPLES AND APPLICATIONS

821. The Stationary Transformer is a device (operating by virtue of the principle of mutual induction, Art. 454) whereby the energy of an alternating-current circuit may be received at one voltage and delivered at a higher or lower voltage. It is really an alternating-current induction coil. A transformer is one of the most important electrical devices—if not the most important. It is remarkably simple in elementary principle and in construction, in that it involves no moving parts. However, some of the reactions that occur within a transformer are extremely complicated and tedious of explanation. Fortunately, these complicated phenomena are of little importance to the practical man.

NOTE.—The essential features of the theory of transformer operation, an understanding of which will enable one to solve the most frequently encountered transformer problems, are relatively simple and readily understood. These will be explained in the following articles. The transformer will operate only with alternating current and will not work with direct current. There is no electrical connection between the primary and secondary windings. They are electrically independent (except in the balance coil) but are magnetically interconnected by the alternating flux.

822. Transformers Are Essential in the Transmission of Electrical Energy.—If electrical energy is to be transmitted with economy over any considerable distance, the transmission voltage must be high, so that the line losses will be a minimum. However, it is not feasible nor desirable to utilize high-voltage electrical energy for electric lamps, motors, and other receiving appliances. Furthermore, it is not feasible to generate electrical energy at the high voltages that must be used for transmitting large amounts of energy over great distances. Therefore, where the energy is generated at a low voltage, a step-up transformer, *A*, Fig. 513, is used for raising the generator voltage to one suitable for energy transmission and then a step-down transformer, *B*, is applied for again decreasing the voltage to one suitable for utilization.

823. The Three Principal Parts of a Transformer are, Fig. 514: (1) an iron core, which provides a circuit of low reluctance for the magnetic flux; (2) the primary winding, that which receives the energy from the supply circuit; and (3) the secondary winding, that which receives energy by induction from the primary and delivers it to the secondary circuit.

NOTE.—Where possible or feasible, misunderstandings are eliminated if the windings are referred to, respectively, as the high-tension or high-voltage

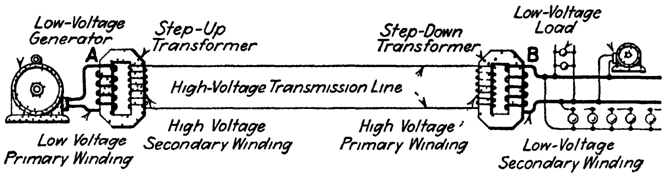


FIG. 513.—Illustrating the application of the potential transformer to electrical energy transmission.

winding and the low-tension or low-voltage winding. Either the high or the low may, Fig. 513, constitute the primary or the secondary winding of a transformer, depending upon its arrangement in the circuit

824. The Iron Cores of Transformers are in practice, to insure maximum economy, laminated to minimize eddy-current losses and are usually of specially prepared silicon steels because of the low hysteresis losses which occur in these special steels.

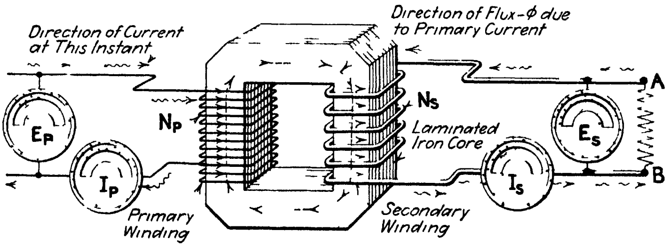


FIG. 514. —Showing the principle of the potential transformer.

Furthermore, these special steels can be used for long—probably indefinite—periods in transformer cores without the losses in them increasing, whereas if ordinary irons or steels are used, the losses in the cores increase as the term of service of the transformer lengthens, that is, such steels are subject to "aging."

825. Current Transformers and Voltage Transformers, Fig. 515, constitute two classes into which these devices may in

general be grouped. With voltage transformers, the primary windings are connected, Fig. 515,*I*, in multiple across the supply mains. With current transformers, Fig. 515,*II*, the primary windings are connected in series in the primary circuit. The characteristics of these two different types will be discussed in the articles that follow. Inasmuch as the voltage transformer is the more frequently used, it will be considered more in detail.

826. How a Voltage Transformer Transforms Voltages can be understood from a consideration of the principles of mutual induction. If an alternating voltage, E_p, Fig. 514, is impressed on the primary winding of a transformer—assuming that the secondary circuit is open—an alternating current, I_p, will be impelled in this winding and it will produce in the core of the transformer an alternating flux, ϕ. This flux (as in building up it expands out of and, as in dying down it contracts back into

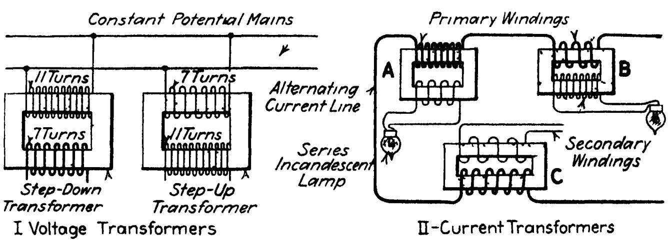


FIG. 515.—Illustrating the difference between a voltage or potential transformer and a current transformer.

the primary turns, as the primary alternating current increases and decreases in intensity and changes in direction) will cut the turns of the secondary winding. Thereby an e.m.f., E_s, will be induced in this secondary winding. If the secondary circuit be closed as at *AB*, Fig. 514, then the secondary e.m.f., E_s, induced as above, will impel a current, I_s, in the secondary circuit. The intensity of this current will be directly proportional to the secondary e.m.f. induced as described, and inversely proportional to the impedance of the secondary circuit, that is: $I_s = E_s \div Z_s$, wherein Z_s is the impedance of the secondary circuit.

827. How the Voltage Transformer Operates will now be explained by referring to Fig. 516. With all of the switches, K_1 to K_5 (Fig. 516) open, that is, with the secondary circuit open, the primary current I_p will be so small as to be relatively

negligible. Hence, it may be stated that, when the secondary circuit of a transformer is open, there is practically no current in its primary circuit. Now if K_1 be closed connecting one lamp across the circuit, the secondary e.m.f. E_s induced as above described will impel a current I_s in the secondary circuit, and furthermore, the ammeter, A_p, will indicate current in the primary. But the primary current, I_p, will always in a step-down transformer, be as many times smaller than the secondary current, I_s, as the number of primary turns is greater than the number of secondary turns, as explained below. In a step-up transformer, the primary current will be correspondingly greater than the secondary current.

If K_2 be also closed, doubling the secondary current, the primary current will likewise be doubled. If all five switches, K_1 to K_5, are closed, increasing the secondary current five times,

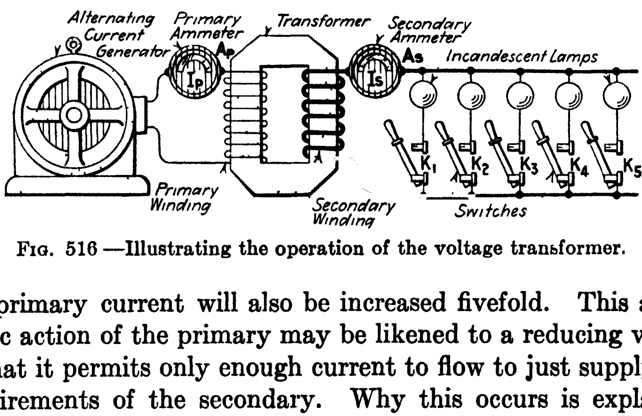


FIG. 516—Illustrating the operation of the voltage transformer.

the primary current will also be increased fivefold. This automatic action of the primary may be likened to a reducing valve, in that it permits only enough current to flow to just supply the requirements of the secondary. Why this occurs is explained below.

828. As to the Theory of Transformer Operation, it might appear that the primary winding, because it has relatively low resistance, would short-circuit the primary circuit, and so it would, if a direct e.m.f. were applied.

EXPLANATION.—Although the resistance of the primary winding is low, its inductance, since it is a coil wound on an iron core, is high. Hence, when an alternating e m.f. is applied across the primary winding it induces a counter e.m.f., practically equal in intensity to the applied e.m.f., E_p. This prevents current—except a very small "magnetizing" current—from flowing in the primary. This magnetizing current is sufficient, however, to develop an alternating flux, ϕ, in the core. so that an e.m.f. is produced in the secondary by ϕ cutting the secondary turns and so that the counter

e.m.f. of self-induction is maintained in the primary. The primary winding is so designed (see formula 287,*A* below) that it has a sufficient number of turns that its counter e.m.f. of self-induction will equal practically the impressed e.m.f., E_p. This is the reason why the current which flows in the primary winding of a transformer, the secondary of which is open, is of very small intensity. Furthermore, since the primary winding is largely inductive, this magnetizing current involves almost no energy.

However, when the secondary circuit is closed by connecting a load across it, as at *AB*, Fig. 514, the secondary e.m.f., E_s, induced as above suggested, in the secondary turns, N_s, impels the secondary current, I_s, in the secondary circuit. As this current, I_s, flows, it also tends to produce an alternating flux in the transformer core. But the flux due to I_s would be (Lenz's law, see index) in a direction opposite to that of the flux, ϕ, produced by the primary winding. The result of this demagnetizing effect is that the total flux, ϕ, in the magnetic circuit is slightly decreased with every increase in secondary load, which decreases correspondingly the counter e.m.f. induced in the primary turns. Thereby a suitably greater primary current is permitted to flow. Thus, the primary current always is automatically maintained at such a value that the secondary circuit will be properly supplied.

828A. The Actual Change in Flux and in Primary Counter E.m.f. in an Actual Transformer Is, Between Full-load and No-load, Very Small, inasmuch as the change seldom exceeds 1 per cent., so that for practical purposes it may be assumed that the flux in the transformer core is constant at all loads and that the primary counter e.m.f. is also constant. Furthermore, it may be assumed that, if the primary or impressed voltage remains constant, the secondary voltage of a well-designed transformer also remains practically constant at all loads. Actually there is a slight drop in the secondary voltage from no-load to full-load, possibly from 1 per cent. to 3 per cent., depending on the design and characteristics of the transformer under consideration.

829. The Ratio of the Number of Primary Turns to the Number of Secondary Turns Determines the Ratio of the Primary to the Secondary Voltage or E.m.f.—It follows from the equation (287*a*) given below for determining the counter e.m.f. of a coil, that for any given coil and with a given frequency, the counter e.m.f. induced in the coil is proportional to the product of *flux* $\times$ *turns*, that is, to $\phi \times N$. The counter e.m.f. of the primary of a transformer equals (practically) the e.m.f. impressed on it. Hence, the primary e.m.f. of a transformer, E_p, is proportional to $\phi \times N_p$. Also, it follows that the e.m.f. induced in the secondary turns must be proportional to $\phi \times N_s$. Now, since the same flux, ϕ, which produces the counter e.m.f. in the primary

also induces the e.m.f. in the secondary turns. It is obvious then that the following proportion must hold:

$$(273) \qquad \frac{\text{primary voltage}}{\text{secondary voltage}} = \frac{\text{primary turns}}{\text{secondary turns}}$$

that is:

$$(274) \qquad \frac{E_p}{E_s} = \frac{N_p}{N_s}$$

or

$$(275) \qquad E_p = \frac{N_p \times E_s}{N_s} \qquad \text{(volts)}$$

$$(275a) \qquad E_s = \frac{E_p \times N_s}{N_p} \qquad \text{(volts)}$$

$$(275b) \qquad N_p = \frac{E_p \times N_s}{E_s} \qquad \text{(turns)}$$

$$(275c) \qquad N_s = \frac{E_s \times N_p}{E_p} \qquad \text{(turns)}$$

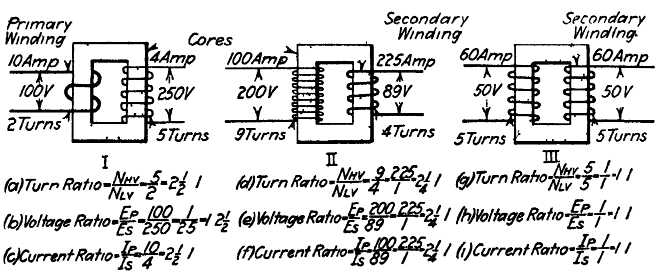


FIG. 517—Illustrating examples of "ratios of transformation."

Wherein, E_p = e.m.f. impressed on primary, in volts, effective. E_S = e.m.f. delivered by secondary, in volts, effective. N_p = number of turns in the primary winding. N_s = number of turns in the secondary winding.

EXAMPLE.—The transformer of Fig. 517,*II*, has 9 turns in its primary and 4 turns in its secondary winding. If an e.m.f. of 200 volts is impressed on the primary, what would be the secondary voltage? SOLUTION.—Substituting in equation (275*a*): $E_s = (E_p \times N_s) \div N_p = (200 \times 4) \div 9 = 89$ *volts*, which will, then, be the secondary e.m.f.

EXAMPLE.—A certain transformer is designed for a high-tension e.m.f. of 3,120 volts and has 400 turns on its high-tension winding. A low-tension voltage of 230 is required. How many turns must be in the secondary winding? SOLUTION.—Substitute in equation (275*c*): $N_s = (E_s \times N_p) \div E_p = (230 \times 1{,}400) \div 3{,}120 = 103$ *turns* in the low-tension winding.

830. The Ratio of a Transformer or the "turn ratio" (A. I. E. E. STANDARDIZATION RULES) is the ratio of the number of turns in the high-voltage winding to the number of turns in the low-voltage winding. The voltage ratio is the ratio of the primary terminal effective voltage to the secondary terminal effective voltage. The current ratio is the ratio of the primary current (effective) to the secondary current (effective). That is:

$$(276) \qquad \text{Turn ratio} = \frac{N_{HV}}{N_{LV}}.$$

$$(276a) \qquad \text{Voltage ratio} = \frac{E_P}{E_S}.$$

$$(276b) \qquad \text{Current ratio} = \frac{I_P}{I_S}.$$

EXAMPLES illustrating the above ratio values are given in Fig. 517.

831. The Power, Current, Voltage and Power-factor Relations in Transformer Primary and Secondary Windings may be readily derived from the fact that the power input to any device must be always equal to its power output, plus the power losses in the device. Now properly designed transformers have very small losses—they are very efficient. Hence, for practical purposes, it can be assumed that the power input to the primary of a transformer equals the power output of its secondary. That is:

$$(277) \qquad P_P = P_S \qquad \text{(watts)}$$

Now, since the power of any alternating-current circuit is: $P = E \times I \times p.f.$, it follows that $P_P = E_P \times I_P \times p.f._P$ Also, $P_S = E_S \times I_S \times p.f._S$ Hence substituting these values in the above (277) equation:

$$(278) \qquad E_P \times I_P \times p.f._P = E_S \times I_S \times p.f._S$$

or,

$$(279) \qquad P_P = E_S \times I_S \times p.f._S$$

and

$$(279a) \qquad P_S = E_P \times I_P \times p.f._P$$

It can be shown (see note following) that $p.f._P = p.f._S$. Hence

$$(280) \qquad E_P \times I_P = E_S \times I_S$$

That is, *primary current* $\times$ *primary voltage* = *secondary current* $\times$ *secondary voltage*. The working formulas become:

$$E_P = \frac{E_S \times I_S}{I_P} \quad \text{(volts)} \tag{281}$$

$$I_P = \frac{E_S \times I_S}{E_P} \quad \text{(amp.)} \tag{281a}$$

$$E_S = \frac{E_P \times I_P}{I_S} \quad \text{(volts)} \tag{281b}$$

$$I_S = \frac{E_P \times I_P}{E_S} \quad \text{(amp.)} \tag{281c}$$

Wherein the symbols have the same meanings above given, except that I_S = current in secondary, in amperes, effective. I_P = current in primary, in amperes, effective. P_P = power input to primary, in watts. P_S = power output of secondary, in watts.

EXAMPLE.—The input to a transformer is 8 kw. The secondary e.m.f. is 220 volts and the power-factor is 80 per cent. What is the secondary current? SOLUTION.—From equation (279) above, $P_P = E_S \times I_S \times p.f._S$, hence, $I_S = P_P \div E_S \times p.f._S$. Then substituting in this equation, $I_S = 8{,}000 \div 220 \times 0.80 = 45.5$ *amp.* Hence, the current in the secondary in this transformer is 45.5 *amp.*

EXAMPLE.—The primary voltage of a certain transformer is 2,200. The primary current is 5 amp. and the secondary current is 100 amp. What is the secondary voltage? SOLUTION.—From equation (281*b*) above, $E_S = E_P \times I_P \div I_S = 2{,}200 \times 5 \div 100 = 110$ *volts.*

NOTE.—THE PROOF THAT THE POWER-FACTOR OF THE PRIMARY CIRCUIT EQUALS THE POWER-FACTOR OF THE SECONDARY CIRCUIT is this: The magnetizing effect of the primary ampere-turns may (if the primary magnetizing current which is relatively very small, be disregarded) be taken as equal to that of secondary ampere-turns. That is: $I_P \times N_P = I_S \times N_S$, or:

$$\frac{N_P}{N_S} = \frac{I_S}{I_P} \tag{282}$$

But from equation (274) above, $N_P \div N_S = E_P \div E_S$, that is:

$$\frac{N_P}{N_S} = \frac{E_P}{E_S} \tag{283}$$

hence

$$\frac{I_S}{I_P} = \frac{E_P}{E_S} \tag{284}$$

or

$$I_S \times E_S = I_P \times E_P \tag{285}$$

Then, dividing equation (278) above by (285), thus:

$$\frac{E_p \times I_p \times p.f._p}{I_p \times E_p} = \frac{E_s \times I_s \times p.f._s}{I_s \times E_s} \tag{286}$$

(287) the result is $p.f._P = p.f._S$ (power-factor)

Which means that the power-factor of the primary circuit of a transformer equals the power-factor of the secondary circuit—if the magnetization current and losses in the transformer be neglected. These can be neglected in practice with little error.

832, 833. The Current in the Primary and Secondary Circuits of a Transformer Varies Inversely as the Voltage of the Winding to Which It is Connected and Inversely as the Number of Turns in the Winding.—This statement logically follows from the previous discussion; in fact it is merely a statement in words of the meanings of equations (280) and (282).

834. The Counter E.m.f. of a Transformer may be computed (see formula, 119, Art. 739) from the equation:

$$(287a)\quad E_{PC} = \frac{4.44 \times f \times \phi \times N}{100{,}000{,}000} = \frac{4.44 \times f \times A \times B_M \times N}{100{,}000{,}000}$$

Wherein, E_{PC} = counter e.m.f. of transformer winding in volts. f = the frequency of the circuit in cycles per second. ϕ = the total flux developed in the core. N = the number of turns in the winding. A = the area of the core in square inches. B_M = the maximum flux density in lines per square inch, that is, the flux density at the instant when the exciting current is a maximum. By using one or the other of the above formulas, and assuming that the counter e.m.f. E_{PC} = the impressed e.m.f. E_P, which it does for all practical purposes, the principal dimensions of a transformer may be determined. The above formulas are very important inasmuch as they are used repeatedly in connection, not alone with transformers, but also with induction and synchronous motors and other alternating-current apparatus.

EXAMPLE.—How many turns should be used in the primary winding of a transformer upon which an e.m.f. of 140 volts is to be impressed at a frequency of 60 cycles? The core upon which the winding is to be served has a sectional area of 1⅝ sq. in., and a maximum permissible flux density of 35,000 lines is assumed. SOLUTION.—Substitute in the equation which has been solved for N:

$$(282)\quad N = \frac{100{,}000{,}000 \times E_{PC}}{4.44 \times f \times A \times B_M} = \frac{100{,}000{,}000 \times 140}{4.44 \times 60 \times 1.625 \times 35{,}000} = 924.$$

Hence the primary winding of this transformer should have 924 turns.

NOTE.—IN DESIGNING A TRANSFORMER the flux density is taken at a value which experience shows will involve iron losses that are as small as is consistent. Usually B_M is taken as 30,000 to 40,000 lines per sq. in. of core area. The number of primary turns is selected upon the basis of what experience has shown desirable and may be computed from an empirical

formula.* Then by substituting these assumed values from B_M and N in the formula, the area, A, of the core required can be computed. To design transformers most effectively, that is, so that they will have a maximum efficiency with a minimum expenditure of material is a specialized art and requires much experience and skill for its successful accomplishment.

835. Magnetic Leakage actually occurs in a transformer, although it has been disregarded in the foregoing discussion because in a well-designed transformer its effects are negligible from a practical standpoint. When there is no load on the secondary of a transformer (Fig. 518) no current flows in the secondary winding, and there is then nothing to oppose the creation of a flux in the core. Hence, at no-load, there is very little leakage. When, however, a load is connected across the secondary circuit and a current flows in the secondary winding, a counter flux ϕ_{SL} (Fig. 518) is created by its secondary current. This counter flux opposes the flux produced by the primary (Lenz's law) as suggested by the arrows in Fig. 518. There is then a tendency toward the creation of magnetic poles (N and S, as shown at one particular instant) at the top and bottom of the core. Under these conditions, a proportion of the flux produced by the primary winding is prevented from following a circuit around through the core. The result is that this primary leakage flux ϕ_{PL} does not cut the secondary turns and the secondary is reduced accordingly. Obviously the primary leakage flux will increase with the load on the secondary which tends to cause a decrease in the secondary voltage as the load increases.

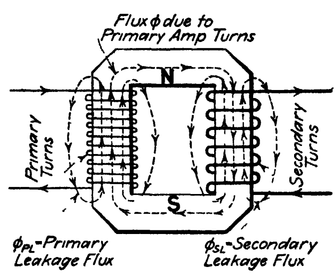


FIG. 518.—Illustrating magnetic leakage in a voltage transformer.

836. Magnetic Leakage May Be Largely Minimized by so disposing the primary and secondary windings that all or most of the flux which is produced by the primary must cut the secondary turns. In practice this is accomplished by winding the primary and secondary coils over one another as suggested in Figs. 519, and 520, or by winding them in sections and interleaving.

837. Core-type and Shell-type Transformers are indicated diagrammatically in Fig. 521. Those of the core type are characterized by a long average length of magnetic circuit and

* See Pender, AMERICAN HANDBOOK FOR ELECTRICAL ENGINEERS, p. 1618.

a short average length of winding. Those of the shell type have a short average length of magnetic circuit and a long average length of winding. The result of this condition is that for a given output and performance a core-type transformer will have a smaller area of core and a larger number of turns than will a corresponding one of the shell type. Either type may be so designed as to be economical, but as a rule the core-type construction is more economical for high-voltage transformers than is the shell-type.

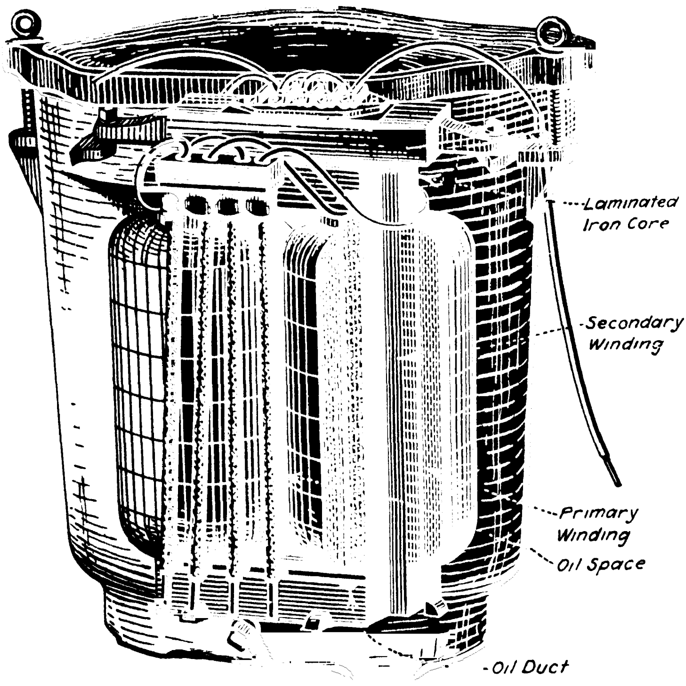


FIG. 519.—Phantom view of a typical distributing transformer assembled.

838. The Losses in the Cores of Transformers are of two kinds: (1) *hysteresis losses;* and (2) *eddy-current losses.* These taken together are called the iron loss, since they both occur in the iron core of the device. The hysteresis loss is due to the fact that the flux in a core is constantly alternating and power is required to effect reversals of polarity in the iron. The eddy-current losses are really $I^2 \times R$ losses in the iron, due to the currents induced in the material by the alternating flux. To

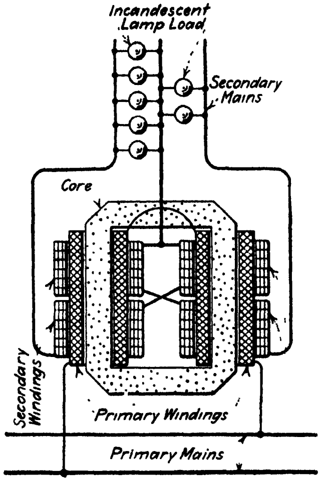


FIG. 520.—Illustrating typical arrangement of transformer primary and secondary coils. (Transformer secondary is feeding a three-wire system.)

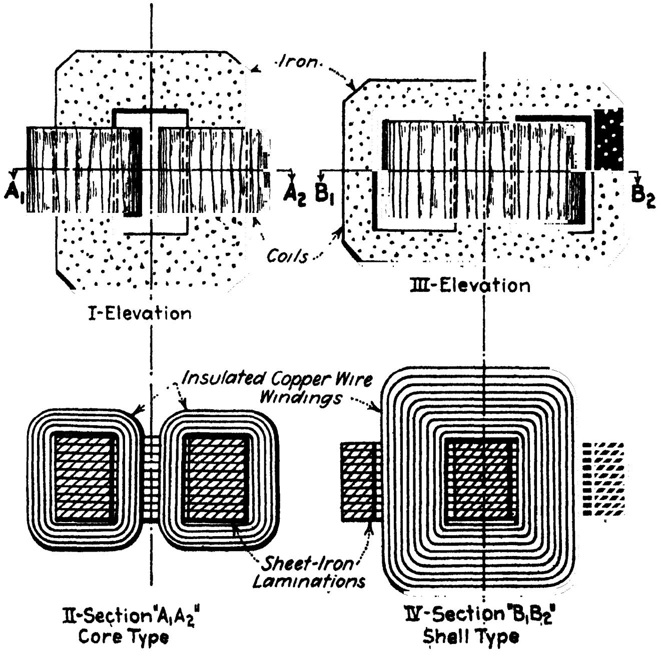


FIG. 521.—Illustrating "core-type" and "shell-type" transformer construction.

minimize the intensities of these eddy currents and consequently the loss due to them, the cores are laminated. Since the alternating flux, ϕ, in the core produces both of the iron losses, and since this flux is practically constant at all loads, it follows that the iron losses are practically constant at all loads, that is, they occur whether or not the transformer is loaded.

839. To Determine the Iron Losses in a Transformer, the power input to the primary, at normal voltage and frequency, with the secondary open, can be measured with a wattmeter. The reading will be the power loss in watts inasmuch as the $I^2 \times R$ copper loss in the primary winding itself will be negligible with only the exciting current flowing in it.

840. The Copper Loss in a Transformer equals the sum of the $I_1 \times R$ losses which occur in the windings. Hence, the copper losses may be readily computed for any load by multiplying the square of the current for that load in the primary or the secondary, respectively, by the resistance of the primary or secondary and then adding together the two values thus obtained. That is, *copper loss, in watts* $= (I_p^2 \times R_p) + (I_s^2 \times R_s)$.

841. The Efficiencies of Transformers are very high, usually being greater than 95 per cent., and with transformers of large capacity greater than 98 per cent. The efficiency of a transformer may be expressed as a formula, thus:

$$(288)\quad Efficiency = \frac{output}{input} = \frac{output}{output + copper\ loss + iron\ loss}$$

EXAMPLE.—The output of a certain transformer is 50 kw. Its iron loss is 320 watts and its copper loss is 520 watts. What is its efficiency? SOLUTION.—Substitute in the equation,

$$efficiency = \frac{output}{output + copper\ loss + iron\ loss}$$

$$= \frac{50{,}000}{50{,}000 + 520 + 320} = \frac{50{,}000}{50{,}840} = 98.3\ per\ cent.$$

842. The Three-phase Transformer is ordinarily arranged substantially as shown in Fig. 522. The windings for all three of the phases are placed on the same core, which usually results in a material saving over the cost of three single-phase transformers of equivalent capacity. Each of the three components operates as if the others were not present. The windings may be connected to the external circuit either in delta or in Y.

843. The Cooling of Transformers is necessary to dissipate the heat developed due to the losses, above referred to, which occur

within them. Special provisions are not ordinarily necessary for the cooling of very small transformers, say those under 1 kva. capacity, because the relatively large surfaces which they expose to the surrounding atmosphere dissipates the heat with sufficient rapidity to prevent the windings of the transformer attaining a temperature which would be injurious. Larger transformers are usually placed within steel or cast-iron tanks, Fig. 523, filled with oil. The oil transfers the heat produced within the core and windings to the iron case which is usually corrugated, for capacities from 50 to 500 kva., or in larger sizes, from 500 to 2,000 kva., provided with radiating pipes, to increase their effective radiating surfaces. Not only does the oil provide a medium for the dissipation of heat, but it also provides additional insulation between parts of the transformer winding which operate at different potentials. The exposed surface of the

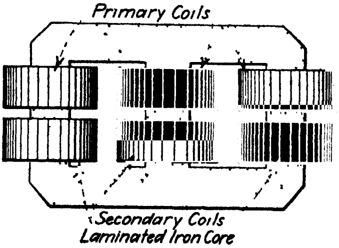


FIG. 522.—Diagram illustrating construction of a three-phase transformer.

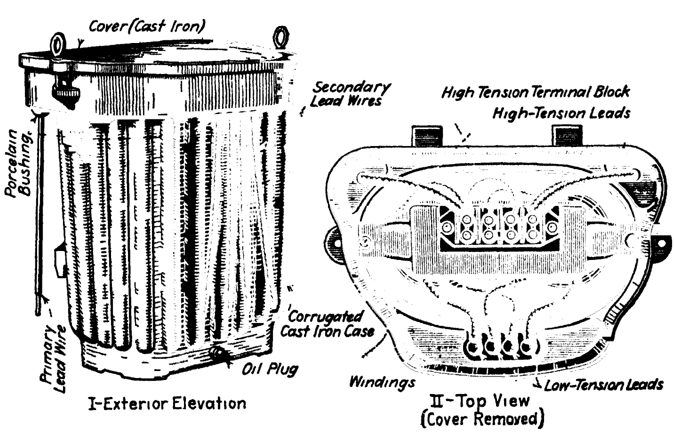


FIG. 523.—Illustrating typical distributing transformer construction.

case can be in many instances made ample and thus effectively dissipate all the heat produced. In very large transformers, capacities greater than 2,000 kva., cooling coils through which water is circulated are submerged in the oil within the tank.

Sometimes where real estate is expensive, and hence floor space must be economized, air-blast transformers are used for pressures less than 30,000 volts. These are cooled with a current of air, circulated past the core and windings by a blower.

844. The Current Transformer is different from the shunt or voltage transformer only in the method of its application. With the current transformer, Figs. 515,*II*, and 524, the impedance of the secondary circuit is ordinarily constant, hence, any change in load that occurs must be due to a simultaneous change of primary current and voltage. In designing a series transformer, little attention is concentrated on the ratio of the primary power input to the secondary power output. However, there should be a definite ratio between the primary amperes and the secondary amperes.

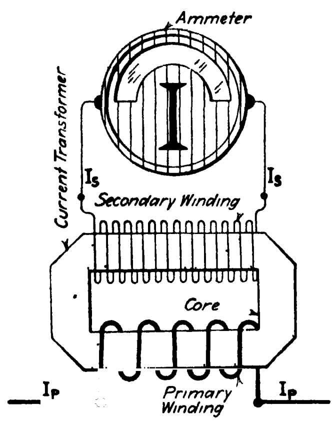


FIG. 524.—Current transformer serving an ammeter

845. The Important Applications of Current Transformers are for electrical instrument circuits, Fig. 524, and for series street lighting. Their function when used with instrument circuits is to produce through the instrument a current I_s (Fig. 524) proportional to, but of much less intensity than, the main current, I_p, which is being metered. Furthermore, where a current transformer is used, the high voltage of the main circuit is not impressed on the instrument. In practice, all instrument current transformers are usually so designed that when full-load current flows in the primary winding, a current of 5 amp. will flow in the secondary. The instruments which are used with them are so calibrated that their pointers will show a full-scale deflection with 5 amp. flowing through the instrument.

EXAMPLE.—A 2,400-amp. current transformer has a ratio of 480 to 1. That is, with a current of 2,400 amp. in its primary circuit, there will flow in the secondary circuit: 2,400 amp. ÷ 480 = 5 amp. The ammeter used with this transformer would have its scale so calibrated that when a current of 5 amp. flowed through it the pointer would indicate 2,400 amp.

846. Current Transformers for Series-incandescent-lighting Circuits are applied as suggested in Fig. 515,*II*, for providing

a current through the lamps of greater or lesser intensity than the constant current flowing in the main curcuit.

847. Distributing Transformers, so-called, are those for capacities up to possibly 200 kva. and primary pressures up to 17,500 volts, which are used for the distribution of energy at secondary low voltages—500 or lower—from high-voltage mains. Figs. 519 and 523 indicate typical construction for transformers of this

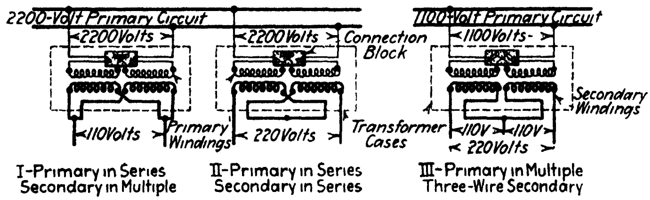


FIG. 525.—Showing standard arrangement of the primary and secondary windings of distributing transformers.

class. They are usually arranged (Fig. 525) with two primary and two secondary coils all on the same core. With this arrangement, a given transformer may, for example, with its primary windings in parallel, be used on a 1,100-volt primary circuit (Fig. 525,*III*) and with the coils in series on a 2,200-volt primary circuit (*I* and *II*). Furthermore, the secondary coils may be

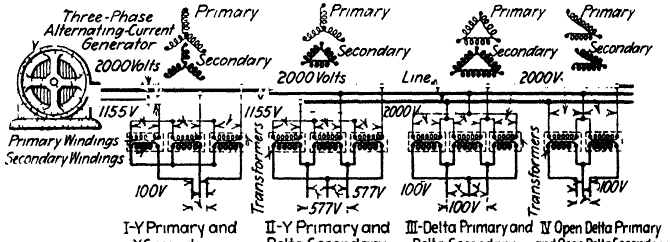


FIG. 526.—Connections of transformers on a three-phase line.

connected either in series or in parallel, providing a secondary e.m.f. of 220 or of 110 volts, or a three-wire e.m.f. of 110 tó 220 volts, see Fig. 525.

848. The Connecting of Transformers is a subject about which a book could easily be written; the possibilities are almost endless. In the author's AMERICAN ELECTRICIANS' HANDBOOK are shown a large number of the different combinations ordinarily

encountered. Fig. 525 shows the connections most frequently used for single-phase distributing transformers. In Fig. 526 are shown four methods of connecting single-phase transformers to three-phase mains, so that three-phase secondary e.m.fs. of different intensities may be obtained.

849. The Auto-transformer is shown diagrammatically in Fig. 527. If a winding, *AD*, on an iron core be connected across an alternating-current supply main of an e.m.f. E_1, any lower e.m.f., E_2 or E_3, may be obtained from the coil by tapping it as suggested. The e.m.f. E_2 will then be proportional to the number of turns between *C* and *D*. The e.m.f. E_3 will be proportional to the number of turns between *D* and *B*. It follows then that:

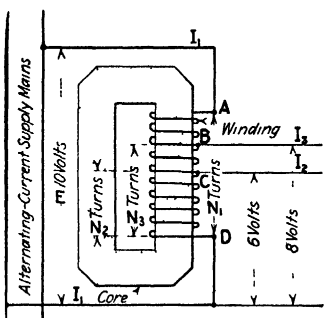


Fig. 527.—The auto transformer.

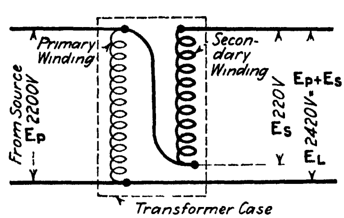


Fig. 528. —A transformer booster.

(289) $\qquad E_1$ *is to* E_2 *as* N_1 *is to* N_2, and *as* I_2 *is to* I_1.

Also:

(290) $\qquad E_1$ *is to* E_3 *as* N_1 *is to* N_3, and *as* I_3 *is to* I_1.

The auto-transformer is very efficient and is economical where the ratio of transformation is not great. The principles of its operation and its applications are more fully discussed in the author's American Electricians' Handbook.

Example.—The auto-transformer winding *AD* in Fig. 527 has 10 turns, and an e.m.f. of 100 volts impressed across it. If 6 of the turns, *CD*, are tapped, what would be the voltage across *CD*, and if a current I_2 of 20 amp. flows in the secondary, what will be the current in the primary circuit, I_1? Solution.—$E_2 : E_1 :: N_2 : N_1$; therefore, $E_2 = N_2 \times E_1 \div N_1 = 6 \times 100 \div 10 = 6.0$ *volts*. That is, the pressure across *CD* would be 6.0 volts. Also $I_1 : I_2 :: N_2 : N_1$. Then, $I_1 = N_2 \times I_2 \div N_1 = 6 \times 20 \div 10 = 12$ *amp*. That is, the current in the primary circuit will be 12 amp.

850. A Booster Transformer is diagrammed in Fig. 528. If a transformer be arranged in a circuit with its primary winding connected across a constant voltage of the circuit E_P and its secondary winding in series with the circuit, the voltage impressed on the load will be increased by the amount of the e.m.f. E_S induced in the secondary winding. That is, the voltage impressed on the load $E_L = E_P + E_S$.

EXAMPLE —If a transformer having a primary winding designed for 2,200 volts and a secondary winding designed to deliver 220 volts be connected in a 2,200-volt alternating-current circuit as a booster as shown in Fig. 528, what would be the e m f impressed on the load? SOLUTION —The secondary winding will develop 220 volts This added to the voltage impressed on the line = (2,200 + 220) = 2,420 *volts*, which will be the pressure on the load.

851. The Induction Regulator, Fig. 529, is really a booster transformer, so arranged that the e.m.f. induced in its secondary

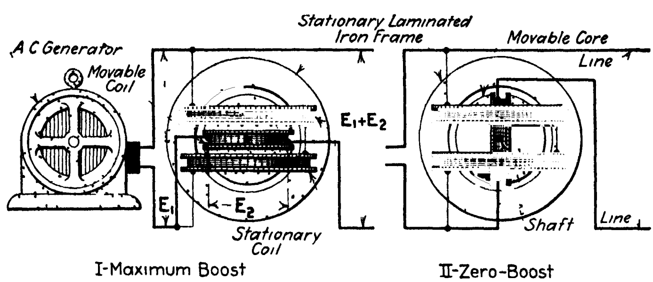


FIG. 529.—The induction regulator and its action.

winding may be varied at will within the range for which it is designed. The device is shown diagrammatically in Fig. 529, which indicates top views. The stationary windings are arranged on a hollow cylindrical laminated iron core. The movable winding is mounted on a laminated iron cylinder within the stationary windings. With the movable coil in the position shown at Fig. 529,*I*, the voltage induced in the secondary, E_2, is a maximum, because then practically all of the flux induced by the stationary coils passes through the movable coil. If, however, the movable coil be turned to position *II*, practically none of the flux produced by the stationary coils cuts the movable coil. Then the voltage induced by the movable coils is zero. Hence, with this device, E_2, the voltage added to the impressed voltage E_1, may be made any desired value between zero and a

maximum. Automatic induction regulators can be obtained wherein the movable core is turned by a small motor with a relay. The arrangement is so adjusted that when the line current increases, the "boosting" effect of the regulator is increased and when the line current decreases, its boosting effect is decreased. Thus, with a constant or varying e.m.f. impressed on the regulator, the e.m.f. impressed on the load can be maintained constant, even though the load varies through a wide range.

852. A Constant-current Regulating Transformer is a device for maintaining a current of constant intensity—6.6 amp. for example—in a secondary circuit. Constant-current transformers are utilized widely for series and incandescent arc lighting circuits. Fig. 530 indicates diagrammatically the construction.

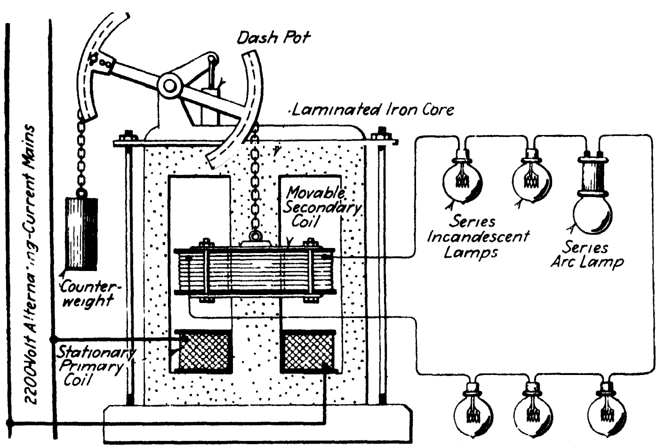


FIG. 530.—Constant-current regulating transformer.

EXPLANATION.—The primary coil is stationary and has impressed on it the constant voltage of the supply circuit. The secondary coil is free to move and is counterbalanced. A dashpot prevents sudden movement and jumping. With the secondary circuit open the coils lie close together, and practically all of the flux generated by the primary coil cuts the turns of the secondary coil. Now, if the secondary circuit be closed, current will flow through it and through the secondary coil. There will then be a repulsive action (Lenz's law, see index) between the two coils and the secondary will be forced away from the primary coil. As it is forced away the magnetic leakage between the coils increases. Fewer turns cut the secondary coil and it induces a smaller e.m.f. It will move away from the primary until it is at such a distance therefrom that its voltage is just sufficient to force the current, which the regulating transformer is designed to maintain,

through the secondary circuit. If lamps are cut out of the secondary circuit, the secondary current tends to increase so that the coils are forced further apart to another position at which the voltage induced in the secondary will be just sufficient to impel the normal secondary current. If lamps are connected into the secondary circuit, it tends to decrease the secondary current, thus permitting the coils to come closer together until they assume a new position in which the flux that cuts them will just be sufficient to induce a voltage that will force the normal current through the circuit.

853. The Principle of the Induction Furnace is shown in Fig. 531. The primary winding is arranged on an iron core as with any other transformer. The secondary winding in this case is merely a ring of the metal which is to be melted. The resistance of this metal ring is exceedingly low, hence a relatively enormous current will be impelled in it by the voltage induced. This current will heat the metal to fusion. By varying the impressed voltage, the intensity of the heat produced may be controlled.

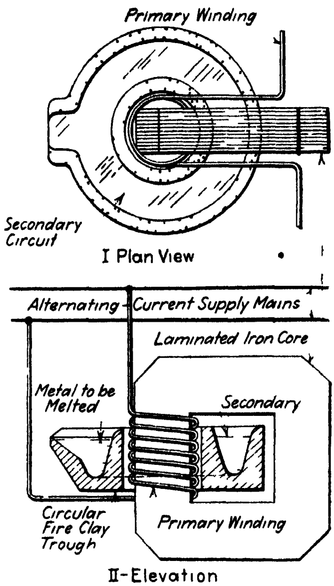


FIG. 531.—Illustrating the principle of the induction furnace.

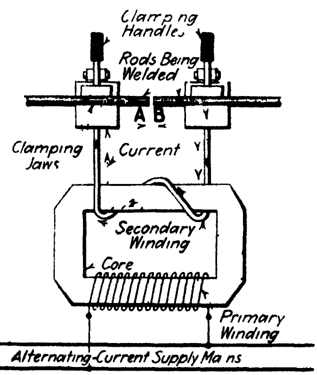


FIG. 532.—Principle of the electric welder.

854. The Electric Welder operates on the principle diagrammed in Fig. 532. The metal pieces to be welded together, *A* and *B*, are so clamped in jaws that they form a part of the path of the secondary circuit. When the two pieces are forced together with pressure in the directions indicated by the arrows, intense heat is developed at their junction. This fuses the metal to the welding temperature.

SECTION 53

THREE-WIRE DISTRIBUTION AND SYSTEMS

855. The Three-wire System Is Used Because It Saves Copper (see Figs. 533 and 536).—Incandescent lamps for 110 volts—more or less—are more economical and efficient than those for higher voltages. A circuit carrying any considerable load and operating at the low pressure of 110 volts would require very large conductors to maintain the $I \times R$ or line voltage and the $I^2 \times R$ power loss in the line within reasonable limits. With the three-wire system, a low voltage, say 110, is impressed on the receivers—incandescent lamps—while one twice as great, say 220, is used for transmission. Since the weight of the metal in

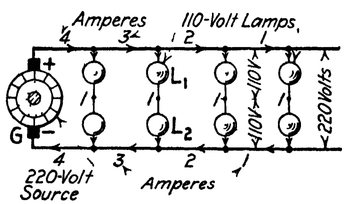


Fig. 533.—110-volt lamps arranged in multiple series across 220 volts.

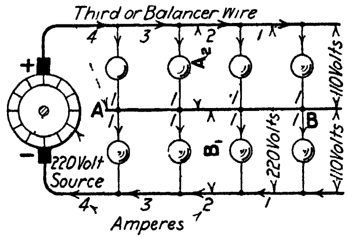


Fig. 534.—Balancing wire "AB" arranged between lamps.

conductors for a given power loss varies inversely as the square of the voltage, it is evident that a considerable saving in conductor material is possible with the three-wire system. In this country the three-wire system is of greatest importance as applied to 110 to 220-volt lighting systems.

856. The Principle of the Three-wire System is illustrated in Figs. 533 to 536. Incandescent lamps for 100 volts could be connected two in series across 220 volts as shown in Fig. 533, and while each lamp would operate at 110 volts, the energy of the group would be transmitted at 220 volts, and the outside conductors could, with equal loss, be but one-fourth the size that would be necessary if the energy were transmitted at 110 volts.

This arrangement (Fig. 533) while it would operate, is not commercially feasible because each lamp (of each pair of lamps in series) must be of the same size and if one lamp $-L_1$, for example—goes out, its partner, L_2, is also extinguished. These disadvantages might be partially corrected by installing a third or balancer wire as at Fig 534. Then one lamp might be turned off and the others would burn. Also, a single lamp might be added to either side of the system, between the third wire and either of the outside wires. But unless the total resistance of all of the lamps connected to one "side circuit," A_2, was practically equal to that of all of the lamps connected to the other "side circuit," B, the voltage across one side circuit would be higher than that across the other. On the side having impressed on it the higher voltage the lamps would burn brightly and on the

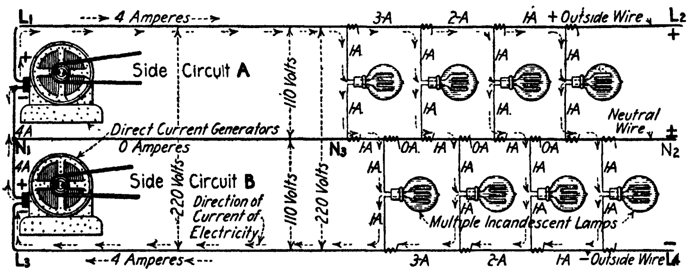


FIG. 535.—Three-wire, direct-current circuit. (Each lamp is supposed to have a resistance of 110 ohms so that it passes 1 amp. when 110 volts is impressed across it.)

side of the lower voltage they would burn dimly. Obviously, it is not feasible in practice to so arrange or "balance" the side circuits that they will have the same resistance. Hence, some other method must be used in practicable three-wire systems whereby the electricity will be transmitted at say 220 volts, and the pressure across the lamps will be, say, 110 volts.

857. Commerical Three-wire Systems consist (Figs. 535 and 536) of: (a) Two outer conductors ($L_1 - L_2$ and $L_3 - L_4$, Fig. 535) having—for lighting installations—a pressure of 220 volts impressed across them; and (b) a neutral wire ($N_1 - N_2$) so connected to sources of voltage that the pressure between it and either of the outside wires is 110 volts. In Fig. 535, direct-current generators are the sources of voltage. The neutral wire joins at the point, N_1 where the generators are connected to-

gether. When the system is perfectly balanced, the neutral wire carries no current and the system is then, in effect, a 220-volt system. Perfect balance seldom obtains in practice. When the balance is not perfect, the neutral wire conveys a current equal to the difference between the current taken by one side circuit and that taken by the other side circuit. Note from Fig. 536, that the current in different parts of the neutral wire may be of different intensities and that it is not necessarily in the same direction in all parts of the neutral wire. Each incandescent lamp in Fig. 536 is assumed to take 1 amp. and the small figures indicate the currents in different parts of the circuit.

NOTE.—A three-wire circuit may, in a sense be considered as a combination of two, two-wire circuits. Thus, the two circuits of Fig. 537 would transmit the electrical energy, to the eight incandescent lamps which they serve, as satisfactorily as does the equivalent three-wire circuit of Fig. 535. However, the Fig. 535 arrangement would, as will be shown, require much less copper in the line conductors for a given watts power loss, and a given percentage voltage drop than would that of Fig. 537. The small numerals indicate the current intensities, in amperes, in the different parts of the circuits in the two illustrations.

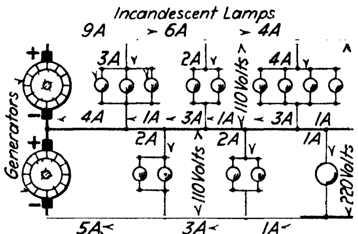


FIG. 536.—Showing the distribution of current in a three-wire system.

858. A Three-wire Circuit with the Neutral the Same Size as the Outers Requires Only Three-eighths the Copper Required for an Equivalent Two-wire Circuit providing the same percentage volts line drop and the same watts power loss. The reason for this may be derived from a consideration of the three-wire circuit of Fig. 535 and the equivalent two two-wire circuits of Fig. 537. It is evident since there is a current of 4 amp. to the *left* in section $L_7 - N_4$ (of Fig. 537) and a current of 4 amp. to the *right* in section $L_9 - N_5$ that there will be no current in section $N_1 - N_3$ of Fig. 535. This follows because the three-wire circuit is "balanced." With the arrangement of Fig. 535, one less wire is required than with that of Fig. 537. Now, the current in the outer line wires is 4 amp., both in the three-wire circuit of Fig. 535, and in the equivalent two two-wire circuits of Fig. 537.

However, in Fig. 535, the 4-amp. line current traverses only

half the length of line conductor that the 4-amp. currents in Fig. 537 traverse, because no—or practically no—current flows in the neutral wire. Hence the line wires in Fig. 535 may be one-half the size of those in Fig. 537, for the same percentage voltage drop. Therefore, since three-fourths the number of wires is required for Fig. 535 as for Fig. 537, and since each of

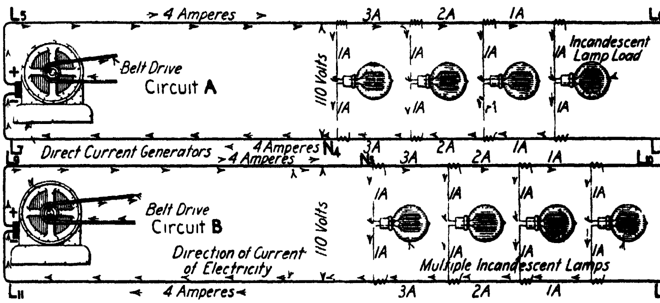


FIG. 537.—Two two-wire direct-current circuits.

these wires may be one-half the size of that necessary in Fig. 537, the amount of copper in Fig. 535 is: $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$ that necessary in Fig. 537 for the same percentage drop—it being assumed that the neutral is made the same size as the outer wires. The neutral may frequently be made smaller than the outer wires, as outlined later.

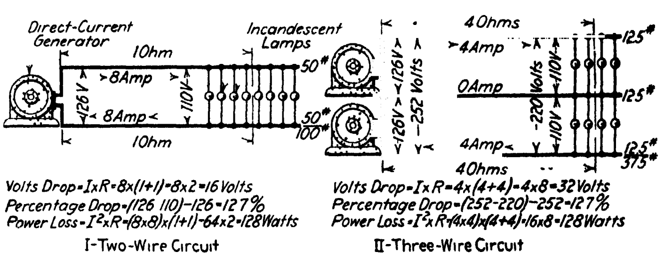


FIG. 538.—Showing a comparison of volts line drop and watts power loss in equivalent two-wire and three-wire circuits

EXAMPLE.—Consider the 110-volt two-wire circuit of Fig. 538,*I*, and the equivalent three-wire circuit of *II*. The transmission distance is the same for each of the two circuits. The load—eight incandescent lamps each taking 110 watts—is the same on *I* as on *II*. Referring to the two-wire circuit of *I*: Each of the line wires is assumed to have a resistance of 1 ohm. Since each lamp takes a current of 1 amp., the line current is 8 amp. Hence, as indicated, the line drop is 16 volts, the percentage line drop is 12.7 per

cent. and the line power loss is 128 watts. If it be assumed that each line wire weighs 50 lb., the total weight of the two line wires will be 100 lb.

If a 110 to 220-volt three-wire circuit is arranged to transmit the same power with the same loss, it will have the characteristics diagrammed in *II*. Since it is in effect a 220-volt circuit, the current will be half of that of the circuit of *I*. Since power loss in a conductor $= I^2R$, if the current is halved, a conductor of four times the resistance will carry it (the current) with the same power loss and with the same percentage voltage loss. Thus, for the same line power loss and the same percentage voltage drop, as in *I*, the outer wires of *II* will each have a resistance of 4 ohms.

There is no drop or loss in the neutral wire since—the load being balanced—it carries no current. Then as indicated, the line drop is 32 volts but the percentage volts drop is 12.7 per cent.—the same as in *I*. The power loss is 128 watts, the same as in *I*. Each outer wire of *II*, since it has four times the resistance of an outer of *I*, will have a weight of: $\frac{1}{4} \times 50 = 12.5$ lb. If the neutral is made the same size as the outers, the total weight will be:

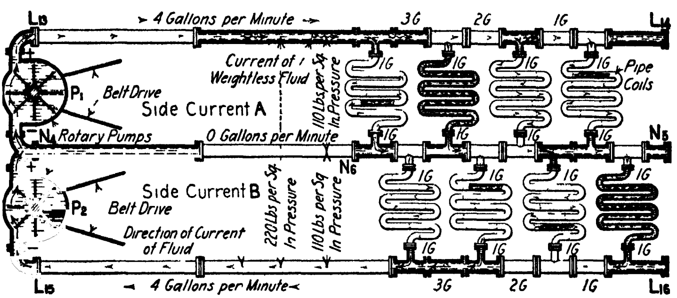


FIG. 539.—Hydraulic analogy to a three-wire, direct-current circuit.

$3 \times 12.5 = 37.5$ lb. Now 37.5 lb. is three-eighths of 100 lb., hence the three-wire circuit of *II* transmits the same power, the same distance with the same power loss and same percentage voltage drop as does the two-wire circuit of *II*—but with three-eighths the weight of conductor.

859. If the Neutral Wire Is Made Half the Size of the Outers, only 31.3 per cent. of the copper is required that would be necessary for a two-wire system transmitting the same power the same distance with the same power-line loss and percentage voltage drop. If the neutral is made one-third the size of the outers, the relative weight of copper is then 29.2 per cent.

860. A Hydraulic Analogy of a Three-wire Circuit is shown in Fig. 539. The two rotary pumps, connected in series, are analogous to two direct-current generators connected in series for three-wire service. The pipe lines L_{13}, L_{14} and L_{15}, L_{16} correspond to the outer wires of a three-wire circuit. The pipe line N_4, N_5 corresponds to the neutral wire in a three-wire circuit. The

current of weightless incompressible fluid in the pipe circuit—analogous to a current of electricity—will be impelled as shown by the arrows. The circulation is due to the hydraulic pressure (voltage) developed by the pumps. No fluid flows in the neutral between N_4 and N_6 because, with the circuit balanced as shown, there is no difference in hydraulic pressure between these two points. The small numerals with the letters "G" following them, indicate the gallons per minute—analogous to amperes—flowing in the different parts of the circuit.

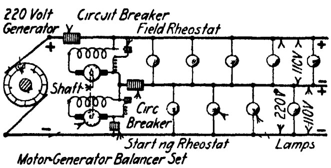


FIG. 540.—A 220-volt generator and a motor-generator balancer.

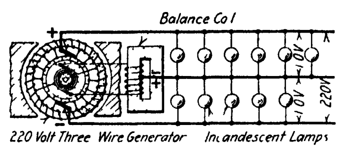


FIG. 541 —Three-wire generator.

861. Where the Balance Is and Always Will Be Perfect, No Neutral Wire Is Necessary.—In out-of-door distribution systems the neutral is often made one-half the size of the outer wires. For interior wiring, the neutral is usually made the same size as the outers, where the outers are of No. 6 A.W.G. or smaller wire. Where the outers are larger than No. 6, the neutral is usually selected so as to have about two-thirds the sectional area of one of the outers. See the author's AMERICAN ELECTRICIANS' HANDBOOK.

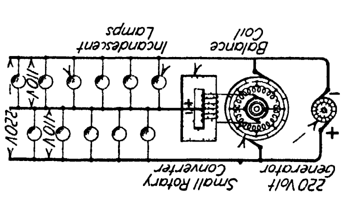


FIG. 542.—Small rotary converter used as a balancer.

862. The Amount of Unbalance That May Come on a Three-wire Circuit depends on local conditions. In ordinary three-wire lighting systems the unbalanced load seldom exceeds 10 per cent. of the total load. Probably 5 per cent. is a fair average for a well-laid-out system. Balancer sets for interior three-wire systems are frequently specified of sufficient capacity to take care of a 10 per cent. unbalance. Sometimes the unbalance on a poorly laid out system may be 20, 30 per cent., or even more.

863. The Methods of Obtaining Three-wire Voltages will now be discussed. That suggested in Fig. 535, while excellent for certain conditions, is not economical for small installations because the operation of two generators is necessary. Its first cost is more and its efficiency is less than with an arrangement requir-

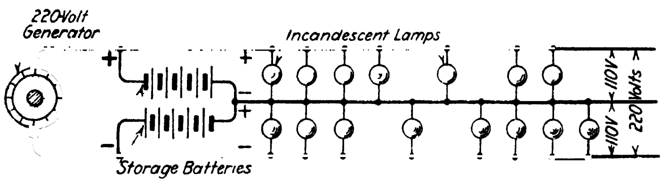


Fig. 543.—A 220-volt generator in combination with two storage batteries for obtaining three-wire voltages.

ing the use of but one generator. The advantage of a two-generator installation (Fig. 535) is that any percentage of unbalance up to the capacity of one generator can be handled by it. The important methods of obtaining three-wire voltages are listed in the following table:

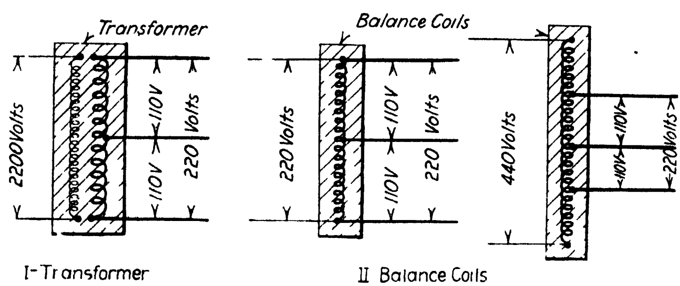


Fig. 544.—Methods of obtaining three-wire voltages with transformers and balance coils.

864. Arrangements for Deriving Three-wire Voltages:—

Direct current	1. A 220-volt generator in combination with a motor-generator balancer set, Fig. 540. 2. A three-wire or Dobrowolsky generator, Fig. 541. 3. A 220-volt generator in combination with a small rotary converter, Fig. 542. 4. A 220-volt generator in combination with storage-battery balancers, Fig. 543.
Alternating current	1. With a transformer, Fig. 544, *I*. 2. With an auto-transformer or balance coil, Fig. 544, *II*.

865. If the Neutral Wire Opens in a Three-wire Circuit, the lamps (Fig. 545) on the side, *A* having the most lamps connected to it, will burn dimly or at under voltage, while the lamps connected to the side having the least number of lamps, *B*, will burn brighter than normal or at over-voltage. Any device, a motor, *C*, for instance, connected to the two outside wires will operate normally.

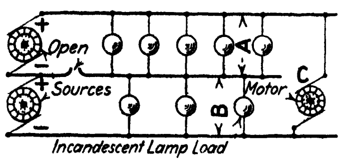


FIG. 545.—Open neutral in a three-wire circuit.

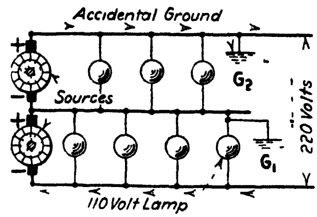


FIG. 546.—Ground on three-wire system outer wire.

866. If One Side of a Lamp on a Three-wire System Becomes Grounded, G_1, and there is another ground, G_2, on the opposite side of the system (Fig. 546) 220 volts will be impressed across the lamp and it will be burned out.

867. If One of the Generators of a Three-wire System Becomes Reversed, as at G_1 (Fig. 547), the lamps connected between both of the outside wires and the neutral will receive nor-

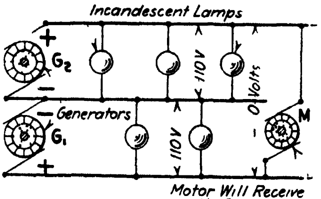


FIG. 547.—Reversal of generators.

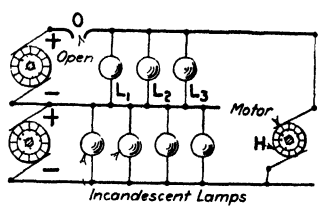


FIG. 548.—Open in the outer of a three-wire system.

mal voltage but the polarity will be reversed. Any receiver connected across the outside wires, *M* for example, will have no difference of potential across it and hence, no current will be forced through it—it will not operate.

868. If One of the Outside Wires of a Three-wire System Opens, *O* (Fig. 548), a device, *H*, connected across the two outside wires on the distant side of the break will receive some cur-

rent at 110 volts through the lamps, L_1, L_2, and L_3, connected to the broken side between the device and the break. These lamps will burn dimly when the device is connected, but the device will not receive enough current to operate it properly.

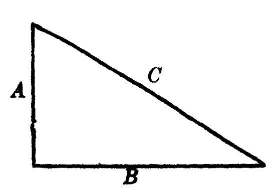
869. The General Arrangement of Three-wire Feeders and Mains may be substantially the same as that for two-wire systems. The important thing is to so balance the circuits as to maintain the currents in the two outside wires approximately equal. The lamps and other devices should be divided between the two side circuits of the system so that the loads will be, as nearly as possible, balanced when the circuit is loaded to its full capacity or when loaded to any fraction of it. All three wires—two outers and the neutral—should be routed to every location where any considerable amount of power is required; to every building in an out-of-door distribution and to every distribution center in distributions within buildings. Where many lamps are to be lighted simultaneously the group should be controlled by three-pole switches to prevent unbalancing. Not only must loads on three-wire systems be balanced on the two side circuits, but the loads on the side circuits must also be distributed along both the circuits in approximately the same manner. If they are not, there may be considerable "extra" drop. This "local unbalancing" is one of the principal causes of the excessive voltage variation that may sometimes occur on three-wire systems.

INDEX

F

TRIGONOMETRIC FUNCTIONS

Angle ϕ or lag angle	Sin or induction factor	Cos or power factor	Tan	Angle ϕ or lag angle	Sin or induction factor	Cos or power factor	Tan
0	000	1 000	000				
1	017	999	017	46	719	695	1 04
2	035	999	035	47	731	682	1 07
3	052	999	052	48	743	669	1 11
4	070	998	070	49	755	656	1 15
5	087	996	087	50	766	643	1 19
6	105	995	105	51	777	629	1 23
7	122	993	123	52	788	616	1 28
8	139	990	141	53	799	602	1 33
9	156	988	158	54	809	588	1 38
10	174	985	176	55	819	574	1 43
11	191	982	194	56	829	559	1 48
12	208	978	213	57	839	545	1 54
13	225	974	231	58	848	530	1 60
14	242	970	249	59	857	515	1 66
15	259	966	268	60	866	500	1 73
16	276	961	287	61	875	485	1 80
17	292	956	306	62	883	469	1 88
18	309	951	325	63	891	454	1 96
19	326	946	344	64	898	438	2 05
20	342	940	364	65	906	423	2 14
21	358	934	384	66	914	407	2 25
22	375	927	404	67	921	391	2 36
23	391	921	424	68	927	375	2 48
24	407	914	445	69	934	358	2 61
25	423	906	466	70	940	342	2 75
26	438	898	488	71	946	326	2 90
27	454	891	510	72	951	309	3 08
28	469	883	532	73	956	292	3 27
29	485	875	554	74	961	276	3 49
30	500	866	577	75	966	259	3 73
31	515	857	601	76	970	242	4 01
32	530	848	625	77	974	225	4 33
33	545	839	649	78	978	208	4 70
34	559	829	675	79	982	191	5 14
35	574	819	700	80	985	174	5 67
36	588	809	727	81	988	156	6 31
37	602	799	754	82	990	139	7 12
38	616	788	781	83	993	122	8 14
39	629	777	810	84	995	105	9 51
40	643	766	839	85	996	087	11 43
41	656	755	869	86	998	070	14 30
42	669	743	900	87	999	052	19 08
43	682	731	933	88	999	035	28 64
44	695	719	966	89	999	017	57 28
45	707	707	1 000	90	1 000	000	Infinity



$\sin\phi = \dfrac{A}{C}$ $\quad\cot\phi = \dfrac{B}{A}$

$\cos\phi = \dfrac{B}{C}$ $\quad\sec\phi = \dfrac{C}{B}$

$\tan\phi = \dfrac{A}{B}$ $\quad\operatorname{cosec}\phi = \dfrac{C}{A}$